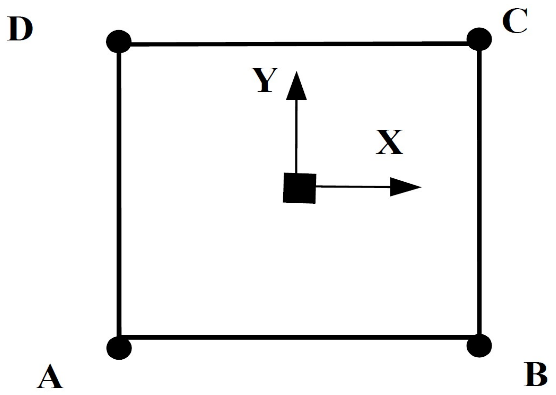
SSLL117 – Validation of the modelizations second Summarized

gradient:

This test makes it possible to validate the modelizations second gradient [R5.04.03] by leaning on analytical solutions.

1 Problem of reference

1.1 Geometry



Appears 1.1-a

Coordinated items	X	Y
A	-1.0	-1.0
B	1.0	-1.0
C	1.0.1.0	
D	-1.0	1.0

the geometry of structure is a square length of with dimensions 1m .

The test is carried out on only one isoparametric finite element of quadratic form, named mesh group ROCHE . The various sides of this square, useful for this modelization, are mesh groups, DA and BC . The mesh group ROCHE , contains the mesh groups in addition SROCHE and MROCHE ; who correspond respectively to the nodes tops and mediums of this same group.

1.2 Properties of the materials

In the context of the mediums of the second gradient, it is necessary to define materials properties for the parts attached to the first and second gradients of the field of displacement. The properties of the material are elastic for these two parts:

- Young modulus: $\text{E}=1000\,\text{Pa}$;
- Poisson's ratio: $\nu=0$;
- Modulate microscopic stiffness: $a_1=10\,\text{Pa.m}^2$.
- Parameter of penalization: $r=1\text{E}8$
- Material parameters of the mediums of the second gradient: $a_2=a_3=a_4=a_5=0\,\text{Pa.m}^2$

1.3 Boundary conditions and loadings

- $Dy=0$ (The modelization brings back itself in the case) 1D on
- $Dx=0$ the segment on BC
- $Dx=0.1\text{m}$ the segment Reference solution AD

2 Method of calculating

2.1

the problem to be solved everywhere is:

$$\int_{\Omega} \sigma\left(\frac{\partial u^{\rightarrow}}{\partial x}\right)+\sum\left(\frac{\partial v^{\rightarrow}}{\partial x}\right)-\lambda\left(\frac{\partial u^{\rightarrow}}{\partial x}-v^{\rightarrow}\right)+\lambda^{\rightarrow}\left(\frac{\partial u}{\partial x}-v\right)+r\left(\frac{\partial u}{\partial x}-v\right)\left(\frac{\partial u^{\rightarrow}}{\partial x}-v^{\rightarrow}\right)=0$$

with $\forall u^{*}, v^{*}, \lambda^{*}$ kinematically admissible.

We have:

$$\begin{cases} \sigma = E\dfrac{\partial u}{\partial x} \\ \Sigma = F\dfrac{\partial v}{\partial x} \end{cases}$$

And the ω let us note the shape functions of the second order N first order shape functions on the linear elements,

Recall by the same occasion the definitions of `SEG2` and `SEG3`:

`SEG2` : segment with 2 nodes
many nodes: 2
many nodes tops: 2

Figure 2.1-a

`SEG3` : segment with 3 nodes
many nodes: 3
many nodes tops: 2

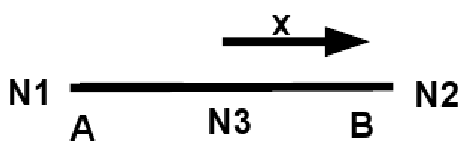


Figure 2.1-b

$N_1=\dfrac{1-x}{2}$ and $N_2=\dfrac{1+x}{2}$ on the element of reference $x\in[-1,+1]$ with N_1 the shape function to the first node.

The shape functions of the segment to the 3 nodes are then:

$\omega_1=-\dfrac{x(1-x)}{2}$ $\omega_2=\dfrac{x(1+x)}{2}$ and $\omega_3=1-x^2$

While posing $u^{*}=v^{*}=0$ we show that:

$\int_{-1}^{+1} \lambda^* \left(\frac{\partial u}{\partial x} - \nu \right) dx = 0$ however $u = \omega_1 . u_1 + \omega_2 . u_2 + \omega_3 . u_3$ with $u_2 = 0$ (C.L.)

from where $\frac{\partial u}{\partial x} = (x - \frac{1}{2}) u_1 - 2 x u_3$ and $\nu = N_1 . \nu_1 + N_2 . \nu_2$ with $\nu_2 = 0$ (C.L.)

We thus have $\nu = \frac{(1-x)}{2} \nu_1$ but, it should be noted after integration that: $\boxed{\nu_1 = -u_1}$ (1)
$\forall u^*, \nu^*, \lambda^*$ kinematically admissible.

While proceeding in the same way and while posing: $\nu^* = \lambda^* = 0$ we find that:

$$\boxed{u_3 = \frac{2(E-r)}{4(E+r)} . u_1 \approx -\frac{u_1}{4}} \quad (2)$$

from where the general formula $S_1 = 3.\mathrm{a}^1 . \frac{\partial \nu}{\partial x} = -\frac{3}{2} . a^1 . \nu_1$ and after simplification $S_1 = \frac{3}{2} . a^1 . u_1$.

2.2 Quantities and results of reference

the fundamental and very general formula for solid is $S_1 = \frac{3}{2} . a^1 . u_1$.

2.3 Uncertainties on the analytical

solution Solution.

3 Modelization A

3.1 Characteristic of the modelization

the characteristics are identical to the reference solution. The modelization bearing on the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of thermal expansion is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

3.2 Characteristics of the mesh

the mesh group ROCHE_REG is obtained by duplication of the named mesh group ROCHE , whose purpose is to accommodate the modelization second gradient of thermal expansion for the regularization.

Number of nœuds8
Many SEG3 4
Many QUAD82
Number of group of mailles6

3.3 Quantities tested and Value

results tested	Urgent	Node	Reference	Criterion	Aster	Tolerance
Displacement GONF	1.0	$N1$	"NON_REGRESSION"	RELATIF	-0,10	0.010
Displacement DX	1.0	$N5$	"NON_REGRESSION"	RELATIF	0,0250	0.010
Reaction force SIG1	1.0		"NON_REGRESSION"	RELATIF	1,5	0.010
Reaction force DEPV	1.0		"NON_REGRESSION"	ABSOLU	1.E-6	1.0E-04

3.4 Remarks

With an aim of validating the option of resolution RIGI_MECA_ELAS of STAT_NON_LINE , a computation identical to the first (option MATRICE=TANGENTE) is carried out by imposing this option of resolution. The results are rigorously identical to those obtained with the first computation option which are presented in the table of results above.

4 Modelization B

4.1 Characteristic of the modelization

In this case, the materials parameters resulting ones from the commands of the tests from non regression generated with the source code are identical in the two modelizations A and B, only differs the quadratic mesh. The purpose of this modelization is to have a validation on a structure with a grid, the analytical solution being difficult to obtain and consequently the solution in NON-regression, is privileged here.
The modelization bearing on the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of thermal expansion is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

4.2 Characteristics of the mesh

the mesh group ROCHE_REG is obtained by duplication of the named mesh group ROCHE , whose purpose is to accommodate the modelization second gradient of thermal expansion for the regularization.

Number of nœuds341
Many SEG340
Many QUAD8200
Number of group of mailles6

4.3 Quantities tested and Value

results tested	Urgent	Standard	Node	Criterion	Aster	Tolerance
Displacement GONF	1.0	$N1$	“NON-REGRESSION”	RELATIF	-0.05500	1.0
Displacement DX	1.0	$N5$	“NON-REGRESSION”	RELATIF	0.09450	1.0
Reaction force DEPV	1.0		“NON-REGRESSION”	ABSOLU	1.E-6	1.0E-04

5 Modelization C

5.1 Characteristic of the modelization

This modelization is identical to modelization A. The modelization bearing on the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of thermal expansion is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

5.2 Characteristics of the mesh

the mesh group ROCHE_REG is obtained by duplication of the named mesh group ROCHE , whose purpose is to accommodate the modelization second gradient of thermal expansion for the regularization.

Number of nœuds9
Many SEG34
Many TRIA64
Number of group of mailles6

5.3 Quantities tested and Value

results tested	Urgent	Standard	Node	Criterion	Aster	Tolerance
Displacement GONF	1.0	*N1*	"ANALYTIQUE"	RELATIF	-0.10	0.010
Displacement DX	1.0	*N5*	"ANALYTIQUE"	RELATIF	0.0250	0.010
Reaction force SIG1	1.0		"ANALYTIQUE"	RELATIF	1.5	0.010
Reaction force DEPV	1.0		"ANALYTIQUE"	ABSOLU	1.E-6	1.0E-04

6 Modelization D

6.1 Characteristic of the modelization

The modelization is identical to the modelization B with a mapping composed of triangles. The modelization bearing on the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of thermal expansion is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

6.2 Characteristics of the mesh

the mesh group ROCHE_REG is obtained by duplication of the named mesh group ROCHE, whose purpose is to accommodate the modelization second gradient of thermal expansion for the regularization.

Number of nœuds441
Many SEG340
Many TRIA6400
Number of group of mailles6

6.3 Quantities tested and Value

results tested	**Urgent**	**Stand ard**	**Node**	**Criterion**	**Aster**	**Tolerance**
Displacement GONF	1.0	N1	"NON-REGRESSION"	RELATIF	-5.5E-2	1.0
Displacement DX	1.0	N5	"NON-REGRESSION"	RELATIF	0.0945	1.0
Reaction force DEPV	1.0		"NON-REGRESSION"	ABSOLU	1.E-6	1.0E-04

7 Modelization E

7.1 Characteristic of the modelization

The modelization bearing on the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of thermal expansion is D_PLAN_DIL with the choice of interpolation P2-P1-P0.
The case present is identical to the modelization A with $r=0$ we have then

$$\begin{cases} v_1=-u_1 \\ u_3=\dfrac{2(E)}{4(E)}.u_1\approx\dfrac{u_1}{2} \\ S_1=\dfrac{3}{2}.a^1.u_1 \end{cases}$$

7.2 Characteristic of the mesh

the mesh group ROCHE_REG is obtained by duplication of the named mesh group ROCHE, whose purpose is to accommodate the modelization second gradient of thermal expansion for the regularization.

Number of nœuds8
Many SEG34
Many QUAD82
Number of group of mailles6

7.3 Quantities tested and Value

results tested	Urgent	Standard	Node	Criterion	Aster	Tolerance
Displacement GONF	1.0	*N1*	"ANALYTIQUE"	RELATIF	-0.10	0.010
Displacement DX	1.0	*N5*	"ANALYTIQUE"	RELATIF	0.0550	0.010
Reaction force SIG1	1.0		"ANALYTIQUE"	RELATIF	1.5	0.010
Reaction force DEPV	1.0		"ANALYTIQUE"	RELATIF	0.0288675	0.010

8 Modelization F

8.1 Characteristic of the modelization

the case present is identical to the modelization A with a generalization of the behavior second complete gradient [R5.04.03]. The modelization bearing on the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of thermal expansion is D_PLAN_2DG with the choice of interpolation P2-P1-P0.

We show as for the modelization E that:

$$u_1=-v_1 \text{ in the same way } u_3=\frac{u_1}{2}$$

For computation of Σ_{111} and Σ_{221} for the definition of the field of double stresses Σ

we thus have:

$$\Sigma_{111}=a^{12345}\chi_{111}+a^{23}\chi_{122}+a^{12}(\chi_{212}+\chi_{221})$$

with:

$$\begin{cases} a^{12345}=2(a^1+a^2+a^3+a^4+a^5) \\ a^{23}=a^2+2a^3 \\ a^{12}=a^1+\frac{a^2}{2} \end{cases}$$

Being given the values of the material parameters and limiting conditions we find:

$$\Sigma_{111}=a^1.u_1$$

in the same way we end to:

$$\Sigma_{221}=\frac{a^1}{2}.u_1$$

8.2 Characteristics of the mesh

the mesh group ROCHE_REG is obtained by duplication of the named mesh group ROCHE, whose purpose is to accommodate the modelization second complete gradient for the regularization.

Number of nœuds8
Many SEG34
Many QUAD82
Number of group of mailles6

8.3 Quantities tested and Value

results tested	Urgent	Stand ard	Node	Criterion	Aster	Tolerance
Displacement V11	1.0	N1	"ANALYTIQUE"	RELATIF	-0.10	0.010
Displacement DX	1.0	N5	"ANALYTIQUE"	RELATIF	0,050	0.010
Reaction force SIG111	1.0		"ANALYTIQUE"	RELATIF	1	0.010
Reaction force SIG221	1.0		"ANALYTIQUE"	RELATIF	0.5	0.010

Reaction force DEPV11	1.0		"ANALYTIQUE"	RELATIF	0.0288675	0.010

9 Modelization G

9.1 Characteristic of the modelization

The modelization G. is identical to the modelization A but with an interpolation without Lagrange multipliers (P2-P1) – to see R5.04.03.

9.2 Characteristics of the mesh

the formula ROCHE_REG is obtained by duplication of the mesh group named ROCHE , whose purpose is to accommodate the modelization second gradient of thermal expansion for the regularization.

Number of nœuds8
Many SEG3 4
Many QUAD82
Number of group of mailles6

9.3 Quantities tested and Value

results tested	Urgent	Standard	Node	Criterion	Aster	Tolerance
Displacement GONF	1.0	N1	"NON-REGRESSION"	RELATIF	-0.1	1.0E-04
Displacement DX	1.0	N5	"NON-REGRESSION"	RELATIF	0.03	1.0E-04
Forced SIG1	1.0		"NON-REGRESSION"	RELATIF	1.5	1,5.1.0E-04

10 Summary of the results

This test makes it possible to check in a very simple way the correct operation of the modelization second gradient, which also coincides with the results with the analytical solution.