SSLL107 - Validation of Summarized

MACR_CARA_POUTRE:
All the modelizations of this test make it possible to validate the macro_commande of computation of the characteristics of section of beam, MACR_CARA_POUTRE, for all the options suggested. The studied sections are different according to the modelizations.

- The modelization A the computation of the characteristics of section of a corner type validates.
- The modelization B the computation of the characteristics of a circular section validates.
- The modelization C the computation of the characteristics of a rectangular section validates.
- The modelization D the computation of the characteristics of an alveolate rectangular section validates.
- The modelization E the computation of the characteristics of an octagonal section validates.
- The modelization F the computation of the characteristics of a circular section with a sequence validates on a computation of beam.
- The modelization G the computation of a network of 2 rectangular beams of section validates.
- The modelization H validates the computation of the characteristics of a mean section in U.
- The modelization I the computation of the constant of torsion for a perforated section validates.
- The modelization J the computation of the shear coefficients for a perforated circular section validates.
- The modelization K the computation of the shear coefficients for a perforated rectangular section validates.

1 Problem of reference

1.1 Geometry

the geometry of the various sections is provided via a plane mesh. It are different for each modelization, and will thus be described in the corresponding paragraphs.

The modelization F also implements the chained computation of the characteristics of a circular section, and the use of these characteristics in a computation of a straight beam, length $L=1\text{m}$ in pure tension.

1.2 Material properties

Without object, except for the modelization F, where the treated beam has a Young modulus of $2.\text{E}11\text{Pa}$ and a Poisson's ratio of 0.3 .

1.3 Boundary conditions and loadings

Without object, except for the modelization F: the straight beam is embedded at an end, and is subjected at the other end with a tractive effort $F=1000\text{N}$.

2 Reference solution

2.1 Method of calculating used for the reference solution

Since the solutions are specific to each modelization, they are described in the corresponding paragraphs. They are drawn mainly from [bib1] and [bib2];

2.2 Results of reference

One describes here the characteristics calculated by MACR_CARA_POUTRE [R3.08.03]:

- Characteristics geometrical of the sections
  - Dans reference OYZ of description of the mesh 2D for the mesh provided by the user
    - aire : A_M
    - position of the center of gravity: CDG_Y_M, CDG_Z_M
    - moments and produced inertia of area, at the center of gravity G in reference GYZ : IY_G_M, IZ_G_M, IYZ_G_M
  - Dans the same total reference, for the mesh obtained by symmetrization if SYME_Y or SYME_Z :
    - aire : A
    - position of the center of gravity: CDG_Y, CDG_Z
    - moments and produced inertia of area, at the center of gravity G in reference GYZ : IY_G, IZ_G, IYZ_G
  - Dans the principal reference of Gyz inertia. cross-section, whose denomination corresponds to that used with the description of the neutral fiber beam elements GX [U4.24.01].
    - moments of inertia of area principal in the Gyz reference, usable for the computation of the flexural rigidity of beam: IY and IZ
    - angle of transition of reference GYZ to the principal reference of Gyz inertia : Characteristic

    ALPHA
    - distances, compared to the center of gravity G of the section for computations of maximum stresses: Y_MAX, Y_MIN, Z_MAX, Z_MIN and R_MAX.
  - Dans the total reference, in a point P provided by the user:
    - Y_P , Z_P : not computation of main moments of inertia
    - IY_P , IZ_P, IYZ_P : main moments of inertia in reference PYZ
    - IY_P , IZ_P : main moments of inertia in the Pyz reference.

- Mechanical characteristics:

Identification	Constant
JX	Meaning of torsion
EY	Position of the center of torsion/shears
EZ	Position of the center of torsion/shears
$PCTY$	Eccentring of the center of torsion in the reference GYZ along the axis Y
$PCTZ$	Eccentring of the center of torsion in the reference GYZ along the axis Z
AY	Shear coefficient
AZ	Shear coefficient
JG	Warping constant

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

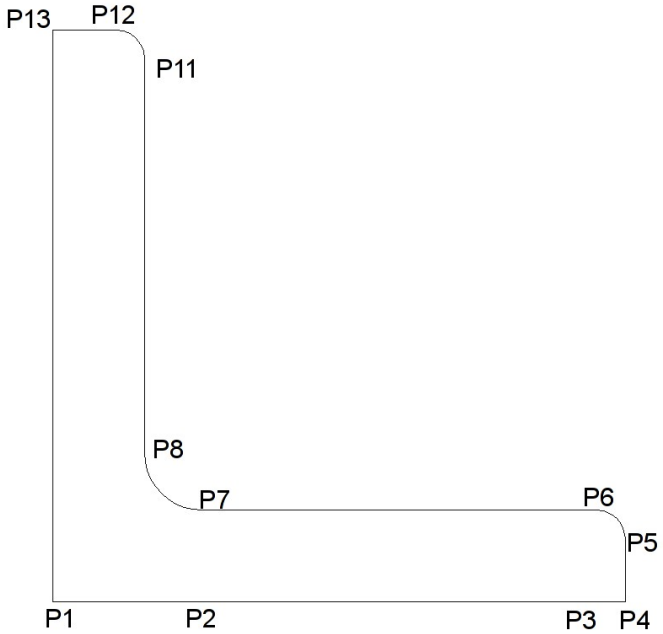
1.PILKEY W.D.: " Formulated for stress, Strain and Structural Matrixes ". Wiley & Idiots, New York, 1994.

2.D. BLEVINS: Formulated for natural frequency and shape mode.

3 Modelization A

3.1 Characteristic of the modelization

corner Section:

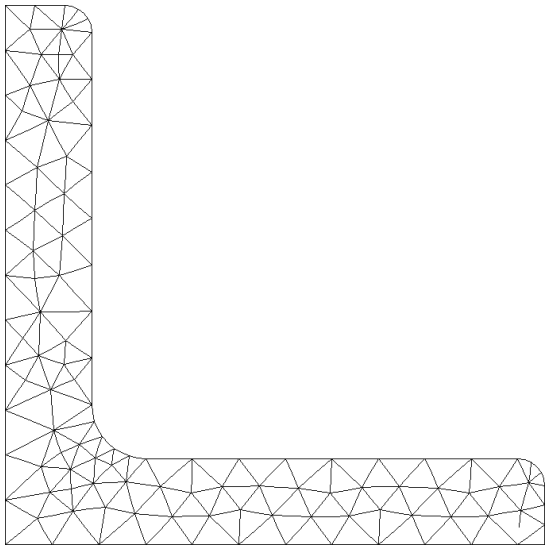


The coordinates of the points are:

P1		
P2	0.0.0.0.1.3E-02	0.0
P3	4.75E-02	0.0
P4	5.0E-02	0.0E+00
P5	5.0E-02	5.5E-03
P6	4.750E-02	8.0E-03
P7	1.30E-02	8.0E-03
P10	8.0E-03	1.300E-02
P11	8.0E-03	4.75E-02
P12	5.5E-03	5.0E-02
P13	0.	5.0E-02

3.2 Characteristic of the mesh

182 meshes TRIA6.



3.3 Reference solution

No exact analytical solution. The values are values of non regression.

3.4 Quantities tested and results

Identification	Reference	Tolerance
A	7.39E-04	1.00E-03
$ALPHA$	1.35E+02	1.00E-03
CDG_Y	1.53E-02	1.50E-03
CDG_Z	1.53E-02	1.50E-03
JX	1.43E-08	1.20E-01
EY	0.00E+00	1.00E-03
EZ	– 1.60E-02	1.00E-03
IY_G	1.64E-07	4.00E-03
$IYR2_P$	1.41E-08	1.40E-02
IYZ_G	– 9.50E-08	1.00E-03
IZ_G	1.64E-07	4.00E-03
IY	6.95E-08	4.00E-03
$IZR2_P$	1.41E-08	1.40E-02
IZ	2.60E-07	4.00E-03
$PCTY$	– 4.00E-03	1.00E-03
$PCTZ$	– 4.00E-03	1.00E-02
R_{MAX}	3.79E-02	1.00E-03
Y_{MAX}	3.54E-02	1.00E-03
Y_{MIN}	– 3.54E-02	1.00E-03
Z_{MAX}	2.17E-002	1.00E-03
Z_{MIN}	– 1.83E-02	1.00E-03

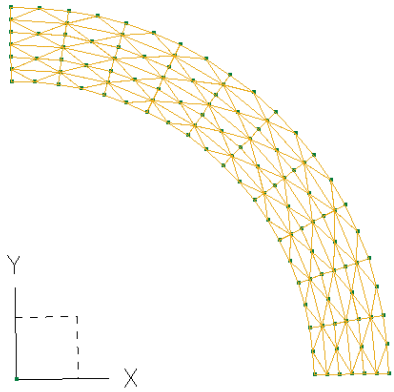
4 Modelization B

4.1 Characteristic of the modelization

It acts of the section of an external circle of radius $R=0.025\mathrm{m}$ and thickness $0.005\mathrm{m}$. Only one quarter of the section is represented. This modelization makes it possible to test key word CARA_GEOM of POST_ELEM, employee also by MACR_CARA_POUTRE to compute: the geometrical characteristics of a plane area.

4.2 Characteristics of the mesh

30 meshes QUAD8.



4.3 Reference solution

For the complete section:

$$A=\pi\left[R^2-(R-ep)^2\right]=1.76714\mathrm{E}\text{-}04\,m^2$$

$$I_y=I_z=\frac{\pi}{4}\left[R^4-(R-ep)^4\right]=1.81132\mathrm{E}\text{-}7\,m^4$$

$$I_p=2\,I_y=2\,I_z$$

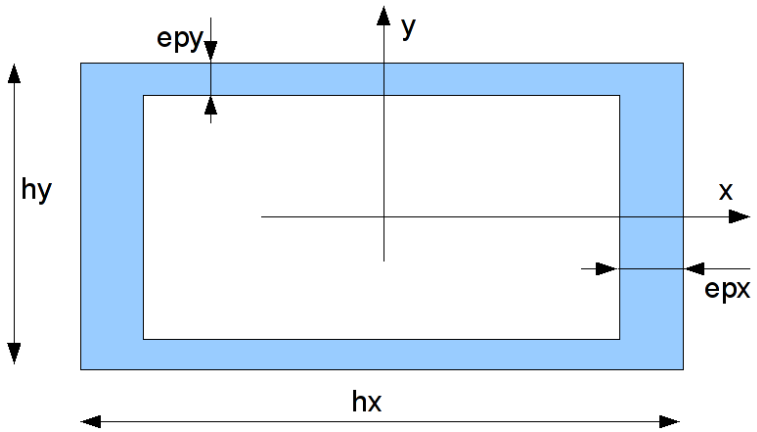
4.4 Quantities tested and results

Identification	Reference	% difference
A_M	1.76714E-04	– 7.76E-05
$CDG_{\text{Y-M}}$	1.438288E-02	– 1.25E-04
$CDG_{\text{Z-M}}$	1.438288E-02	– 1.25E-04
$IY_{\text{G-M}}$	8.7265757E-09	– 2.78E-04
$IZ_{\text{G-M}}$	8.7265757E-09	– 2.78E-04
$IYZ_{\text{G-M}}$	– 7.72837E-09	– 3.83E-04
A	7.0685745E-04	– 7.76E-05
CDG_Y	0.00000E+00	0
CDG_Z	0.00000E+00	0
IY_G	1.81132E-07	– 4.19E-06
IZ_G	1.81132E-07	– 4.19E-06
IYZ_G	0.00000E+00	
IY	1.81132E-07	– 4.19E-06
IZ	1.81132E-07	– 4.19E-06
Y_{MIN}	– 2.50000E-02	0.00E+00
Y_{MAX}	2.50000E-02	0.00E+00
Z_{MIN}	– 2.50000E-02	0.00E+00
Z_{MAX}	2.50000E-02	0.00E+00

5 Modelization C

5.1 Characteristic of the rectangular

modelization Section digs which one represents a quarter. This modelization makes it possible to test key word CARA_GEOM of POST_ELEM, employee also by MACR_CARA_POUTRE to compute: the geometrical characteristics of a plane area.

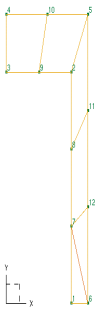


$Hx = 0.02\text{m}$ $Hy = 0.05\text{m}$ $epx = 0.002\text{m}$ $epy = 0.005\text{m}$.

5.2 Characteristics of the mesh

the coordinates of the nodes are:

N1	8.00E-03	0.00E+00
N2	8.00E-03	2.00E-02
N3	0.00E+00	2.00E-02
N4	0.00E+00	2.50E-02
N5	1.00E-02	2.50E-02
N6	1.00E-02	0.00E+00
N7	8.00E-03	6.6667E-03
N8	8.00E-03	1.3333E-02
N9	4.00E-03	2.00E-02
N10	5.00E-03	2.50E-02
N11	1.00E-02	1.66667E-02
N12	1.00E-02	8.3333E-03



5.3 Reference solution

$$A = h_y h_z - (h_y - 2\,ep_y)(h_z - 2\,ep_z) = 3.6\text{E-}04\,m^2$$

$$I_x = \frac{1}{12}\left[h_y h_z^3 - (h_y - 2\,ep_y)(h_z - 2\,ep_z)^3\right] = 1.23\text{E-}7\,m^4$$

$$I_y = \frac{1}{12}\left[h_z h_y^3 - (h_z - 2\,ep_z)(h_y - 2\,ep_y)^3\right] = 1.968\text{E-}8\,m^4$$

5.4 Quantities tested and results

Identification	Reference	% difference
A	3.60000E-04	– 2.26E-13
$ALPHA$	9.00000E+01	0.00E+00
CDG_Y	0.00000E+00	0
CDG_Z	0.00000E+00	0.00E+00
IY_G	1.23000E-07	– 6.46E-14
$IYZ_{\text{G-M}}$	– 1.11111E-09	1.00E-04
IZ_G	1.96800E-08	– 2.52E-13
IY	1.96800E-08	– 2.52E-13
IZ	1.23000E-07	– 6.46E-14
R_{MAX}	2.69258E-02	– 6.54E-04
Y_{MAX}	2.50000E-02	0.00E+00
Y_{MIN}	– 2.50000E-02	0.00E+00
Z_{MAX}	1.00000E-02	0.00E+00
Z_{MIN}	– 1.00000E-02	0.00E+00

6 Modelization D

6.1 Characteristic of the modelization

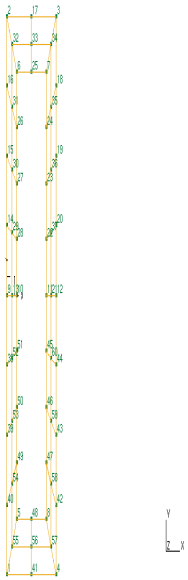
hollow rectangular Section. This modelization makes it possible to test MACR_CARA_POUTRE to compute: the geometrical and mechanical characteristics of a plane area.
Two computations are carried out:

- first with the key word SYME_Z is carried out = "OUI', i.e. that the section considered is obtained by symmetry around the axis Z (alveolate section). Moreover inertias are calculated compared to the point of coordinates $(0,-0.025)$ (key word ORIG_INER),
- second is carried out without symmetry, on the section with a grid, with a computation of inertias in the center of the mesh, C coordinates $(0.005,0)$, and 2 mesh groups different, which correspond each one to the vertical half of the mesh (on both sides of the axis Cz).

6.2 Characteristics of the mesh

40 meshes QUAD4.
The coordinates of the nodes tops of the rectangle are:

$N1$	0.00E+00	– 2.50E-02
$N2$	0.00E+00	2.50E-02
$N3$	1.00E-02	2.50E-02
$N4$	1.00E-02	– 2.50E-02
$N5$	2.00E-03	– 2.00E-02
$N6$	2.00E-03	2.00E-02
$N7$	8.01E-03	2.00E-02
$N8$	8.01E-03	– 2.00E-02
$N9$	0.00E+00	0.00E+00



6.3 Quantities tested and results

For the section symmetrized according to OY, the geometrical characteristics is:

Identification	Reference	% difference
A_M	2.600E-04	– 1.25E-13
A	5.200E-04	– 1.25E-13
$ALPHA$	9.000E+01	0.00E+00
$CDG_{\text{Y-M}}$	5.000E-03	– 5.20E-14
CDG_Y	0.000E+00	0.00E+00
$CDG_{\text{Z-M}}$	0.000E+00	1.40E-18
CDG_Z	0.000E+00	1.40E-18
$IY_{\text{G-M}}$	7.21667E-08	– 4.62E-05
IY_G	1.44333E-07	2.31E-04
IY_P	4.69333E-07	7.10E-05
$IYZ_{\text{G-M}}$	0.000E+00	– 4.33E-26
IYZ_G	0.000E+00	– 4.33E-26
$IZ_{\text{G-M}}$	3.44667E-09	– 9.67E-05
IZ_G	1.98933E-08	1.68E-04
IY	1.98933E-08	1.68E-04
IY_P	1.98933E-08	1.68E-04
IZ	1.44333E-07	2.31E-04
R_{MAX}	2.69260E-02	– 6.54E-04
Y_{MAX}	2.500E-02	0.00E+00
Y_{MIN}	– 2.500E-02	0.00E+00
Z_{MAX}	1.000E-02	1.73E-14
Z_{MIN}	– 1.000E-02	1.73E-14

For the not symmetrized section, the geometrical characteristics is:

Place	Identification	Reference	% difference
$TOUT$	IY_P	3.60833E-08	9.24E-05
$GR1$	IY_P	3.60833E-08	9.24E-05
$GR2$	IY_P	7.21667E-08	– 4.62E-05
$TOUT$	IZ_P	1.72333E-09	1.93E-04
$GR1$	IZ_P	1.72333E-09	1.93E-04
$GR2$	IZ_P	3.44667E-09	– 9.67E-05

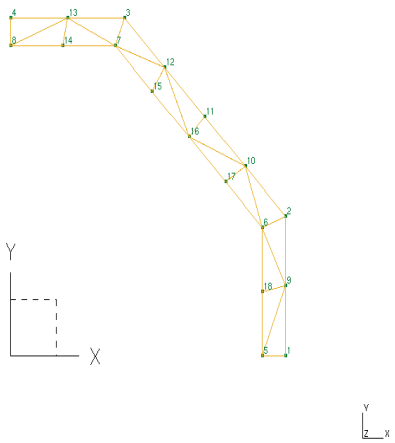
7 Modelization E

7.1 Characteristic of the modelization

hollow octagonal Section, which one nets a quarter.

7.2 Characteristics of the mesh

```
N1   2.30969E-02   0.00000E+00
N2   2.30969E-02   9.56708E-03
N3   9.56708E-03   2.30969E-02
N4   0.00000E+00   2.30969E-02
N5   2.11835E-02   0.00000E+00
N6   2.11835E-02   8.77452E-03
N7   8.77452E-03   2.11835E-02
N8   0.00000E+00   2.11835E-02
```



7.3 Quantities tested and results

For the section symmetrized according to OZ, the geometrical characteristics are:

Identification	Reference	% difference
$ALPHA$	9.00000E+01	0.00E+00
IYZ_G	0.00000E+00	0.00E+00
IZ_G	7.28824E-08	0.003
IY	7.28824E-08	0.003
IZ	7.28824E-08	0.003
R_{MAX}	2.50000E-02	4.58E-13
Y_{MAX}	2.30967E-02	0.001
Y_{MIN}	– 2.30967E-02	0.001
Z_{MAX}	2.30967E-02	0.001
Z_{MIN}	– 2.30967E-02	0.001

8 Modelization F

8.1 Characteristic of the modelization

full circular Section, radius $0.025\mathrm{m}$. The characteristics calculated are then used directly in a computation of straight beam (length $L=1\mathrm{m}$), in pure tension ($F=1000\mathrm{N}$). The Young modulus is worth $2.\mathrm{E}11\ Pa$. The characteristics of the section are given to AFFE_CARA_ELEM via key word TABLE_CARA.

8.2 Characteristics of the mesh

Number of meshes: 52 TRIA6, 299 QUAD8

8.3 Reference solution

$$A=\pi R^2=1.9635\mathrm{E}\text{-}3\,m^2\ ;\quad I_y=I_z=\frac{\pi}{4}R^4=3.06796\mathrm{E}\text{-}7\,m^4\ ;\quad A_y=A_z=\frac{10}{9}\ ;\quad C=I_x=2\,I_y=2\,I_z$$

Pure tension of a beam of full circular section, length $L=1\mathrm{m}$, subjected to a force $F=1000\mathrm{N}$:

$$u(x)=\frac{F\,x}{E\,A}\quad u(L)=\frac{F\,L}{E\,A}=2.54648\mathrm{E}\text{-}6\,m$$

8.4 Quantities tested and results

For the section symmetrized according to OY, the geometrical characteristics are:

Identification	Reference	Value	Tolerance
A	ANALYTIQUE	1.96E-03	0.50%
CDG_Y	ANALYTIQUE	0.00E+00	0.10%
CDG_Z	ANALYTIQUE	0.00E+00	0.10%
IY_G	ANALYTIQUE	3.07E-07	0.90%
IZ_G	ANALYTIQUE	3.07E-07	0.90%
IYZ_G	ANALYTIQUE	0.00E+00	0.10%
IY	ANALYTIQUE	3.07E-07	0.90%
IZ	ANALYTIQUE	3.07E-07	0.90%
Y_{MIN}	ANALYTIQUE	-2.50E-02	0.10%
Y_{MAX}	ANALYTIQUE	2.50E-02	0.10%
Z_{MIN}	ANALYTIQUE	-2.50E-02	0.10%
Z_{MAX}	ANALYTIQUE	2.50E-02	0.10%
JX	ANALYTIQUE	6.14E-07	0.90%
AY	ANALYTIQUE	1.17E+00	0.10%
AZ	ANALYTIQUE	1.17E+00	0.10%
EY	ANALYTIQUE	0.00E+00	0.10%
EZ	ANALYTIQUE	0.00E+00	0.10%
JG	ANALYTIQUE	0.00E+00	0.10%

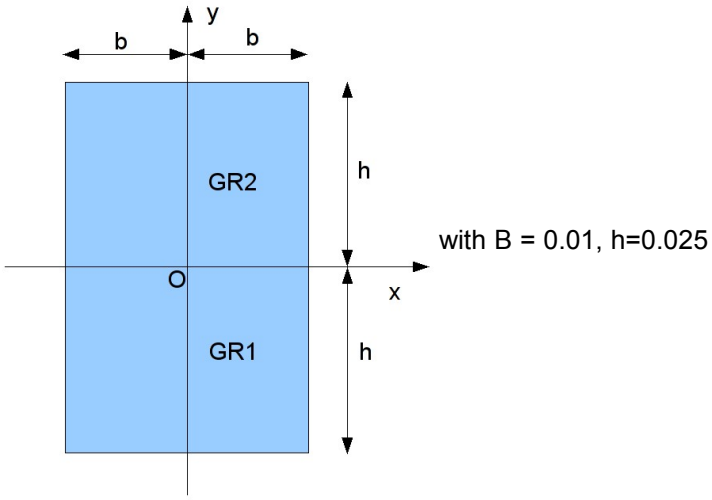
For the computation of tension of beam, result is:

Identification	Reference	Value	Tolerance
DEPL	ANALYTIQUE	2.55E-06	1.00E-03
Forc_nodA	ANALYTIQUE	1.00E+03	1.00E-03

9 Modelization G

9.1 Characteristic of the modelization

full rectangular Section, width $0.02\mathrm{m}$ and height $0.05\mathrm{m}$. It is divided into two rectangles respective heights $0.025\mathrm{m}$, in order to test the computation of the characteristics on mesh groups for a network here made up of two beams parallel, ranging between two bottoms distant of $L=0.0002\mathrm{m}$ (what makes it possible to obtain characteristics (shear coefficient) very close to that of the complete section).



9.2 Characteristic of the mesh

Number of meshes: 32 QUAD8

9.3 Reference solution

geometrical Characteristics for the complete section and each half-section:

```
       LIEUACDG_YCDG_ZIY_GIZ_GIYZ_G
Tout1.00E-       030.00.02.08                              E-073.33E-080.0
GR15.00E-040.0-1.25E-022.60E-081.67E-080.0
GR25.00E-040.01.25E-022.60E-081.67E-080.0

                                                LIEUY_PZ_PIY_PIZ_PIYZ_PIYIZ

Tout0.00E+000.00E+002.08E-073.33E-080.03.33E-082.08E-07

GR10.00E+000.00E+001.04E-071.67E-080.01.67E-081.04E-07

GR20.00E+000.00E+001.04E-071.67E-080.01.67E-081.04E-07
```

Shear coefficients: for each rectangular section: $A_y=A_z=1.2$

9.4 Quantities tested and results

For the complete section, the geometrical and mechanical characteristics are:

Identification	Reference	% difference
A	1.0000000E-03	0.00E+00
$ALPHA$	9.0000000E+01	0.00E+00
AY	1.2000000E+00	– 0.004
AZ	1.2000000E+00	– 0.065
CDG_Y	0.0000000E+00	– 1.03E-19
CDG_Z	0.0000000E+00	– 2.67E-19
JX	9.9805000E-08	– 0.124
EY	0.0000000E+00	1.55E-18
EZ	0.0000000E+00	– 4.79E-18
IY_G	2.0833333E-07	1.60E-06
IYZ_G	0.0000000E+00	– 1.40E-24
IZ_G	3.3333330E-08	1.00E-05
$PCTY$	0.0000000E+00	4.90E-18
$PCTZ$	0.0000000E+00	1.82E-18
Y_{MAX}	2.5000000E-02	0.00E+00
Y_{MIN}	– 2.5000000E-02	0.00E+00
Z_{MAX}	1.0000000E-02	1.73E-14
Z_{MIN}	– 1.0000000E-02	1.73E-14

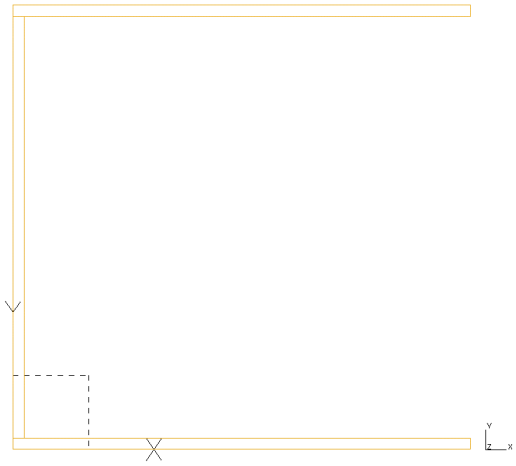
For the two disjoined groups, one obtains:

Place	Identification	Reference	% difference
$GR2$	A	5.00000E-04	2.17E-14
$GR1$	A	5.00000E-04	4.34E-14
$TOUT$	AY	1.20000E+00	– 0.064
$GR1$	AY	1.20000E+00	– 0.065
$GR2$	AY	1.20000E+00	– 0.065
$GR1$	AZ	1.20000E+00	– 0.065
$GR2$	AZ	1.20000E+00	– 0.065
$GR1$	CDG_Y	0.00000E+00	1.59E-19
$GR2$	CDG_Y	0.00000E+00	2.11E-19
$GR1$	CDG_Z	1.25000E-02	– 1.39E-14
$GR2$	CDG_Z	– 1.25000E-02	– 4.16E-14
$GR1$	IY_G	2.60417E-08	– 1.28E-04
$GR2$	IY_G	2.60417E-08	– 1.28E-04
$GR1$	IYZ_G	0.00000E+00	– 1.58E-24
$GR2$	IYZ_G	0.00000E+00	1.98E-24
$GR1$	IZ_G	1.66667E-08	– 2.00E-04
$GR2$	IZ_G	1.66667E-08	– 2.00E-04

10 Modelization H

10.1 Characteristic of the modelization

Section in U, of dimension $l=20\text{mm}$, and thickness $e=0.5\text{mm}$



10.2 Characteristics of the mesh

Number of meshes: 236 QUAD8

10.3 Reference solution

the approximate analytical values result from [bib1].

10.4 Quantities tested and results

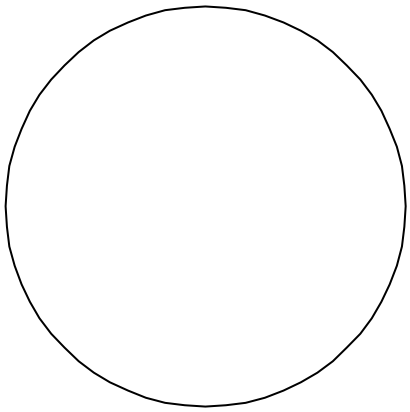
For the complete section, the geometrical and mechanical characteristics are:

Identification	Reference	% difference
A	29.5	0.00E+00
AY	4.25300E+00	5.483
AZ	1.61800E+00	18.472
CDG_Y	6.8602	– 0.015
CDG_Z	10	– 2.31E-13
JX	2.4984	– 1.9
EY	0	– 4.18E-11
EZ	– 15.43	– 0.089
JG	8.69 E+04 [bib4]	0.253

11 Modelization I

11.1 Characteristic of the modelization

hollow circular Section, external radius 10mm , and thickness 1mm .



11.2 Characteristics of the mesh

Number of meshes: 300 QUAD8

11.3 Reference solution

$$C = I_x = \pi\left[\frac{R^4}{2} - \frac{(R-e)^4}{2}\right] = 5401.97\,mm^4$$

11.4 Quantities tested and results

the constant of torsion are worth:

Identification	Reference	% difference
JX	5401.97	– 0.194

12 Modelization J

12.1 Characteristic of the modelization

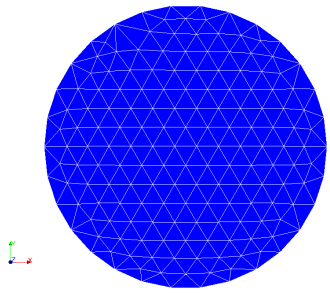
Three sections:

-circular full with radius 2 circular
-Mr. hollow, external radius 2 m and parameter ALPHA = 0.5.
-hollow circular, of external radius 2 m and parameter ALPHA = 0.9.

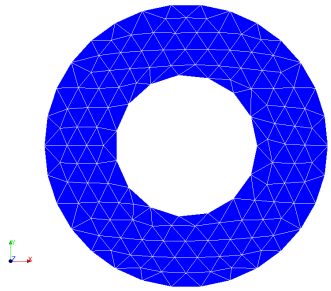
Note: ALPHA = (Rext – thickness)/Rext

12.2 Characteristics of meshes

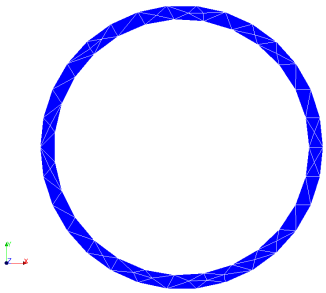
1. Number of meshes: 318 TRIA6



2. Number of meshes: 318 TRIA6



3. Number of meshes: 318 TRIA6



12.3 Quantities tested and results

Of the tests to guarantee the NON-regression of the code are carried out on the calculated shear coefficients.

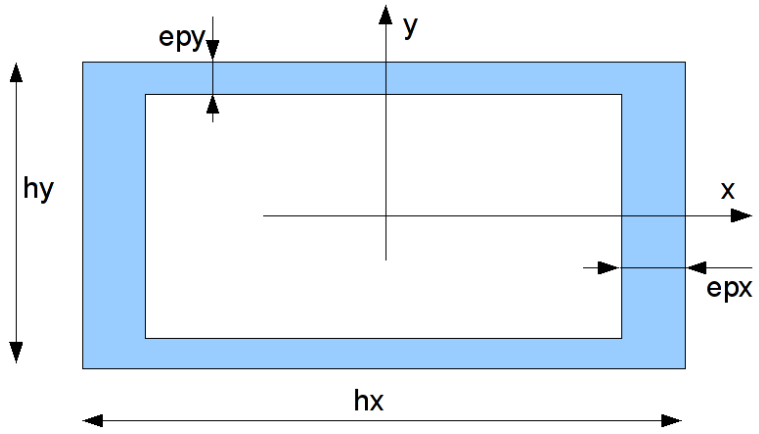
13 Modelization K

13.1 Characteristic of the modelization

Three sections:

-square full on side 4 square
-Mr. digs, on side 4 m and of parameter ALPHA = 0.5/BETA = 0.525.
-square digs, on side 4 m and parameter ALPHA = 0.9/BETA = 0.945.

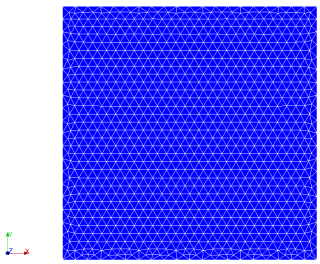
Note:



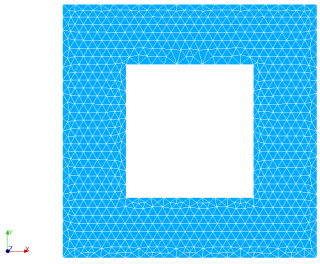
*ALPHA = (HX – 2*EPX) /HX*
*BETA = (HY – 2*EPY)/HY*

13.2 Characteristic of meshes

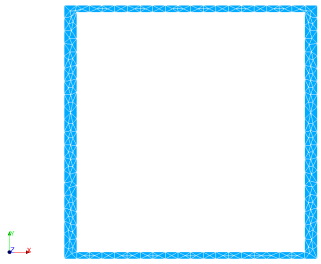
1. Number of meshes: 1838 TRIA6



2. Number of meshes: 1418 TRIA6



3. Number of meshes: 442 TRIA6



13.3 Quantities tested and results

Of the tests to guarantee the NON-regression of the code are carried out on the calculated shear coefficients.

14 Summary of the results

This test makes it possible to check of the command simultaneously good performance MACR_CARA_POUTRE for various types of sections.