SSLL102 - clamped Beam subjected to unit forces

Summarized:
This test allows a simple checking of computations of straight beams and shell $1D$ in linear static structural mechanics. The model is linear.

The modelizations A, B, C, D, F, G, J and K make it possible element types to test the different ones from straight beams in *Code_Aster*. For each modelization, one calculates simultaneously 3 beams of different sections: rectangle, circle, angle.

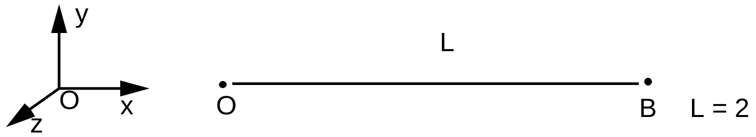
The modelization A allows of more than test the change of reference: the beam is directed according to the trisecting one with the total reference.
The modelization E the distributed loading tests on voluminal edges of elements.
The modelization F corresponds to a distributed loading varying linearly with modelization POU_D_E.
The modelization G corresponds to a distributed loading varying linearly with modelization POU_D_TG.
The modelization H allows to test shell element $1D$ (COQUE_C_PLAN) subjected to unit stresses.
The modelization I allows to test a distributed loading varying linearly with modelization TUYAU_3M.

The values tested are displacements, the generalized forces and the forced.

1 Problem of reference

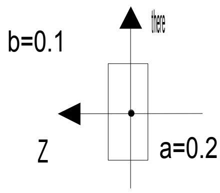
1.1 Geometry

Straight beam length L , direction x . Dimensions are expressed in meters $[\mathrm{m}]$.



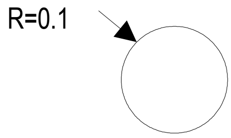
One calculates simultaneously 3 types of different cross sections:

1 rectangular section



1 corner section with equal wings

1 circular section



1.2 Material properties

Modulus Young: $E=2.10^{11}\,Pa$
Poisson's ratio: $\nu=0.3$

1.3 Boundary conditions and loadings

Fixed support in O
6 unit loadings in B :

$Fx=1$	$Mx=1$
$Fy=1$	$My=1$
$Fz=1$	$Mz=1$

1 loading combined bending plus tension: $Fx=1$; $My=1$; $Mz=1$;
1 loading combined shears plus torsion: $Fy=1$ $Fz=1$ $Mx=1$
1 linear distributed loading: $Fy=1000.\mathrm{x}$ circular section (modelizations F, G, I) (with simple bearing in A and B in this case)

1.4 Notation of the characteristics of cross sections

the geometrical characteristics of the cross sections are noted:

A	area of the section
I_y, I_z	geometrical main moments of inertia compared to the principal axes of inertia of the constant
JX	section of torsion
ay, az	shear coefficients in the directions Gy and Gz
$A'_y=\frac{A}{ay}$ $A'_z=\frac{A}{az}$	equivalent reduced areas
e_y, e_z	eccentricity of the center of torsion
JG	warping constant

2 Reference solution

2.1 Method of calculating used for the analytical reference solution

Solution [bib1] and [bib2]: displacements in B

Tension simple	$u_x=\frac{F_x L}{E S}$		
Bending pure	$u_y=\frac{F_y L^3(4+\phi_y)}{12 E I_z}$	$\theta_z=\frac{L^2 F_y}{2 E I_z}$	$\phi_y=\frac{12 E I_y}{L^2 G A'_y}$
Bending pure	$u_z=\frac{F_z L^3(4+\phi_z)}{12 E I_y}$	$\theta_y=\frac{-L^2 F_z}{2 E I_y}$	$\phi_z=\frac{12 E I_z}{L^2 G A'_z}$
Torsion		$\theta_x=\frac{M_x L}{G J_x}$	
Pure bending	$u_z=-\frac{M_y L^2}{2 E I_y}$	$\theta_y=\frac{M_y L}{E I_y}$	
Pure bending	$u_y=\frac{M_z L^2}{2 E I_z}$	$\theta_z=\frac{M_z L}{E I_z}$	

Notices 1:

For the corner section, as the shear center is not confused with the center of gravity $(e_y \neq 0)$ *, it is necessary to add the twisting moment:* $M_x=F_z e_y$ *with the loading* $F_z=1$ *.*
This modifies displacement:

$$u_z=\frac{F_z L^3(4+\phi_z)}{12 E I_y}+\theta_x e_y \quad \theta_x=\frac{M_x L}{G J_x}$$

In the same way, the loading $M_x=1$ *involves a displacement* $u_z=\theta_x e_y$ *.*

Linear distributed loading:

$$u_y(x)=\frac{p x}{360 L E I}(3 x^4-10 L^2 x^2+7 L^4) \quad u_y^{max}=\frac{0.00652\, p L^4}{E I}$$

$$en\ x=0.519 L$$

Notice 2:

With regard to the modelization A, the beam is carried by the vector $e_1=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$*. The other vectors of the local coordinate system are:* $e_2=\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$ and the $e_3=\frac{1}{\sqrt{6}}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$

component from the vector displacement in the total reference are obtained by:

$$u_G=\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}}\end{pmatrix} u_{local}$$

Generalized forces and stresses in O :

$N(O)=F_x \qquad \sigma_{xx}=\frac{N}{S}$

$M_z(O)=T_y L \qquad T_y=F_y \qquad \sigma_{xx}(y)=\frac{M_z y}{I_z} \qquad \sigma_{xy}=\frac{T_y}{k_y S}$

$M_y(O)=-T_z L \qquad T_z(O)=F_z \qquad \sigma_{xx}(y)=\frac{-M_y z}{I_y} \qquad \sigma_{xz}=\frac{T_z}{k_z S}$

$M_x(0)=M_x(B) \qquad \sigma_{xy}=\sigma_{xz}=\frac{M_x R_T}{J_x}$

$M_y(0)=M_y(B) \qquad \sigma_{xx}(z)=\frac{M_y z}{I_y}$

$M_z(0)=M_z(B) \qquad \sigma_{xx}(y)=\frac{M_y y}{I_z}$

Linear distributed loading:

$$M_z(x)=\frac{-1000}{6}\left(L^2 x-x^3\right) \quad V_y(x)=\frac{1000\,L^2}{6}-\frac{1000\,x^2}{2} \quad \sigma_{xx}^{max}=\frac{M_z^{max} R}{I_z} \quad en\ x=\frac{L\sqrt{3}}{3}$$

2.2 Results of reference

Displacement of the point B ,
Forces generalized at the point O ,
Stresses of the point O .

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

1.J.L. BATOZ, G. DHATT: “Modelization of structures by finite elements” - Volume 2 ED. HERMES.
2.N.D. PIKLEY: “Formulated for Stress, Stain & Structural Matrixes” ED. John Wiley & Sounds.

3 Modelization A

3.1 Characteristic of the modelization

2 elements `POU_D_E` $k_y = k_z = 1$ $\phi = 0$ by type of section

$S1$: Rectangular section modelled by `SECTION: "GENERALE"`

$$A = 0.02 \quad Iy = 0.1666\text{E}-4 \quad Iz = 0.6666\text{E}-4 \quad J_x = 0.45776\text{E}-4$$
$$Ry = 0.1 \quad Rz = 0.05 \quad R_T = 0.0892632$$
$$Point\ de\ calcul\ des\ contraintes$$

$S2$: Corner section

$$A = 1.856\text{E}-3 \quad I_y = 4.167339\text{E}-4 \quad I_z = 1.045547\text{E}-4$$
$$J_x = 03.9595\text{E}-8 \quad e_y = 41.012\text{E}-3 \quad e_z = 0.0$$

$S3$: Rectangular section modelled by `SECTION: RECTANGLE`

$Hy = 0.2$ $Hz = 0.1$

$S4$: Section cercle $R = 0.1$

$$I_Y = I_Z = \frac{\pi R^4}{4} = \frac{\pi}{4} 10^{-4}$$

3.2 Characteristic of the mesh

4×2 elements `POU_D_E`. The beam is directed according to the vector $(1,1,1)$.

3.3 Quantities tested and results

Loading case	Beam	Identification	Reference
$F_x = 1$	$S1 = S3$	$u_x(B)$	2.887E-10
		$\theta_{xx}(0)$	50.
	$S2$	$u_x(B)$	3.11E-9
	$S4$	$u_x(B)$	1.838E-10
		σ_{xx}	31.83
$F_y = 1$	$S1 = S3$	$u_y(B)$	+1.414E-7
		$\theta_z(B)$	1.225E-7
		$\sigma_{xx}(0)$	3000
	$S2$	$u_y(B)$	9.017E-8
	$S4$	$\sigma_{xx}(0)$	2546.479
$F_z = 1$	$S1 = S3$	$u_z(B)$	6.532E-7
		$\theta_y(B)$	-4.243E-7
		$\sigma_{xx}(0)$	6000
		$\sigma_{xz}(0)$	50
	$S2$	$u_z(B)$	9.279E-7
		$\theta_y(B)$	1.553E-5
		$\theta_x(B)$	1.555E-5
	$S4$	$u_z(B)$	1.386E-7
		$\theta_y(B)$	9th-8

		$\sigma_{xx}(0)$	2546.479
		$\sigma_{xz}(0)$	31.831
$M_x=1$	$S1=S3$	$\theta_x(B)$	3.279E-7
		$\sigma_{xy}=\sigma_{xz}(0)$	1950.0
	$S2$	$\theta_x(B)$	3.791E-4
		$u_z(B)$	2.199E-5
	$S4$	$\theta_x(B)$	9.556E-8
		$\sigma_{xy}=\sigma_{xz}(0)$	636.62
$M_y=1$	$S1=S3$	$u_z(B)$	-4.899E-7
		$\theta_y(B)$	4.243E-7
		$\sigma_{xx}(0)$	3000
	$S2$	$u_z(B)$	-1.959E-8
		$\theta_y(B)$	1.697E-8
	$S4$	$u_z(B)$	-1.04E-7
		$\theta_y(B)$	9.0E-8
		$\sigma_{xx}(0)$	1273.2395
$M_z=1$	$S1=S3$	$u_y(B)$	1.061E-7
		$\theta_z(B)$	1.225E-7
		$\sigma_{xx}(0)$	1500.0
	$S2$	$u_y(B)$	6.763E-8
		$\theta_z(B)$	7.809E-8
	$S4$	$u_y(B)$	9.0E-7
		$\sigma_z(B)$	1.04E-7
		$\sigma_{xx}(0)$	1273.2395
$M_y=1$ $M_z=1$ $F_x=1$	$S1=S3$	$\sigma_{xx}\,max(0)$	4550.0
		$\sigma_{xx}\left(\frac{a}{2},\frac{b}{2}\right)$	1550.0
	$S4$	$\sigma_{xx}\,max(0)$	1832.4636
$F_y=1$ $F_z=1$ $M_x=1$	$S1, S3$	$\sigma_{xy}(0)$	2000.0
		$\sigma_{xz}(0)$	2000.0
		$\sigma_{xx}\,max(0)$	9000.0
	$S1, S3$	$\sigma_{xx}\left(\frac{a}{2},\frac{b}{2}\right)$	-9000.0
	$S4$	$\sigma_{xx}\,max(0)$	3601.27
		$\sigma_{xy}(0)$	668.451

4 Modelization B

4.1 Characteristic of the modelization

2 elements `POU_D_T`.
The shear coefficients are:
$S1$: Rectangular section

$$AY=AZ=1.2=\frac{1}{k_y}$$

$S2$: Corner section

$$AY=AZ=\frac{1}{0.358}$$

$S4$: Section cercle

$$AY=AZ=\frac{10}{9}$$

4.2 Characteristic of the mesh

4×2 elements `POU_D_T`

4.3 Quantities tested and results

One gives only the values which differ from the modelization A (because of the taking into account of the transverse shears).

Loading	Section	Identification	Reference
$F_y=1$	$S1=S3$	$u_y(B)$	2.0156E-7
		$\sigma_{xy}(0)$	60.
	$S2$	$u_y(B)$	1.666552E-7
	$S4$	$u\,y(B)$	1.707308E-7
		$\sigma_{xy}(0)$	37.13615
$F_z=1$	$S1, S3$	$u_z(B)$	8.0156E-7
		$\sigma_{xz}(0)$	60.
	$S2$	$u_z(B)$	1.17559754E-6
	$S4$	$u_z(B)$	1.707308E-7
		$\sigma_{xz}(0)$	37.13615
$F_y=1$ $F_z=1$ $M_x=1$	$S4$	$\sigma_{xz}(0)$	673.75592
		$\sigma_{xy}(0)$	673.75592

5 Modelization C

5.1 Characteristic of the modelization

2 elements POU_D_TG.
Warping is not constrained.
The shear coefficients are identical to those of the modelization B.

5.2 Characteristic of the mesh

4×2 elements POU_D_TG

5.3 Quantities tested and results

Loading	Section	Identification	Reference
$F_y=1$	$S1=S3$	$u_y(B)$	2.0156E-7
		$\theta_{xy}(0)$	60.
	$S2$	$u_y(B)$	1.666552E-7
	$S4$	$u_y(B)$	1.70684E-7
		$\theta_{xy}(0)$	35.367765
$F_z=1$	$S1, S3$	$u_z(B)$	8.0156E-7
		$\theta_{xz}(0)$	60.
	$S2$	$u_z(B)$	1.17559754E-6
	$S4$	$u_z(B)$	1.70684E-7
		$\theta_{xz}(0)$	35.367765

5.4 Remark

warping is not constrained. The results are thus identical to those of the modelization B.

6 Modelization D

6.1 Characteristic of the modelization

Elements `POU_D_TG`, torsion obstructed

$$JG=\begin{cases}5.5556\text{E}-8 \; pour \; S_1\\ 4.439822\text{E}-11 \; pour \; S_2\end{cases}$$

in 0 $GRX=0$

6.2 Characteristics of the mesh

- 10 elements,
- refinement towards the fixed support.

6.3 Quantities tested and results

Same results as for the modelization C, except those which relate to the effects of warping.

Loading	Section	Identification	Reference
$F_z=1$	$S2$	$\theta_x=DRX$	2.62034E-5
		$u_z=DZ$	1.14578E-6
		GRX	1.34652E-5
$M_x=1$	$S1$	$u_z=DZ$	5.52E-7
		GRX	2.84E-7
	$S2$	u_z	2.6203E-5
		θ_x	6.3892E-4
		GRX	3.28324E-4

6.4 Remarks

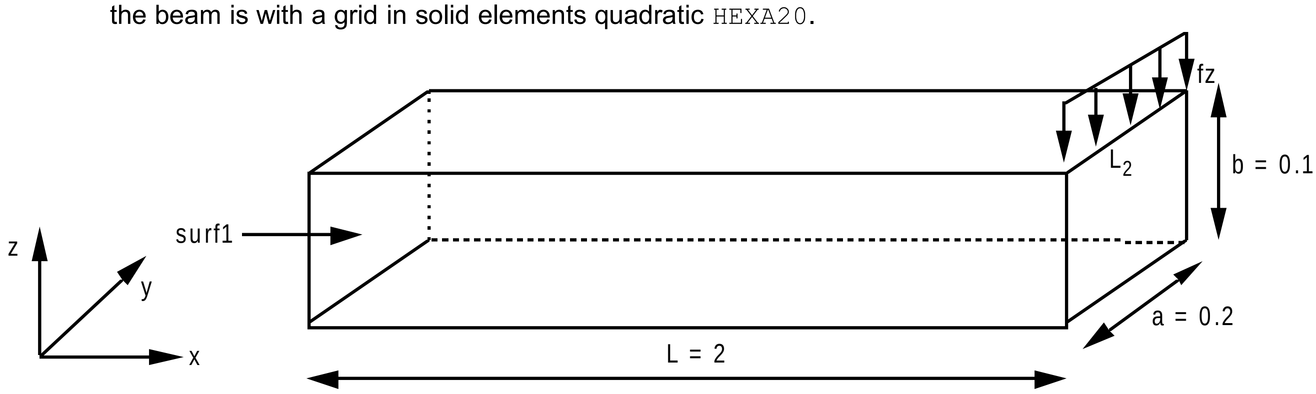
For θ_x the solution is (cf [bib1]):

$$\theta_x=\frac{M_x L}{G J_x}+\frac{M_x\left(1-e^{2\alpha L}-2 e^{\alpha L}\right)}{\alpha^3 E J G\left(1+e^{2\alpha L}\right)} \quad \alpha^2=\frac{G J}{E J G}$$

7 Modelization E

7.1 Characteristic of the modelization

the beam is with a grid in solid elements quadratic HEXA20.



The beam is embedded on the level of the section $surf1$. It is subjected to unit shears which are modelled by one density linear of load fz applying to the 4 meshes SEG3 constituting the higher edge $L2$.

7.2 Characteristics of the mesh

the beam is with a grid with 640 solid elements quadratic HEXA20.
The model comprises 3665 nodes.

7.3 Quantities tested and results

One tests the value of the deflection according to z medium node of the section where one applies the loading (node $N62$).

Identification	Reference	*Aster*	% difference
dz of the node $N62$	-8.0E-7	-7.9523E-7	-0.596

7.4 Remarks

the value of reference corresponds to the value given by Strength of materials.

8 Modelization F

8.1 Characteristic of the modelization

The model is composed of 10 elements straight beam of Eulerian. The section is circular full, of radius 0.1m.

8.2 Characteristics of the mesh

It consists of 10 elements POU_D_E. The length of the beam is $L=6m$

8.3 Quantities tested and results

8.3.1 analytical

	Internal forces Results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ\ (2\sqrt{3})$	-1.3856E+04

8.3.2 Forced

	analytical Results
$SIXX\ (2\sqrt{3})$	1.7642E+07

9 Modelization G

9.1 Characteristic of the modelization

The model is composed of 10 elements straight beam of Timoshenko with warping. The section is circular full, of radius $0.1\mathrm{m}$.

9.2 Characteristics of the mesh

It consists of 10 elements POU_D_TG. The length of the beam is $L=6\,m$

9.3 Quantities tested and results

9.3.1 analytical

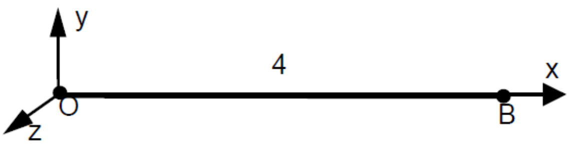
	Internal forces Results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ\ (2\sqrt{3})$	-1.3856E+04

9.3.2 Displacement (deflection close to the medium of the beam)

	Aster *Results* (non regression)	Tolerance (%)
$DY(3.115977734)$	3.23499E-03	1.E-4

10 Modelization H

10.1 Characteristic of the modelization



Modélisation COQUE_C_PLAN

- Section rectangulaire
- Conditions limites : Point O $u = v = \theta_z = 0$
- Chargement unitaire : Point B F_x , F_y et M_z

10.2 Characteristics of the mesh

Many nodes: 9
Number of meshes and types: 4 SEG3

10.3 Quantities tested and results

Loading case	Beam	Identification	Reference
$F_x=1$	S1	$u_x(B)$	5.E-10
		$\sigma_{xx}(0)$	5.0
$F_y=1$	S1	$u_y(B)$	2.E-7
		$\theta_z(B)$	1.5E-7
		$\sigma_{xx}(0)$	300.0
$M_z=1$	S1	$u_y(B)$	1.5E-7
		$\theta_z(B)$	1.5E-7
		$\sigma_{xx}(0)$	150.0

Loading cases	Beam	Identification	Value *Aster*	% difference
$OMEGA=100$	S1	$u_x(B)$	0.0104	NON_REGRESSION
		$\sigma_{xx}(0)$	1.576E+8	NON_REGRESSION

10.4 Remarks

the width for modelization COQUE_C_PLAN is imposed on 1 in *Code_Aster*. Consequently, we multiplied by 0.1 the Young modulus to take account of the real width of the beam. This width of 1 modifies the inertia of the beam and consequently the value of the stress σ_{xx} which is 10 times lower than the value of reference. Moreover, for displacements, the results differ from the modelization A because of change of reference.
For the loading in rotation, one compares a computation where the rotational axis is confused with the origin with same computation where the mesh and the rotational axis are relocated (to test key word CENTER).
The computed values quite equal and are tested in NON_REGRESSION.

11 Modelization I

11.1 Characteristic of the modelization

The model is composed of 21 elements `TUYAU_3M` leaning on meshes `SEG4`.
The force distributed is imposed along the axis y. bending thus takes place around z.

11.2 Characteristics of the mesh

It consists of 21 meshes `SEG3`. The length of the pipe is $L=6\,m$

11.3 Quantities tested and results

11.3.1 analytical

	Results Displacements
$D_y\,maxi$	9.38888E-03

11.3.2 analytical

	Internal forces Forced
$V_y(x=0)$	Results
$V_y(x=L=6)$	6.0000E+03
$MFZ\,2\sqrt{3}$	-1.2000E+04

11.3.3 -1.3856E+04

It are calculated at the point of X-coordinate $x=\dfrac{L\sqrt{3}}{3}$ which corresponds to the maximum moment:

$$M_z(x)=\frac{-1000}{9\sqrt{(3)}}L^3=-13856.41\,N.m$$

For angle 0 on the circumference of the pipe (the origin of the angles being the axis z), the stresses are null, and for angle 90, they are maximum: $\sigma_{xx}^{max}=\dfrac{M_z^{max}(R-e/2)}{I_z}=-4.87363\mathrm{E}+07\,Pa$

	Reference	Tolerance
$\sigma_{xx}(\alpha=0)$	0	0.10%
$\sigma_{xx}(\alpha=90)$	-4.87363E+07	1.00%
MFZ	-1.3856E+04	1.%

12 Modelization J

12.1 Characteristic of the modelization

The model is composed of 2 elements POU_D_EM.
The loading is similar to that of the modelization A (twisting moment only)

12.2 Characteristic of the mesh

It consists of 2 meshes SEG2. The length of the beam is $L=2\,m$

the beam is directed according to the vector $(1,1,1)$.

The section is rectangular, identical to that of the modelization A.

12.3 Quantities tested and results

12.3.1 Displacement (rotation due to the twisting moment)

	Aster *Results* (non regression)	**Tolerance (%)**
$DX=DY=DZ$	3.2792525E-07	1.E-6

13 Modelization K

13.1 Characteristic of the modelization

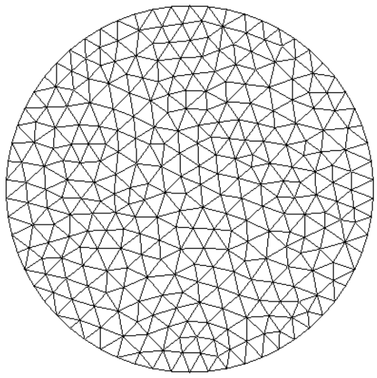
The model is composed of 10 elements POU_D_EM.
One applies a distributed force of $6000 \mathrm{N}/m$ to all the beam.

13.2 Characteristics of the mesh

It consists of 10 meshes SEG2. The length of the beam is $L=6m$

The mesh of the section consists of:

- 373 nodes
- 62 SEG2
- 682 TRIA3



13.3 Quantities tested and results

13.3.1 Internal forces

Internal forces	analytical Results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
MFZ	-1.3856E+04

13.3.2 Displacement (deflection close to the medium of the beam)

	Aster *Results* (non regression)	**Tolerance (%)**
$DY(3.115977734)$	3.23499E-03	1.E-4

14 Summary of the results

This test make it possible simultaneously to check the correct operation of elements POU_D_E, POU_D_T and POU_D_TG on 3 types of different sections. The perfect coincidence of the results with the analytical solutions (RDM) is normal, and must always be observed, since the solution is contained in the shape functions of the elements.

Moreover, the modelization E makes it possible to test the distributed loading on voluminal edges of elements. The variation with the analytical solution (RDM) is lower than 0.6% .

The modelizations F, G, I and K make it possible to test the distributed loading (linear variation) for beam elements POU_D_E, POU_D_TG, POU_D_EM and the pipe sections . The variation with the analytical solution (Strength of materials) is lower than 0.6% .

For modelization COQUE_C_PLAN the results are satisfactory (displacements and stresses) for the unit loadings of standard extension and bending (imposed moment). For the loading of bending (load imposed at an end) the error on displacement is weak and lower than 0.5% . It is more important on the stress: 3.6% .