SSLL104 - Predeformations in a straight beam

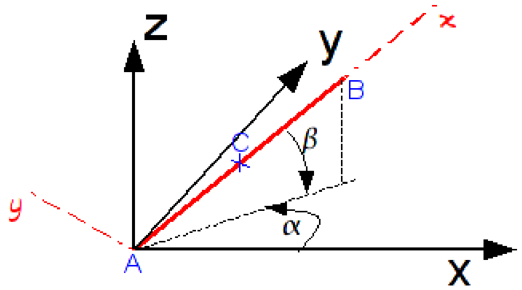
Summarized:

This test validates the taking into account of predeformations in the elastic design of a straight beam. The characteristics of computation are:

- static analysis,
- linear behavior,
- model linear,
- 1 only modelization testing elements POU_D_E, POU_D_T, POU_D_TG and POU_D_EM
- the solution is analytical.

1 Problem of reference

1.1 Geometry



a beam AB length $l=100\,mm$ is located on trisecting trihedron (X, Y, Z) : the coordinates of the point B are: $B=\left(\frac{100}{\sqrt{3}}, \frac{100}{\sqrt{3}}, \frac{100}{\sqrt{3}}\right)$

One C defines also a point medium of A, B.

the local coordinate system (A, x, y, z) results from the total reference (A, X, Y, Z) by the nautical angles $\begin{cases} \alpha=45° \\ \beta=-35.26°\, solution\, de \cos\beta=\sqrt{\frac{2}{3}} \end{cases}$

1.2 Material properties

the material is elastic linear.

Young modulus $E=1.0\,MPa$ (without influence on result).

Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

Fixed support in A : $DX=DY=DZ=DRX=DRY=DRZ=0$.

Loading: predeformation in the local coordinate system (A, x, y, z)

- elongation according to x : $\epsilon_X^0=0.001$
- curvature around y : $\chi_y^0=0.002$
- curvature around z : $\chi_z^0=0.003$

1.4 Characteristics of the section of beam

All the characteristics (area, inertias,…) are taken equal to 1. They are without influence on result.

2 Reference solution

2.1 Method of calculating used for the reference solution

the solution is analytical. It is calculated in the local coordinate system.
that is to say:

$U=(u, v, w, \theta_x, \theta_y, \theta_z)$ the displacement of the beam and $E=(\epsilon_x, \chi_y, \chi_z, \gamma_{xy}, \gamma_{xz})$ generalized strain.

That is to say the solution:

$$u=\alpha x \qquad v=\gamma \frac{x^2}{2} \qquad w=-\beta \frac{x^2}{2} \qquad \theta_x=0 \qquad \theta_y=\beta x \qquad \theta_z=\gamma x$$

then:

$$\epsilon_x=u_{,x}=\alpha \qquad \chi_y=\theta_{y,x}=\beta \qquad \chi_z=\theta_{z,x}=\gamma \qquad \gamma_{xy}=v_{,x}-\theta_z=0 \qquad \gamma_{xz}=w_{,x}+\theta_y=0$$

If one chooses $\alpha=\epsilon_X(=0.001)$ $\beta=\chi_y^0(=0.002)$, $\gamma=\chi_z^0(=0.003)$ then $E-E_{init}=0$ and the forces are null: the equilibrium is thus checked. In addition, the solution checks the boundary conditions of fixed support in A. It is thus the solution of the posed problem.

2.2 Results of reference

the results expressed in the local coordinate system are:

In B :

$$Dx=0.10\,mm\,;\,Dy=15.0\,mm\,;\,Dz=-10.0\,mm\,;\,DRx=0.0\,rd\,;\,DRy=0.2\,rd\,;\,DRz=0.30\,rd$$

In C :

$$Dx=0.05\,mm\,;\,Dy=3.75\,mm\,;\,Dz=-2.50\,mm\,;\,DRx=0.0\,rd\,;\,DRy=0.1\,rd\,;\,DRz=0.15\,rd$$

In the total reference, one finds at the points B and C :

$$DX(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}(-3\sqrt{6}+2\sqrt{2})[mm] \qquad DX(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}(-3\sqrt{6}+2\sqrt{2})[mm]$$

$$DY(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}(3\sqrt{6}+2\sqrt{2})[mm] \qquad DY(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}(3\sqrt{6}+2\sqrt{2})[mm]$$

$$DZ(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}(-4\sqrt{2})[mm] \qquad DZ(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}(-4\sqrt{2})[mm]$$

$$DRX(B)=\frac{1}{20}(-\sqrt{6}-2\sqrt{2})[rd] \qquad DRX(C)=\frac{1}{40}(-\sqrt{6}-2\sqrt{2})[rd]$$

$$DRY(B)=\frac{1}{20}(-\sqrt{6}+2\sqrt{2})[rd] \qquad DRY(C)=\frac{1}{40}(-\sqrt{6}+2\sqrt{2})[rd]$$

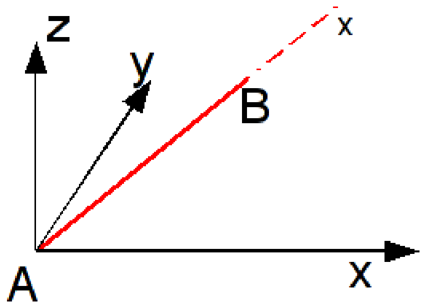
$$DRZ(B)=\frac{1}{20}(2\sqrt{6})[rd] \qquad DRZ(C)=\frac{1}{40}(2\sqrt{6})[rd]$$

2.3 Uncertainty on the solution

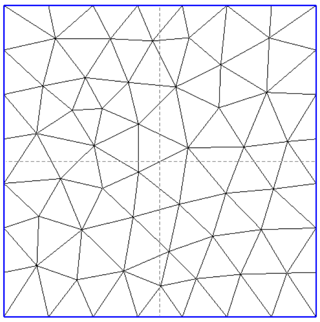
the solution is exact for the theory of the beams of Eulerian (or of Timoshenko because there are no shears). Torsion not intervening, the solution is also valid for elements POU_D_TG.

3 Modelization A

3.1 Characteristic of the modelization



- The segment AB is cut out in 10 of the same elements length (10.). (Only one element would be sufficient).
- 4 identical calculations are successively done on this mesh with 4 different modelizations:
  - with 10 elements POU_D_E
  - with 10 elements POU_D_T
  - with 10 elements POU_D_TG
  - with 10 elements POU_D_EM



Mesh of the section:
- 77 nodes
- 124 TRIA3

3.2 Characteristic of the mesh

Many nodes: 11
Number of meshes and types: 10 SEG2

3.3 Quantities tested and results

Modelization	Identification	Reference	% difference
POU_D_E	BDX	– 6.4664E+00	< 1.0E-9
	DY	1.4747E+01	< 1.0E-9
	DZ	– 8.1072E+00	< 1.0E-9
	DRX	– 2.6390E-01	< 1.0E-9
	DRY	1.8947E-02	< 1.0E-9
	DRZ	2.4495E-01	< 1.0E-9
	CDX	– 1.6022E+00	< 1.0E-9
	DY	3.7011E+00	< 1.0E-9
	DZ	– 2.0124E+00	< 1.0E-9
	DRX	– 1.3195E-01	< 1.0E-9
	DRY	9.4734E-03	< 1.0E-9
	DRZ	1.2247E-01	< 1.0E-9
POU_D_T	BDX	– 6.4664E+00	< 1.0E-9
	DY	1.4747E+01	< 1.0E-9
	DZ	– 8.1072E+00	< 1.0E-9
	DRX	– 2.6390E-01	< 1.0E-9
	DRY	1.8947E-02	< 1.0E-9
	DRZ	2.4495E-01	< 1.0E-9
	CDX	– 1.6022E+00	< 1.0E-9
	DY	3.7011E+00	< 1.0E-9
	DZ	– 2.0124E+00	< 1.0E-9
	DRX	– 1.3195E-01	< 1.0E-9
	DRY	9.4734E-03	< 1.0E-9
	DRZ	1.2247E-01	< 1.0E-9
POU_D_TG	BDX	– 6.4664E+00	< 1.0E-9
	DY	1.4747E+01	< 1.0E-9
	DZ	– 8.1072E+00	< 1.0E-9
	DRX	– 2.6390E-01	< 1.0E-9
	DRY	1.8947E-02	< 1.0E-9
	DRZ	2.4495E-01	< 1.0E-9
	CDX	– 1.6022E+00	< 1.0E-9
	DY	3.7011E+00	< 1.0E-9
	DZ	– 2.0124E+00	< 1.0E-9
	DRX	– 1.3195E-01	< 1.0E-9
	DRY	9.4734E-03	< 1.0E-9
	DRZ	1.2247E-01	< 1.0E-9
POU_D_EM	BDX	– 6.4664E+00	< 1.0E-6
	DY	1.4747E+01	< 1.0E-5
	DZ	– 8.1072E+00	< 1.0E-5
	DRX	– 2.6390E-01	< 1.0E-5
	DRY	1.8947E-02	< 1.0E-4
	DRZ	2.4495E-01	< 1.0E-5
	CDX	– 1.6022E+00	< 1.0E-6
	DY	3.7011E+00	< 1.0E-5
	DZ	– 2.0124E+00	< 1.0E-5
	DRX	– 1.3195E-01	< 1.0E-5
	DRY	9.4734E-03	< 1.0E-4
	DRZ	1.2247E-01	< 1.0E-5

4 Summary of the results

As one could expect it, the results are very precise. They validate the good taking into account of the predeformations in the beam elements.

The test does not relate to curved beams (POU_C_T) because one does not have reference solution.