SSLL103 - Elastic buckling of an angle

Summarized:

A straight beam (corner with equal wings) biarticulée is subjected to a normal force (excentré or not) or to one bending moment.

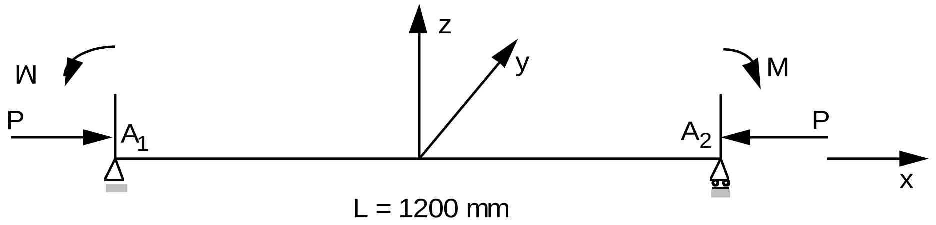
One seeks the critical loads of elastic buckling.

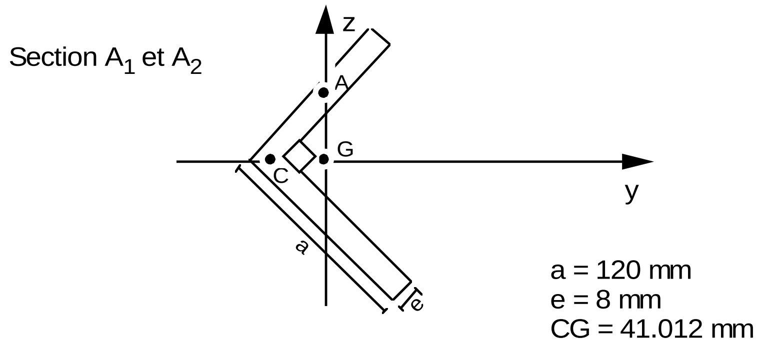
- linear elastic mechanics,
- buckling of a beam,
- eccentring of the center of torsion,
- interest of the test: computation of the geometrical stiffness matrix of elements POU_D_TG and POU_D_T,
- 2 modelizations.

An uncertainty persists on the number of buckling modes of the reference solution [§5].

1 Problem of reference

1.1 Geometry





Caractéristiques de la section

$A =$ 1856 mm^2
$I_y =$ 4167339 mm^4
$I_z =$ 1045547 mm^4
$J =$ 39595 mm^4
$I_\omega =$ 44398819 mm^6
$I_{yr2} =$ 84948392 mm^5
$y_c =$ –41.012 mm
$z_c =$ 0

1.2 Material properties

Modulus Young: $E=2.10\mathrm{E}-5\,MPa$
Poisson's ratio: $\nu=0.3$

1.3 Boundary conditions and loadings

$$A1 \;:\; DX=DY=DZ=DRX=0$$
$$A2 \;:\; DY=DZ=DRX=0$$

Loading
- case 1: axial load P in G
- case 2: axial load P in C
- case 3: axial load P in A
- case 4: bending moment M

1.4 Remarks

For cases 2 and 3, one applies in $A2$ a force in G, then one superimposes in $A1$ and $A2$ a bending moment (according to oz for case 2, following oy for case 3) to offset the force in C (or A).

2 Reference solution

2.1 Méthode de calcul used for the reference solution

With taking into account of warping, the calculations done by V. Of City De Goyet [bib1] give:
that is to say:

$$I_y=\int_A z^2 dA \qquad I_y=\int_A y^2 dA \qquad I_{yr^2}=\int_A y\left(y^2+z^2\right)dA \qquad I_{yr^2}=\int_A z\left(y^2+z^2\right)dA$$

$$P_{cry}=\frac{\pi^2 E I_z}{L^2} \qquad P_{crz}=\frac{\pi^2 E I_y}{L^2} \qquad P_{crx}=\left(\frac{GJ+\pi^2 E I_\omega}{L^2}\right)Ar_a$$

$$Ar_c=\frac{\left(I_y+I_z\right)}{A}+y_c^2+z_c^2+y_c\left(\frac{I_{yrz}}{Iz}-2y_c\right)+z_c\left(\frac{I_{zr^2}}{Iz}-2z_c\right)$$

$$Ar_a=\frac{\left(I_y+I_z\right)}{A}+y_c^2+z_c^2+y_a\left(\frac{I_{yrz}}{Iz}-2y_c\right)+z_a\left(\frac{I_{zr^2}}{Iz}-2z_c\right)$$

with:

$\left(y_a, z_a\right)$: coordinates of the point of application of the force
$\left(y_c, z_c\right)$: coordinates of the center of torsion

Case 1,2,3:
One obtains 3 critical loads by solving the equation of the 3° degree in P :

$$Ar_a\left(P_{cry}-P\right)\left(P_{crz}-P\right)\left(P_{crx}-P\right)-P^2\left(P_{crz}-P\right)(z_c-z_a)^2-P^2\left(P_{cry}-P\right)(y_c-y_a)^2=0$$

Case 4:
The critical moment Mcr (around the axis y) is worth:

$$Mcr=\pm\left(\left(GJ+\frac{\pi^2 E I_\omega}{L^2}\right)P_{cry}\right)^{1/2}$$

By neglecting warping: the analytical solution of reference is given in [bib2] [bib3].

2.2 Results of reference

Values of the critical loads corresponding to the first modes of buckling for the various loading cases.

2.3 Uncertainty on the analytical

solution Solution. The values of reference are obtained using NAG (routine $C0SAGF$, $EPS=10^{-8}$).

2.4 Bibliographical references

1. V. OF TOWN OF GOYET "nonlinear Static analysis by the finite element method of formed spatial structures by beams to asymmetric section" - Thesis of doctorate University of Liege, MSM, academic year (1988-1989).

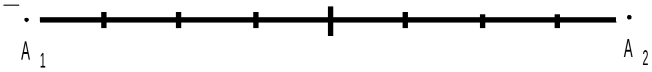
2. P. PENSERINI "elastic Instability of the beams with open mean profile: theoretical and numerical aspects" Notes EDF/DER/HM77/112.

3. J. CHERRY TREE "Propagation of two cases tests of modelization of the computation of the beams in elastic buckling in *Code_Aster*" HM77/184

3 Modelization A

3.1 Characteristic of the modelization

8 elements POU_D_TG



3.2 Characteristics of the mesh

Many nodes: 9
Number of meshes and types: 8 SEG2

3.3 Quantities tested and results

Identification	Reference
Case 1	
fashion 1	– 6.92531E+05
mode 2	– 1.50487E+06
mode 3	– 1.00589E+07
Cases 2	
mode 1	– 1.50487E+06
mode 2	– 5.99812E+06
mode 3	1.47904E+06
Cases 3	
mode 1	– 5.72260E+05
mode 2	– 2.45950E+06
mode 3	– 1.85673E+07
Cases 4	
mode 1	7.00631E+07

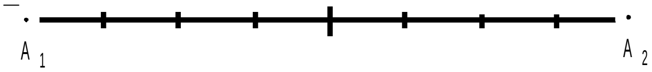
3.4 Remarks

the accuracy is excellent with 8 elements in the length.

4 Modelization B

4.1 Characteristic of the modelization

8 elements POU_D_T



4.2 Characteristics of the mesh

Many nodes: 9
Number of meshes and types: 8 SEG2

4.3 Quantities tested and results

Identification	Reference
Case 1	
fashion 1	– 6.796E+05
mode 2	– 1.505E+06
mode 3	– 1.0055E+07
Cases 2	
mode 1	– 1.505E+06
mode 2	– 5.998E+06
Cases 3	
mode 1	– 5.638E+05
mode 2	– 2.453E+06
mode 3	– 1.8525E+07
Cases 4	
mode 1	6.9376E+07

4.4 Remarks

the accuracy is rather good with 8 elements in the length. The solution differs a little that obtained with warping (modelization A).

5 Summary of the results

the analytical solution gives us 3 buckling modes of which the critical loads are roots of an equation of the 3° degree.

Y-a it of other critical loads intercalated between the 3 found values?

Aster finds the good critical loads, but in the middle of much of others… for example for case 3, the 3 sought critical loads correspond to `the NUME_MODE` : 1,10 and 19.

This is true for the two modelizations.