

SSLL111 - Static response of a beam reinforced concrete (section in T) with linear behavior

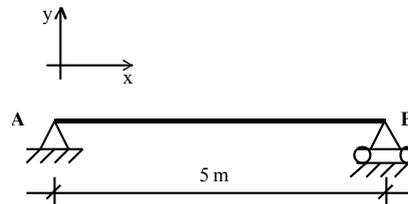
Abstract:

The problem consists in a beam modelization analyzing the response of a reinforced concrete beam via multifibre. This test corresponds to a static analysis of a beam having a linear behavior. Three successive loading cases are tested: a specific force, the inertia loading and a rise in temperature. For the first loading case, two meshes of the section, one coarse and the other finer are tested.

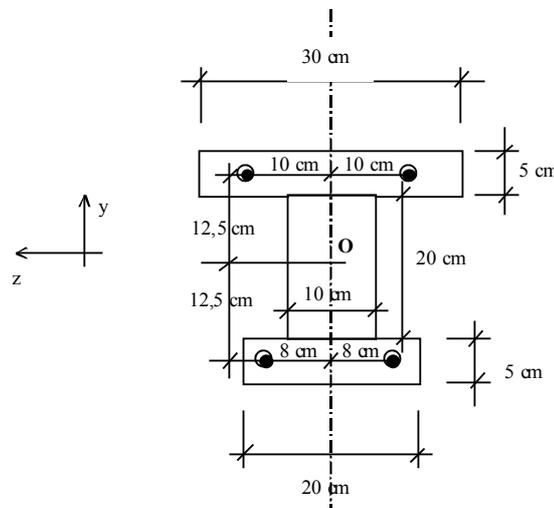
1 Problem of reference

1.1 Geometry

Beam in bending three points, defined by:



With a section in double T :



On this diagram, O is located at middle height of the section.

The total section of higher steels is $3 \cdot 10^{-4} m^2$ and that of lower steels is $4 \cdot 10^{-4} m^2$.

1.2 Material properties

- concrete: $E = 2 \cdot 10^{10} Pa$; $\nu = 0.2$; $\rho = 2400 kg \cdot m^{-3}$; $\alpha = 10^{-5} K^{-1}$
- steel : $E = 2,1 \cdot 10^{11} Pa$; $\nu = 0.33$; $\rho = 7800 kg \cdot m^{-3}$; $\alpha = 10^{-5} K^{-1}$

1.3 Boundary conditions

simple Bearing in B : $dy = 0$

Bearing "doubles" in A : $dx = dy = dz = 0$ just as $rx = ry = 0$.

1.4 Loadings

Three loading cases are tested successively:

- Loading 1: force concentrated in the mediums of the beam, $F = 10000 N$
- Loading 2: inertia loading of the beam, $g = 9,8 m \cdot s^{-2}$
- Loading 3: homogeneous heating of the beam $\Delta T = 100 K$

2 Reference solution

computations of reference are carried out from a simple elastic design in RdM.

2.1 Center elastic

In pure bending, for an elastic behavior, the neutral axis passes by the elastic center (barycenter of the sections balanced by the moduli of the materials):

$$C \text{ such as } \int_S E \overrightarrow{CM} dS = \vec{0}$$

One determines initially the position of the centers of gravity of the concrete alone G_b and steel alone G_a compared to the point O .

$$y_{G_b} = \frac{0.125 \times 0.3 \times 0.05 - 0.125 \times 0.2 \times 0.05}{0.2 \times 0.05 + 0.1 \times 0.2 + 0.3 \times 0.05} = 1.38888 \cdot 10^{-2} \text{ m}$$

$$y_{G_a} = \frac{0.125 \times 3 - 0.125 \times 4}{3 + 4} = -1.78571 \cdot 10^{-2} \text{ m}$$

$$z_{G_a} = z_{G_b} = 0 \text{ m}$$

One can then determine the position compared to O elastic center C .

$$\overrightarrow{OC} = \frac{E_a S_a \overrightarrow{OG_a} + E_b S_b \overrightarrow{OG_b}}{E_a S_a + E_b S_b}$$

The concrete section S_b is $0,045 \text{ m}^2$ and the steel formula section S_a is $7 \cdot 10^{-4} \text{ m}^2$. The Young modulus of the concrete is $2 \cdot 10^{10} \text{ MPa}$ and that of steel $21 \cdot 10^{10} \text{ MPa}$. There is thus

$$y_c = \frac{2 \times 0.045 \times 1.38888 - 21 \times 7 \cdot 10^{-4} \times 1.78571}{2 \times 0.045 + 21 \times 7 \cdot 10^{-4}} = 0.94317 \cdot 10^{-2} \text{ m}$$

$$z_c = 0 \text{ m}$$

2.2 quadratic Moments

the quadratic moments of the rectangular concrete sections are calculated by the following formula:

$$\frac{bh^3}{12} + b \times h \times d^2$$

Where, B represents the width, H the height and D the distance from the center of gravity of the section compared to the axis for which one calculates the moment.

One then obtains the quadratic moment of the concrete section compared to the axis z passing by the elastic center:

$$I_{\text{béton}} = \frac{0.3 \times 0.05^3}{12} + (0.3 \times 0.05) \cdot (0.125 - 0.94317 \cdot 10^{-2})^2 + \frac{0.1 \times 0.2^3}{12} + (0.1 \times 0.2) \cdot (0.94317 \cdot 10^{-2})^2$$

$$+ \frac{0.2 \times 0.05^3}{12} + (0.2 \times 0.05) \cdot (0.125 + 0.94317 \cdot 10^{-2})^2 = 0.4547 \cdot 10^{-3} \text{ m}^4$$

Inertias of steels are calculated by the following formula:

$$\frac{\pi \phi^4}{64} + S \times d^2 \approx S \times d^2$$

Where, ϕ the diameter of steel represents, S the section of steel and D the distance from the center of gravity of the section compared to the axis for which one calculates the moment. The diameter of steels being small, one neglects the first term.

One then obtains the quadratic moment of the steel sections compared to the axis z passing by the elastic center:

$$3 \cdot 10^{-4} \times (0.125 - 0.94317 \cdot 10^{-2})^2 + 4 \cdot 10^{-4} \times (0.125 + 0.94317 \cdot 10^{-2})^2 = 0.1124 \cdot 10^{-4} \text{ m}^4$$

For the complete section of the beam, the quadratic moment balanced by the Young moduli of the materials is:

$$EI = 2 \cdot 10^{10} \times 0.4547 \cdot 10^{-3} + 21 \cdot 10^{10} \times 0.1124 \cdot 10^{-4} = 11.4544 \cdot 10^6 \text{ Pa} \cdot \text{m}^4$$

2.3 Loading case 1

For loading case 1 (loading concentrated in the middle of the beam), the deflection is calculated by the formula of following RDM:

$$f = \frac{F \times l^3}{48 EI}$$

What gives the deflection:

$$f = \frac{10000 \times 5^3}{48 \times 11.4544 \cdot 10^6} = 2.2735 \cdot 10^{-3} \text{ m}$$

One can also calculate the following generalized forces:

- the shears at the beginning of the beam (left left) are worth $\frac{F}{2} = 5000 \text{ N}$,
- the bending moment in the middle of the beam is worth: $\frac{F \times l}{4} = 1.25 \cdot 10^4 \text{ N.m}$.

2.4 Loading case 2

For loading case 2 (inertia loading of the beam), the deflection is calculated by the formula of following strength of materials:

$$f = \frac{5 \times p \times l^4}{384 EI}$$

where p is the linear load due to the weight of the materials:

$$p = g(\rho_a S_a + \rho_b S_b) = 9.8 \times (2800 \times 7 \cdot 10^{-4} + 2400 \times 0.045) = 1111.9 \text{ N.m}^{-1}$$

What gives the deflection:

$$f = \frac{5 \times 1111.9 \times 5^4}{384 \times 11.4544 \cdot 10^6} = 7.9 \cdot 10^{-4} \text{ m}$$

2.5 Loading case 3

For loading case 3 (homogeneous rise in temperature), the beam being statically determinate and the coefficients of thermal expansion of the concrete and steel being identical, the solution is simple:

The generalized stresses and forces are null.

The lengthening of the beam is: $\Delta l = \alpha \times l \times \Delta T$

What gives with the values of our case:

$$\Delta l = 10^{-5} \times 5 \times 100 = 5 \cdot 10^{-3} \text{ m}$$

3 Modelization A

3.1 Characteristic of the modelization

One uses a modelization `POU_D_EM`.

3.2 Characteristics of the mesh

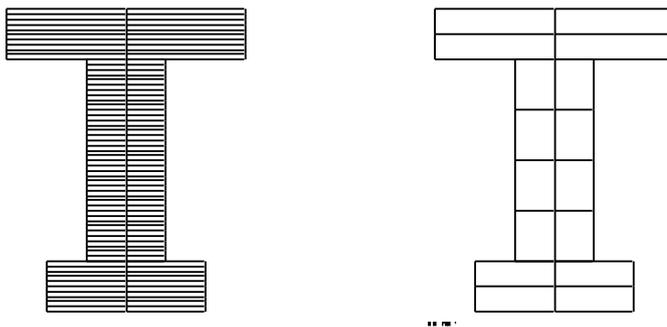
longitudinal Mesh of beam:

We have 3 nodes and two elements (`POU_D_EM`).



The concrete part of the cross section of the beam is with a grid (`DEFI_GEOM_FIBRE/ SECT`) while steels are given directly in the form of 4 specific fibers in `DEFI_GEOM_FIBRE/ FIBER`.

Two meshes of the concrete part are tested for loading case 1. The mesh fine consists of 120 fibers and the coarse mesh consists of 16 fibers:



Note:

The problem being 2D, only one fiber in the width could seem sufficient (multi-layer), but that would result in having null terms in the stiffness matrix (the own inertia of fibers not being taken into account) and with an error during the resolution of the system of equations.

3.3 Quantities tested and results

3.3.1 Loading case 1

	Reference
Marks with arrows (fine mesh)	2,2735 10-3
Deflection (coarse mesh)	2,2735 10-3
Shears (bearings A)	5000
Bending moment (Medium)	1,25 104

- 1) Computations are carried out without taking into account the own inertia of each fiber. The results show that it is not nevertheless very useful to take account of it because the difference between a coarse mesh and a fine mesh is not obvious.
The mesh of the section need does not have to be very fine to have precise results (in elasticity).
- 2) Option `EFGE_NOEU` used to compute: the forces generalized with the nodes makes an average of the generalized forces of all the elements connected to the node. In our case, we have 2 superimposed beam elements (for the concrete, for steel), the calculated forces are thus divided by 2.
If one adds the values with forces by element (`EFGE_ELNO`) of the element concrete and the element steel, one finds the theoretical values well.

Note:

If one does a calculation of deflection while taking O (middle height) as reference axis instead of elastic center (`COOR_AXE_POUTRE`), the relative error on the deflection is of 0,2% (because here the elastic center is practically with middle height (see 1.2.1)).

3.3.2 Loading cases 2

	Reference
Mark with arrows (fine mesh)	7,900 10-4

3.3.3 Loading cases 3

	Reference
Lengthening	5,00 10-3
Forces	0,00

4 Summary of the results

the got results are in concord with the results of reference.