SHLV100 - Harmonic response of a hollow roll in plane strains

Summarized:

This axisymmetric three-dimensional test makes it possible to validate computations of the stiffness matrixes, mass and the vectors of pressure on all the axisymmetric 3D elements 2D and plane strains and (10 modelizations).

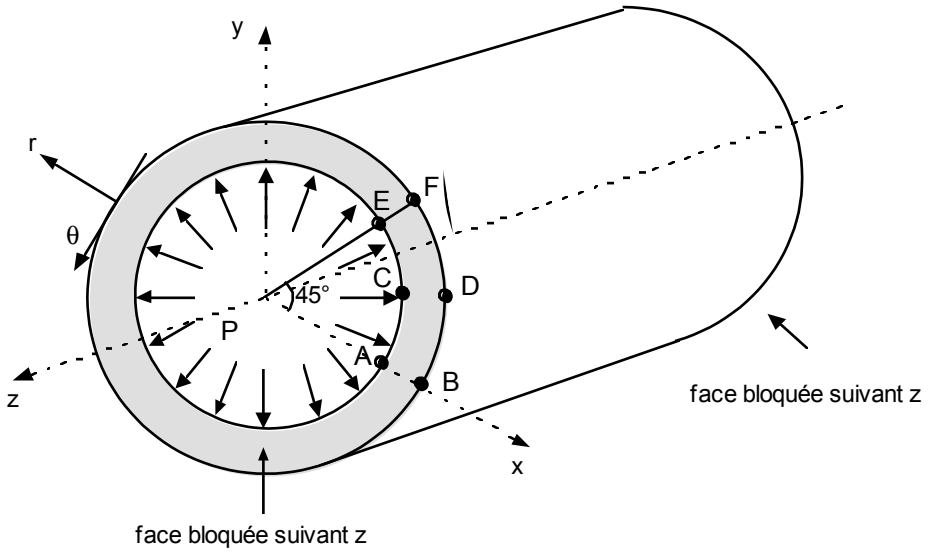
Displacements are imposed:

- either by degrees of freedom,
- or by face of element.

For the four modelizations 3D, the pressures applied are provided with the minus sign, because the sides of elements 3D are badly directed in the mesh files used.

1 Problem of reference

1.1 Geometry



radius interns $a=0.1\,m$
external radius $b=0.2\,m$

Coordinates of the points:

	A	B	C	D	E	F
x	0.100	0.200	$0.1\cos(22.5)$	$0.2\cos(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
y	0.	0.	$0.1\sin(22.5)$	$0.2\sin(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
z	0.	0.	0.	0.	0.	0.

1.2 Material properties

$E=26\,N/m^2$
$\nu=0.3$
$\rho=35\,Kg/m^3$

the very low value of the Young modulus does not have anything physics.

1.3 Boundary conditions and loadings

Pressure interns $P=pe^{j\omega t}$ with $p=1\,Mpa$ and $\omega=0.2\,rad/s$

1.4 Initial conditions

- without initial conditions,
- computation direct of the harmonic solution.

2 Reference solution

2.1 Method of calculating used for the reference solution

$$u_r = A J_1\left(k_L r\right) + B Y_1\left(k_L r\right) \qquad u_\theta = u_z = 0$$

$$\sigma_{rr} = 2\mu K_L \left[A\left[\left(2\gamma^2 - 1\right) J_0\left(k_L r\right) - \frac{1}{k_L r} J_1\left(k_L r\right)\right] + B\left[2\gamma^2 Y_0\left(k_L r\right) - \frac{1}{k_L r} Y_1\left(k_L r\right)\right]\right]$$

$$\sigma_{\theta\theta} = 2\mu K_L \left[A\left[\left(2\gamma^2 - 1\right) J_0\left(k_L r\right) + \frac{1}{k_L r} J_1\left(k_L r\right)\right] + B\left[2\gamma^2 Y_0\left(k_L r\right) + \frac{1}{k_L r} Y_1\left(k_L r\right)\right]\right]$$

$$\sigma_{zz} = 2\mu K_L \left(2\gamma^2 - 1\right)\left[A J_0\left(k_L r\right) + B Y_0\left(k_L r\right)\right]$$

$$\sigma_{r\theta} = \sigma_{rz} = \sigma_{\theta z} = 0$$

avec : $\gamma^2 = \frac{\lambda + 2\mu}{4\mu} = \frac{1-\nu}{2(1-\nu)} = \frac{1}{4\beta^2}$ $\qquad k_L = \frac{\omega}{C_L} = \omega\sqrt{\frac{\rho}{\lambda + 2\mu}}$

J_1, J_0, Y_1, Y_0 : Fonctions de Bessel.

the constants A and B are calculated by solving the linear system obtained while writing:

$$\sigma_{rr}(a) = -p \quad \sigma_{rr}(b) = 0$$

One obtains:

For $r = 0.1$	$u_r = 7.3398\,10^{-3}$	For $r = 0.2$	$u_r = 4.6816\,10^{-3}$
	$\sigma_{rr} = -1$		$\sigma_{rr} = 0.$
	$\sigma_{\theta\theta} = 1.6685$		$\sigma_{\theta\theta} = 0.66738$
	$\sigma_{zz} = 0.20055$		$\sigma_{zz} = 0.20031$

Transition in the system of Cartesian axes:

$$\begin{aligned}
\sigma_{xx} &= \sigma_{rr}\cos^2\theta + \sigma_{\theta\theta}\sin^2\theta - 2\sigma_{r\theta}\sin\theta\cos\theta \\
\sigma_{yy} &= \sigma_{rr}\sin^2\theta + \sigma_{\theta\theta}\cos^2\theta + 2\sigma_{r\theta}\sin\theta\cos\theta \\
\sigma_{xy} &= \sigma_{rr}\sin\theta\cos\theta - \sigma_{\theta\theta}\sin\theta\cos\theta - 2\sigma_{r\theta}(\cos^2\theta - \sin^2\theta)
\end{aligned}$$

with:

$\theta = 0°$ at the points A and B

$\theta = 22.5°$ the points C and D

$\theta = 45°$ the points E and F

2.2 Results of reference

Displacements (u, v) and forced $\left(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}\right)$ to the points A, B, C, D, E, F.

2.3 Uncertainty on the solution

Accuracy of the computation of the Functions of Bessel.

2.4 Bibliographical references

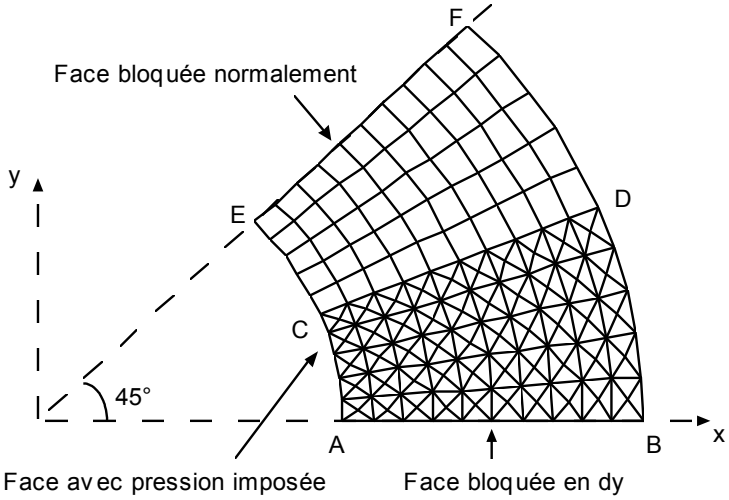
1) Mr. BONNET: Methods of the integral equations regularized into elastodynamic - Bulletin of DER - Series C - N°1/2 - (1987).

2) ERINGEN - SUHUBI - Elastodynamics, Vol.2: linear theory Academic Close (1975).

3 Modelization A

3.1 Characteristic of the modelization

Elements $3D$ (PENTA6 and HEXA8) (resulting from the mesh $2D$ below).



along the axis Z : 2 layers of elements total thickness: 0.01

limiting Conditions:

	DDL_IMPO:	(All: "yes"	Dz: 0.)
face AB		(Group_no: BordAB	Dy: 0.)
face EF	FACE_IMPO:	(Group_ma: FaceEF	Dnor: 0.)
pressure on face AE	PRES_REP:	(Group_ma: FaceAE	Near: -1.)

Names of the nodes: A=No1 B=No119 C=No36 D=No166 E=No41 F=No171

3.2 Characteristics of the mesh

Many nodes: 513
Number of meshes and types: 400 PENTA6 100 HEXA8 40 QUAD4

3.3 Remarks

the pressure have a negative sign (instead of positive) because the sides of the elements $3D$ are badly directed.

3.4 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3243 10-3	– 0.21	10-2
	v	0.	eps	–	
	σ_{xx}	– 1.	– 0.8789	12.1	0.2.0.2
	σ_{yy}	1.6685	1.6241	– 2.66	
	σ_{zz}	0.20055	0.2235	11.75	
	σ_{xy}	0.	– 0.0922	–	
C	u	6.78109 10-3	6.7670 10-3	– 0.21	10-2
	v	2.80882 10-3	2.8012 10-3	– 0.27	0.3.0.3.0.3
	σ_{xx}	– 0.60921	– 0.5121	15.94	
	σ_{yy}	1.27771	1.3300	4.09	
	σ_{zz}	0.20055	0.2454	22.39	0.3.0.3
	σ_{xy}	– 0.94346	– 0.8567	9.20	
E	u	5.19002 10-3	5.1784 10-3	– 0.22	10-2
	v	5.19002 10-3	5.1784 10-3	– 0.22	0.6.0.6.0.6
	σ_{xx}	0.33425	0.4319	29.23	
	σ_{yy}	0.33425	0.5315	59.04	
	σ_{zz}	0.20055	0.289	44.50	0.6
	σ_{xy}	– 1.33425	– 1.269	4.87	
B	u	4.6716 10-3	4.6641 10-3	– 0.16	10-2
	v	0.	eps	–	
	σ_{xx}	0.	– 0.0132	–	
	σ_{yy}	0.66738	0.6724	0.75	2.10-2
	σ_{zz}	0.20021	0.1977	– 1.25	–
	σ_{xy}	0.	0.0219	–	–
D	u	4.32523 10-3	4.3084 10-3	– 0.39	10-2
	v	1.79157 10-3	1.7854 10-3	– 0.34	0.3.0.3.0.3
	σ_{xx}	0.09774	0.0739	– 24.39	
	σ_{yy}	0.56964	0.5728	0.56	
	σ_{zz}	0.20021	0.1941	– 3.05	0.3.0.3
	σ_{xy}	– 0.23595	– 0.2348	0.49	
F	u	3.31039 10-3	3.2974 10-3	– 0.39	10-2
	v	3.31039 10-3	3.2974 10-3	– 0.39	0.2.0.2.0.2
	σ_{xx}	0.33369	0.2977	– 10.78	
	σ_{yy}	0.33369	0.3245	2.75	
	σ_{zz}	0.20021	0.1866	– 6.80	0.2.0.2
	σ_{xy}	– 0.33369	– 0.3415	– 2.34	

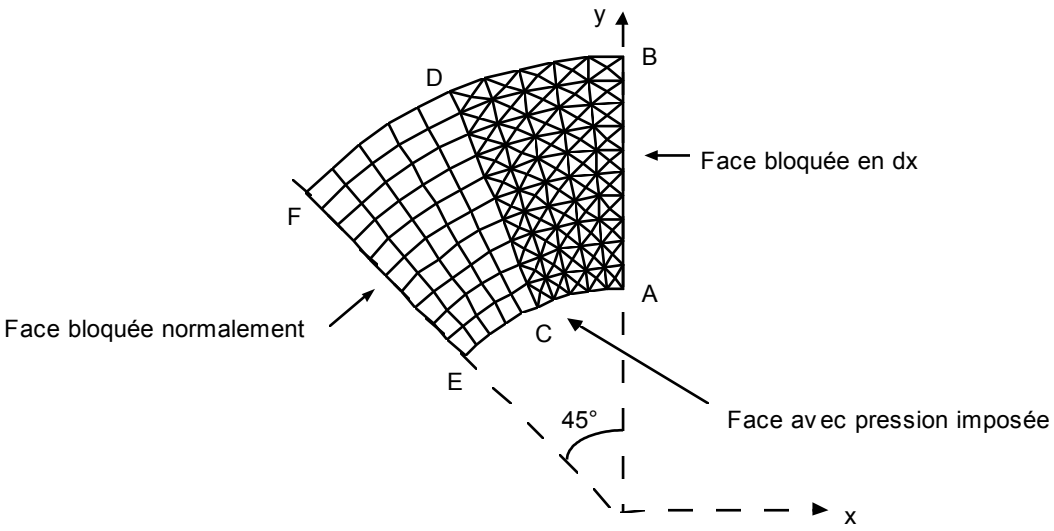
3.5 Remarks

The mesh is insufficient for linear elements.

4 Modelization B

4.1 Characteristic of the modelization

Elements $3D$ (PENTA15 and HEXA20) (resulting from the mesh $2D$ below).



along the axis Z : 2 layers of elements total thickness: 0.01

limiting Conditions:

	DDL_IMPO:	(All: "yes"	Dz: 0.)
face AB		(Group_no: BordAB	Dx: 0.)
face EF	FACE_IMPO:	(Group_ma: FaceEF	Dnor: 0.)
pressure on face AE	PRES_REP:	(Group_ma: FaceAE	Near: -1.)

Names of the nodes: A=No2 B=No361 C=No121 D=No584 E=No155 F=No503

4.2 Characteristics of the mesh

Many nodes: 2115
Number of meshes and types: 400 PENTA15 100 HEXA20 40 QUAD8

4.3 Remarks

the pressure have a negative sign (instead of positive) because the sides of the elements $3D$ are badly directed.

4.4 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A0	u	.	eps	–	
	v	7.3398 10-3	7.3326 10-3	– 0.10	10-2
	σ_{xx}	1.6685	1.6669	– 0.09	10-2
	σ_{yy}	– 1.	– 0.9959	0.41	10-2
	σ_{zz}	0.20055	0.2013	0.37	10-2
	σ_{xy}	0.	3.3234 10-3	–	10-2
C	u	– 2.80882 10-3	– 2.8063 10-3	– 0.09	10-2
	v	6.78109 10-3	6.7745 10-3	– 0.10	10-2
	σ_{xx}	1.27771	1.278	0.02	10-2
	σ_{yy}	– 0.60921	– 0.6078	0.23	10-2
	σ_{zz}	0.20055	0.20107	0.26	10-2
	σ_{xy}	0.94346	0.94027	0.34	10-2
E	u	– 5.19002 10-3	– 5.1851 10-3	– 0.09	10-2
	v	5.19002 10-3	5.1851 10-3	– 0.10	10-2
	σ_{xx}	0.33425	0.3346	0.10	10-2
	σ_{yy}	0.33425	0.3340	– 0.07	10-2
	σ_{zz}	0.20055	0.2006	0.02	10-2
	σ_{xy}	1.33425	1.331	– 0.24	10-2
B	u	0.	eps	–	
	v	4.6716 10-3	4.6682 10-3	– 0.07	10-2
	σ_{xx}	0.66738	0.6675	0.02	10-2
	σ_{yy}	0.	3.2779 10-4	–	10-2
	σ_{zz}	0.20021	0.2003	0.04	10-2
	σ_{xy}	0.	– 5.0918 10-4	–	10-2
D	u	– 1.79157 10-3	– 1.7864 10-3	– 0.29	10-2
	v	4.32523 10-3	4.3129 10-3	– 0.29	10-2
	σ_{xx}	0.56964	0.56957	– 0.01	10-2
	σ_{yy}	0.09774	0.09803	0.30	10-2
	σ_{zz}	0.20021	0.20027	0.03	10-2
	σ_{xy}	0.23595	0.23623	0.12	10-2
F	u	– 3.31039 10-3	– 3.3009 10-3	– 0.29	10-2
	v	3.31039 10-3	3.3009 10-3	– 0.29	10-2
	σ_{xx}	0.33369	0.3337	– 0.003	10-3
	σ_{yy}	0.33369	0.3337	0.003	10-3
	σ_{zz}	0.20021	0.2002	0.	10-3
	σ_{xy}	0.33369	0.3339	0.06	10-3

5 Modelization C

5.1 Characteristic of the modelization

Elements 3D (TETRA4)

along the axis Z : 2 layers of elements total thickness: 0.01

limiting Conditions:

	DDL_IMPO:	(All: "yes"	Dz: 0.)
face AB		(Group_no: BordAB	Dy: 0.)
face EF	FACE_IMPO:	(Group_ma: FaceEF	Dnor: 0.)
pressure on face AE	PRES_REP:	(Group_ma: FaceAE	Near: -1.)

Names of the nodes:	A=No3	B=No7	C=No4	D=No8	E=No154	F=No156
plane $z=0.005$	A2=No1	B2=No5	C2=No2	D2=No6	E2=No153	F2=No155
plane $z=0.01$	A3=No283	B3=No285	C3=No284	D3=No286	E3=No359	F3=No360

5.2 Characteristics of the mesh

Many nodes: 423
Number of meshes and types: 1416 TETRA4 72 TRIA3

5.3 Remarks

the pressure have a negative sign (instead of positive) because the sides of the elements $3D$ are badly directed.

5.4 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3331 10-3	– 0.10	10-2
	v	0.	eps	-	
	σ_{xx}	– 1.	– 0.9000	+10.00	0.02
	σ_{yy}	1.6685	1.6809	0.74	0.02
	σ_{zz}	0.20055	0.2343	16.83	0.02
	σ_{xy}	0.	0.1016	-	0.02
C	u	6.78109 10-3	6.7783 10-3	– 0.04	10-2
	v	2.80882 10-3	2.8077 10-3	– 0.04	
	σ_{xx}	– 0.60921	– 0.5061	16.92	0.04
	σ_{yy}	1.27771	1.3184	3.18	0.04
	σ_{zz}	0.20055	0.2437	21.51	0.04
	σ_{xy}	– 0.94346	– 0.9123	3.30	0.04
E	u	5.19002 10-3	5.1853 10-3	– 0.09	10-2
	v	5.19002 10-3	5.1853 10-3	– 0.09	
	σ_{xx}	0.33425	0.2888	– 13.60	0.5.0.5.0.5
	σ_{yy}	0.33425	0.4920	47.19	
	σ_{zz}	0.20055	0.2343	16.83	
	σ_{xy}	– 1.33425	– 1.2905	3.28	0.5
B	u	4.6716 10-3	4.6634 10-3	– 0.18	10-2
	v	0.	eps	-	
	σ_{xx}	0.	0.0146	-	
	σ_{yy}	0.66738	0.6570	– 1.55	5.10-2
	σ_{zz}	0.20021	0.1976	– 1.30	5.10-2
	σ_{xy}	0.	– 0.0159	-	5.10-2
D	u	4.32523 10-3	4.2960 10-3	– 0.68	10-2
	v	1.79157 10-3	1.7795 10-3	– 0.67	
	σ_{xx}	0.09774	0.0824	– 15.69	0.2.0.2.0.2
	σ_{yy}	0.56964	0.5809	1.97	
	σ_{zz}	0.20021	0.1921	– 4.05	
	σ_{xy}	– 0.23595	– 0.2378	– 7.84	0.2
F	u	3.31039 10-3	3.2976 10-3	– 0.39	10-2
	v	3.31039 10-3	3.2975 10-3	– 0.39	
	σ_{xx}	0.33369	0.3052	– 8.54	0.1.0.1.0.1
	σ_{yy}	0.33369	0.3371	1.02	
	σ_{zz}	0.20021	0.1921	– 4.05	
	σ_{xy}	– 0.33369	– 0.3358	– 0.63	0.1

5.5 Remarks

One notes a variation ($< 0.24\%$) displacements for the points of the plane $z = 0.005$.
The mesh is insufficient for linear elements.

6 Modelization D

6.1 Characteristic of the modelization

Elements 3D (TETRA10)

along the axis Z: 2 layers of elements total thickness: 0.01

limiting Conditions:

	DDL_IMPO:	(All: "yes"	Dz: 0.)
face AB		(Group_no: BordAB	Dy: 0.)
face EF	FACE_IMPO:	(Group_ma: FaceEF	Dnor: 0.)
pressure on face AE	PRES_REP:	(Group_ma: FaceAE	Near: -1.)

Names of the nodes:	A=No3	B=No7	C=No4	D=No8	E=No1228	F=No230
z plan = 0.005	A2=No1	B2=No5	C2=No2	D2=No6	E2=No227	F2=No229
z plan = 0.01	A3=No420	B3=No422	C3=No421	D3=No423	E3=No573	F3=No574

6.2 Characteristics of the mesh

Many nodes: 703
Number of meshes and types: 356 TETRA10 36 TRIA6

6.3 Remarks

the pressure have a negative sign (instead of positive) because the sides of the elements 3D are badly directed.

6.4 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3522 10-3	0.10	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9925	0.75	5.10-2
	σ_{yy}	1.6685	1.6725	0.24	5.10-2
	σ_{zz}	0.20055	0.2040	– 1.72	5.10-2
	σ_{xy}	0.	– 0.0365	-	5.10-2
C	u	6.78109 10-3	6.7836 10-3	0.04	10-2
	v	2.80882 10-3	2.8099 10-3	0.04	10-2
	σ_{xx}	– 0.60921	– 0.5977	1.89	5.10-2
	σ_{yy}	1.27771	1.294	1.28	5.10-2
	σ_{zz}	0.20055	0.2088	4.11	5.10-2
	σ_{xy}	– 0.94346	– 0.9457	– 0.24	5.10-2
E	u	5.19002 10-3	5.1988 10-3	0.17	10-2
	v	5.19002 10-3	5.1988 10-3	0.17	10-2
	σ_{xx}	0.33425	0.3035	– 9.20	0.15
	σ_{yy}	0.33425	0.3766	12.67	0.15
	σ_{zz}	0.20055	0.2040	1.72	0.15
	σ_{xy}	– 1.33425	– 1.332	0.17	0.15
B	u	4.6716 10-3	4.6711 10-3	– 0.01	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	8.597 10-4	-	10-2
	σ_{yy}	0.66738	0.6679	0.08	10-2
	σ_{zz}	0.20021	0.2006	0.19	10-2
	σ_{xy}	0.	1.0181 10-3	-	10-2
D	u	4.32523 10-3	4.3134 10-3	– 0.28	10-2
	v	1.79157 10-3	1.7867 10-3	– 0.28	10-2
	σ_{xx}	0.09774	0.09418	– 3.64	5.10-2
	σ_{yy}	0.56964	0.5652	– 0.78	5.10-2
	σ_{zz}	0.20021	0.1978	– 1.20	5.10-2
	σ_{xy}	– 0.23595	– 0.2355	0.19	5.10-2
F	u	3.31039 10-3	3.3029 10-3	– 0.23	10-2
	v	3.31039 10-3	3.3029 10-3	– 0.23	10-2
	σ_{xx}	0.33369	0.3357	0.60	10-2
	σ_{yy}	0.33369	0.3334	– 0.09	10-2
	σ_{zz}	0.20021	0.2007	0.24	10-2
	σ_{xy}	– 0.33369	– 0.3336	-	10-2

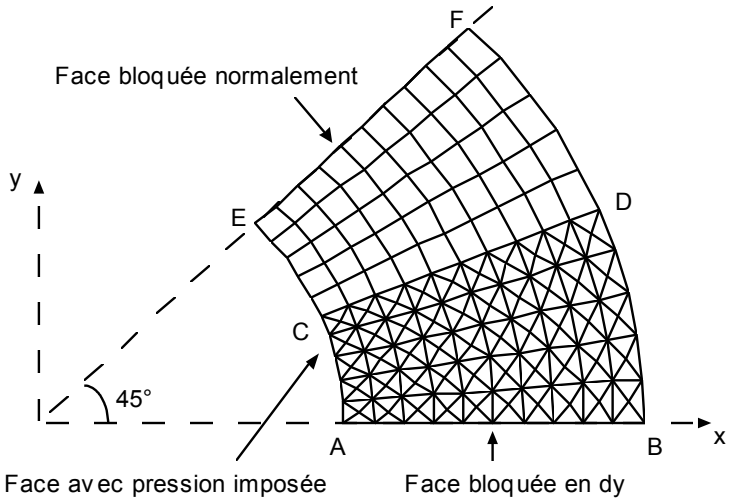
6.5 Remarks

One notes a variation ($< 0.23\%$) displacements for the points of the plane $z = 0.005$.

7 Modelization E

7.1 Characteristic of the modelization

Elements D_PLAN (TRIA3 + QUAD4)



limiting Conditions:

side AB	DDL_IMPO:	(Group_no: GRNM11	Dy: 0.)
side EF	FACE_IMPO:	(Group_ma: GRMA12	Dnor: 0.)
pressure on AE	PRES_REP:	(Group_ma: GRMA13	Near: 1.)

Names of the nodes: $A=N1$ $B=N119$ $C=N36$ $D=N166$ $E=N41$ $F=N171$

7.2 Characteristics of the mesh

Many nodes: 171
Number of meshes and types: 200 TRIA3 50 QUAD4

7.3 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3243 10-3	– 0.21	10-2
	v	0.	eps	-	
	σ_{xx}	– 1.	– 0.8790	12.10	0.15
	σ_{yy}	1.6685	1.6241	– 2.66	0.15
	σ_{zz}	0.20055	0.2235	11.44	0.15
	σ_{xy}	0.	– 0.0922	-	0.15
C	u	6.78109 10-3	6.7670 10-3	– 0.21	10-2
	v	2.80882 10-3	2.8012 10-3	– 0.27	
	σ_{xx}	– 0.60921	– 0.5122	– 15.92	0.3.0.3.0.3
	σ_{yy}	1.27771	1.3302	4.11	
	σ_{zz}	0.20055	0.2454	22.36	
	σ_{xy}	– 0.94346	– 0.8567	– 9.19	0.3
E	u	5.19002 10-3	5.1784 10-3	– 0.22	10-2
	v	5.19002 10-3	5.1784 10-3	– 0.22	
	σ_{xx}	0.33425	0.4318	29.18	0.6.0.6.0.6
	σ_{yy}	0.33425	0.5315	59.01	
	σ_{zz}	0.20055	0.2890	44.10	
	σ_{xy}	– 1.33425	– 1.2686	4.92	0.6
B	u	4.6716 10-3	4.6641 10-3	– 0.16	10-2
	v	0.	eps	-	
	σ_{xx}	0.	– 1.3198 10-2	-	0.05
	σ_{yy}	0.66738	0.6723	0.74	0.05
	σ_{zz}	0.20021	0.1977	–1.25	0.05
	σ_{xy}	0.	– 0.0219	-	0.05
D	u	4.32523 10-3	4.3084 10-3	– 0.39	10-2
	v	1.79157 10-3	1.7854 10-3	– 0.39	
	σ_{xx}	0.09774	0.07393	– 24.36	0.3.0.3.0.3
	σ_{yy}	0.56964	0.5728	0.55	
	σ_{zz}	0.20021	0.1940	– 3.10	
	σ_{xy}	– 0.23595	– 0.2347	0.53	0.3
F	u	3.31039 10-3	3.2974 10-3	– 0.39	10-2
	v	3.31039 10-3	3.2974 10-3	– 0.39	
	σ_{xx}	0.33369	0.2976	– 10.81	0.15
	σ_{yy}	0.33369	0.3245	– 2.75	0.15
	σ_{zz}	0.20021	0.1866	– 6.80	0.15
	σ_{xy}	– 0.33369	– 0.3415	– 2.34	0.15

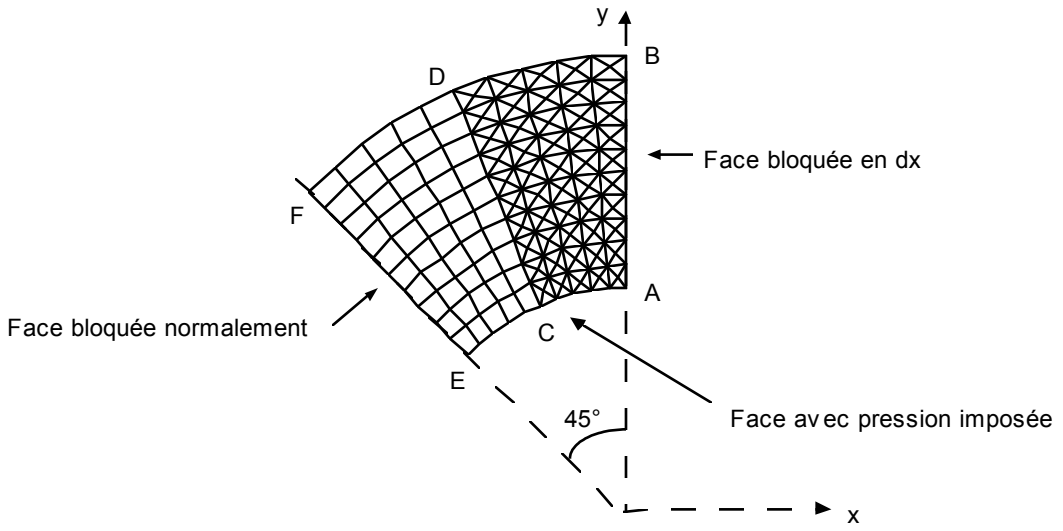
7.4 Remarks

The mesh is insufficient for linear elements.

8 Modelization F

8.1 Characteristic of the modelization

Elements D_PLAN (QUAD8 + TRIA6)



limiting Conditions:

side AB	DDL_IMPO:	(Group_no: GRNM11	Dy: 0.)
side EF	FACE_IMPO:	(Group_ma: GRMA12	Dnor: 0.)
pressure on AE	PRES_REP:	(Group_ma: GRMA13	Near: 1.)

Names of the nodes: $A=N2$ $B=N361$ $C=N121$ $D=N584$ $E=N155$ $F=N503$

8.2 Characteristics of the mesh

Many nodes: 591
Number of meshes and types: 200 TRIA6 50 QUAD8

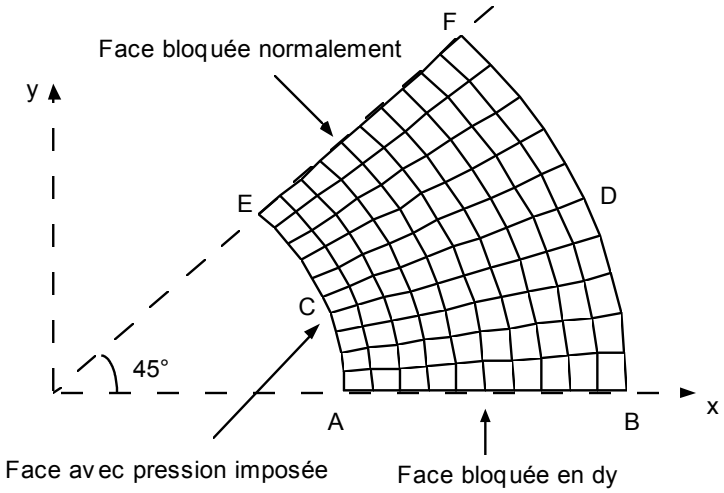
8.3 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A0	u	.	0.	-	10-2
	v	7.3398 10-3	7.3326 10-3	– 0.10	10-2
	σ_{xx}	1.6685	1.6669	0.09	10-2
	σ_{yy}	– 1.	– 0.9959	0.41	10-2
	σ_{zz}	0.20055	0.20129	0.37	10-2
	σ_{xy}	0.	0.00332	-	10-2
C	u	– 2.80882 10-3	– 2.8063 10-3	– 0.09	10-2
	v	6.78109 10-3	6.7745 10-3	– 0.10	10-2
	σ_{xx}	1.27771	1.27799	0.02	10-2
	σ_{yy}	– 0.60921	– 0.60779	0.23	10-2
	σ_{zz}	0.20055	0.20106	0.25	10-2
	σ_{xy}	0.94346	0.94027	– 0.34	10-2
E	u	– 5.19002 10-3	– 5.1851 10-3	– 0.09	10-2
	v	5.19002 10-3	5.1851 10-3	– 0.09	10-2
	σ_{xx}	0.33425	0.33462	0.11	10-2
	σ_{yy}	0.33425	0.33403	– 0.066	10-2
	σ_{zz}	0.20055	0.20059	0.02	10-2
	σ_{xy}	1.33425	1.33117	– 0.23	10-2
B	u	0.	eps	-	10-2
	v	4.6716 10-3	4.6682 10-3	– 0.07	10-2
	σ_{xx}	0.66738	0.66758	0.03	10-2
	σ_{yy}	0.	0.00033	-	10-2
	σ_{zz}	0.20021	0.20037	0.08	10-2
	σ_{xy}	0.	– 5.1132 10-4	-	10-2
D	u	– 1.79157 10-3	– 1.7865 10-3	– 0.28	10-2
	v	4.32523 10-3	4.3129 10-3	– 0.28	10-2
	σ_{xx}	0.56964	0.56962	– 0.003	10-2
	σ_{yy}	0.09774	0.09805	0.32	10-2
	σ_{zz}	0.20021	0.200298	0.044	10-2
	σ_{xy}	0.23595	0.23623	0.12	10-2
F	u	– 3.31039 10-3	– 3.3009 10-3	– 0.29	10-2
	v	3.31039 10-3	3.3009 10-3	– 0.29	10-2
	σ_{xx}	0.33369	0.33371	0.006	10-2
	σ_{yy}	0.33369	0.33366	– 0.009	10-2
	σ_{zz}	0.20021	0.20021	0.	10-2
	σ_{xy}	0.33369	0.33392	0.069	10-2

9 Modelization G

9.1 Characteristic of modelization

D_PLAN (QUAD9)



limiting Conditions:

side AB	DDL_IMPO:	(Group_no: GRNM11	Dy: 0.)
side EF	FACE_IMPO:	(Group_ma: GRMA12	Dnor: 0.)
pressure on AE	PRES_REP:	(Group_ma: GRMA13	Near: 1.)

Names of the nodes: $A=N1$ $B=N347$ $C=N21$ $D=N432$ $E=N39$ $F=N229$

9.2 Characteristics of the mesh

Many nodes: 441
Number of meshes and types: 100 QUAD9

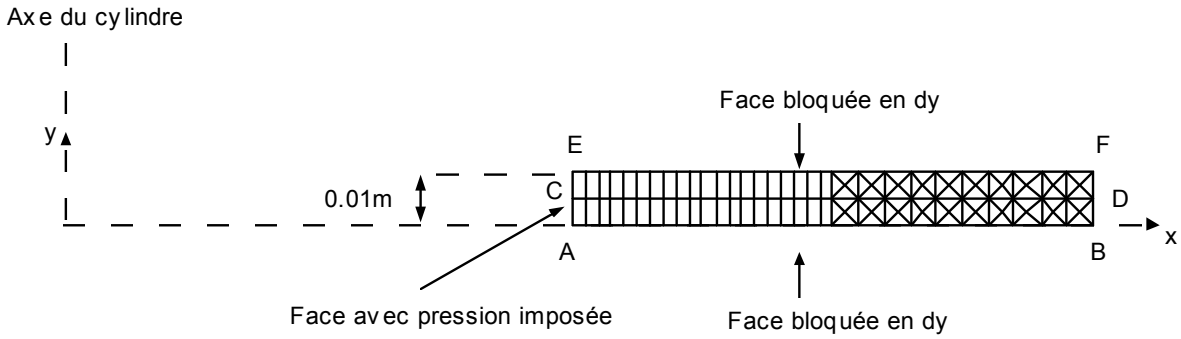
9.3 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3329 10-3	– 0.09	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9968	0.32	10-2
	σ_{yy}	1.6685	1.6655	– 0.18	10-2
	σ_{zz}	0.20055	0.20059	0.02	10-2
	σ_{xy}	0.	– 2.97 10-4	-	10-2
C	u	6.78109 10-3	6.7747 10-3	– 0.09	10-2
	v	2.80882 10-3	2.8062 10-3	– 0.09	10-2
	σ_{xx}	– 0.60921	– 0.60695	0.37	10-2
	σ_{yy}	1.27771	1.27563	– 0.16	10-2
	σ_{zz}	0.20055	0.20060	0.02	10-2
	σ_{xy}	– 0.94346	– 0.94128	– 0.23	10-2
E	u	5.19002 10-3	5.1851 10-3	– 0.09	10-2
	v	5.19002 10-3	5.1851 10-3	– 0.09	10-2
	σ_{xx}	0.33425	0.33403	– 0.06	10-2
	σ_{yy}	0.33425	0.33463	0.11	10-2
	σ_{zz}	0.20055	0.20059	0.02	10-2
	σ_{xy}	– 1.33425	– 1.33117	0.23	10-2
B	u	4.6716 10-3	4.6682 10-3	– 0.07	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	– 2.394 10-3	-	10-2
	σ_{yy}	0.66738	0.66759	0.03	10-2
	σ_{zz}	0.20021	0.200207	– 0.001	10-2
	σ_{xy}	0.	– 2.65 10-5	-	10-2
D	u	4.32523 10-3	4.3128 10-3	– 0.29	10-2
	v	1.79157 10-3	1.7864 10-3	– 0.29	10-2
	σ_{xx}	0.09774	0.09756	– 0.18	10-2
	σ_{yy}	0.56964	0.56979	0.02	10-2
	σ_{zz}	0.20021	0.200206	– 0.002	10-2
	σ_{xy}	– 0.23595	– 0.23611	– 0.07	10-2
F	u	3.31039 10-3	3.3009 10-3	– 0.29	10-2
	v	3.31039 10-3	3.3009 10-3	– 0.29	10-2
	σ_{xx}	0.33369	0.33366	– 0.009	10-2
	σ_{yy}	0.33369	0.33371	0.006	10-2
	σ_{zz}	0.20021	0.20021	0.	10-2
	σ_{xy}	– 0.33369	– 0.33392	– 0.07	10-2

10 Modelization H

10.1 Characteristic of the modelization

Elements axis (TRIA3 + QUAD4)



limiting Conditions:

side AB	DDL_IMPO:	(Group_no: GRNM11	Dy: 0.)
side EF	FACE_IMPO:	(Group_ma: GRMA12	Dnor: 0.)
pressure on AE	PRES_REP:	(Group_ma: GRMA13	Near: 1.)

Names of the nodes: $A=N111$ $B=N1$ $C=N112$ $D=N3$ $E=N113$ $F=N4$

10.2 Characteristics of the mesh

Many nodes: 113
Number of meshes and types: 40 QUAD4 80 TRIA3

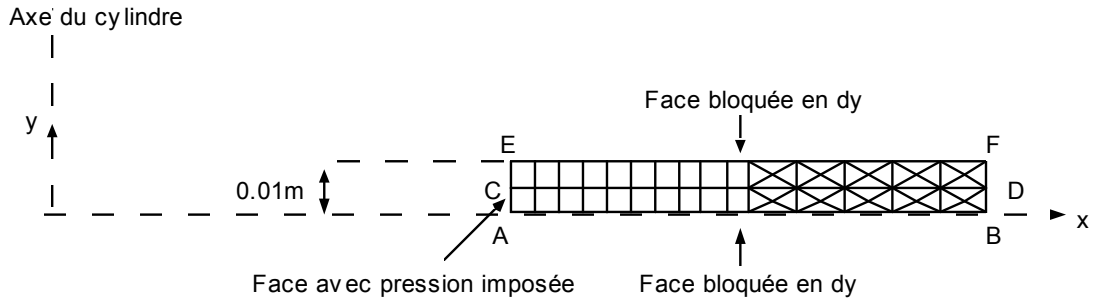
10.3 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3390 10-3	– 0.01	10-2
	v	0.	eps	-	
	σ_{xx}	– 1.	– 0.9430	– 5.72	0.2.0.2.0.2
	σ_{yy}	0.20055	0.2248	12.19	
	σ_{zz}	1.6685	1.6923	1.46	
	σ_{xy}	0.	eps	-	0.2
C	u	7.3398 10-3	7.3390 10-3	– 0.01	10-2
	v	0.	eps	-	
	σ_{xx}	– 1.	– 0.9430	– 5.72	0.2.0.2.0.2
	σ_{yy}	0.20055	0.2248	12.19	
	σ_{zz}	1.6685	1.6923	1.46	
	σ_{xy}	0.	eps	-	0.2
E	u	7.3398 10-3	7.3390 10-3	– 0.01	10-2
	v	0.	0·	-	
	σ_{xx}	– 1.	– 0.9430	– 5.72	0.2.0.2.0.2
	σ_{yy}	0.20055	0.2248	12.19	
	σ_{zz}	1.6685	1.6923	1.46	
	σ_{xy}	0.	eps	-	0.2
B	u	4.6716 10-3	4.6713 10-3	– 0.01	10-2
	v	0.	eps	-	
	σ_{xx}	0.	– 0.0110	-	0.05
	σ_{yy}	0.20021	0.1954	– 2.35	0.05
	σ_{zz}	0.66738	0.6625	– 0.72	0.05
	σ_{xy}	0.	– 0.0011	-	0.05
D	u	4.6716 10-3	4.6713 10-3	– 0.01	10-2
	v	0.	eps	-	
	σ_{xx}	0.	– 0.0110	-	0.05
	σ_{yy}	0.20021	0.1954	– 2.35	0.05
	σ_{zz}	0.66738	0.6625	– 0.72	0.05
	σ_{xy}	0.	eps	-	0.05
F	u	4.6716 10-3	4.6713 10-3	– 0.01	10-2
	v	0.	eps	-	
	σ_{xx}	0.	– 0.0110	-	0.05
	σ_{yy}	0.20021	0.1954	– 2.35	0.05
	σ_{zz}	0.66738	0.6625	– 0.72	0.05
	σ_{xy}	0.	+0.0011	-	0.05

11 Modelization I

11.1 Characteristic of modelization

Elements axis (TRIA6 + QUAD8)



limiting Conditions:

side AB	DDL_IMPO:	(Group_no: GRNM11	Dy: 0.)
side EF	FACE_IMPO:	(Group_ma: GRMA12	Dnor: 0.)
pressure on AE	PRES_REP:	(Group_ma: GRMA13	Near: 1.)

Names of the nodes: $A=N8$ $B=N174$ $C=N5$ $D=N170$ $E=N3$ $F=N159$

11.2 Characteristics of the mesh

Many nodes: 175
Number of meshes and types: 20 QUAD8 40 TRIA6

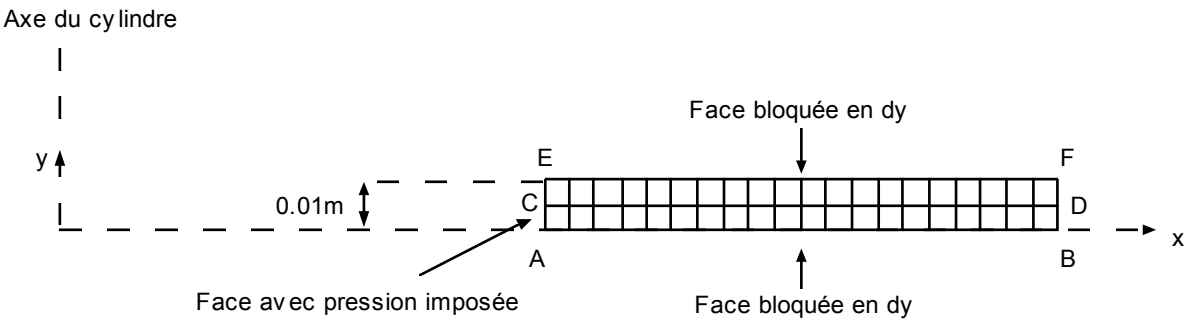
11.3 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3397 10-3	– 0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9984	– 0.16	10-2
	σ_{yy}	0.20055	0.20055	-	10-2
	σ_{zz}	1.6685	1.669	– 0.57	10-2
	σ_{xy}	0.	eps	-	10-2
C	u	7.3398 10-3	7.3397 10-3	– 0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9984	– 0.16	10-2
	σ_{yy}	0.20055	0.20055	-	10-2
	σ_{zz}	1.6685	1.669	– 0.57	10-2
	σ_{xy}	0.	eps	-	10-2
E	u	7.3398 10-3	7.3397 10-3	– 0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9984	– 0.16	10-2
	σ_{yy}	0.20055	0.20055	-	10-2
	σ_{zz}	1.6685	1.669	– 0.57	10-2
	σ_{xy}	0.	eps	-	10-2
B	u	4.6716 10-3	4.6716 10-3	0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	3.8 10-4	-	10-2
	σ_{yy}	0.20021	0.2002	-	10-2
	σ_{zz}	0.66738	0.66716 10-5	– 0.03	10-2
	σ_{xy}	0.		-	10-2
D	u	4.6716 10-3	4.6716 10-3	0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	3.8 10-4	-	10-2
	σ_{yy}	0.20021	0.2002	-	10-2
	σ_{zz}	0.66738	0.66716	– 0.03	10-2
	σ_{xy}	0.	eps	-	10-2
F	u	4.6716 10-3	4.6716 10-3	0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	3.8 10-4	-	10-2
	σ_{yy}	0.20021	0.2002	-	10-2
	σ_{zz}	0.66738	0.66716	– 0.03	10-2
	σ_{xy}	0.	10-5	-	10-2

12 Modelization J

12.1 Characteristic of the modelization

Elements axis (QUAD9)



limiting Conditions:

side AB	DDL_IMPO:	(Group_no: GRNM11	Dy: 0.)
side EF	FACE_IMPO:	(Group_ma: GRMA12	Dnor: 0.)
pressure on AE	PRES_REP:	(Group_ma: GRMA13	Near: 1.)

Names of the nodes: $A = N196$ $B = N1$ $C = N200$ $D = N5$ $E = N202$ $F = N7$

12.2 Characteristics of the mesh

Many nodes: 205
Number of meshes and types: 40 QUAD9

12.3 Quantities tested and results

Localization	Quantities	Reference	*Aster*	% difference	tolerance
A	u	7.3398 10-3	7.3397 10-3	– 0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9984	+0.16	10-2
	σ_{yy}	0.20055	0.2005	-	10-2
	σ_{zz}	1.6685	1.667	– 0.57	10-2
	σ_{xy}	0.	eps	-	10-2
C	u	7.3398 10-3	7.3397 10-3	– 0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9984	+0.16	10-2
	σ_{yy}	0.20055	0.2005	-	10-2
	σ_{zz}	1.6685	1.667	– 0.57	10-2
	σ_{xy}	0.	eps	-	10-2
E	u	7.3398 10-3	7.3397 10-3	– 0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	– 1.	– 0.9984	+0.16	10-2
	σ_{yy}	0.20055	0.2005	-	10-2
	σ_{zz}	1.6685	1.667	– 0.57	10-2
	σ_{xy}	0.	eps	-	10-2
B	u	4.6716 10-3	4.6716 10-3	0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	1.1 10-4	-	10-2
	σ_{yy}	0.20021	0.20021	-	10-2
	σ_{zz}	0.66738	0.66727	– 0.04	10-2
	σ_{xy}	0.	eps	-	10-2
D	u	4.6716 10-3	4.6716 10-3	0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	1.1 10-4	-	10-2
	σ_{yy}	0.20021	0.20021	-	10-2
	σ_{zz}	0.66738	0.66727	– 0.04	10-2
	σ_{xy}	0.	eps	-	10-2
F	u	4.6716 10-3	4.6716 10-3	0.00	10-2
	v	0.	eps	-	10-2
	σ_{xx}	0.	1.1 10-4	-	10-2
	σ_{yy}	0.20021	0.20021	0.02	10-2
	σ_{zz}	0.66738	0.66727	– 0.04	10-2
	σ_{xy}	0.	eps	-	10-2

13 Summary of the Summary

results of errors		3D			D_PLAN			Axis		
max in %	MOD A	MOD B	MOD C	MOD D	MOD E	MOD F	MOD G	MOD H	MOD I	MOD J
Displacements										
A, C, E B, D, F	– 0.27	– 0.10	– 0.10	0.17	– 0.27	– 0.10	– 0.09	– 0.01	0.00	– 0.00
	– 0.39	– 0.02	– 0.68	– 0.28	– 0.39	– 0.29	– 0.29	– 0.01	0.00	– 0.00
Stresses	σ_{xx}									
A, C, E B, D, F	29.23	0.10	16.92	– 9.20	29.18	0.11	0.37	5.72	0.27	0.27
	– 24.39	0.02	– 15.69	– 3.64	– 24.36	0.03	– 0.18	-	-	-
Stresses	σ_{yy}									
A, C, E B, D, F	59.04	0.41	47.19	12.67	59.01	0.41	– 0.18	12.19	0.09	0.09
	2.75	0.30	1.97	– 0.78	– 2.75	0.32	0.03	– 2.35	0.09	0.02
Stresses	σ_{zz}									
A, C, E B, D, F	44.50	0.37	21.51	4.11	44.10	0.37	0.02	1.46	0.57	0.57
	– 6.80	0.04	– 4.05	– 1.20	– 6.80	0.08	– 0.001	– 0.72	0.03	0.04
Stresses	σ_{xy}									
A, C, E B, D, F	9.20	0.34	3.30	– 0.24	– 9.19	– 0.34	0.23	-	-	-
	– 2.34	0.12	– 7.84	0.19	– 2.34	0.12	– 0.07	-	-	them

- meshes for the elements of order 1 are not fine enough.
- The results are more precise with elements of order 2.
- The problem is adapted more to an axisymmetric modelization (H, I, J) - > better results.
- The results of the elements 3D and the plane elements having spaces of interpolation in correspondence are identical.
- The results of axisymmetric elements QUAD8 and QUAD9 are identical.