# SHLL101 - Straight beam. Harmonic analysis

**Summarized:**

This two-dimensional problem consists in calculating the forces present in a beam subjected to a tension or a bending during a harmonic analysis. The reference solution is obtained starting from the discretized equations.

This test comprises two modelizations.

For the first modelization, four requests are tested:

- tensile force,
- tensile force and material presenting a damping,
- strength flexural,
- flexural strength and material presenting a damping.

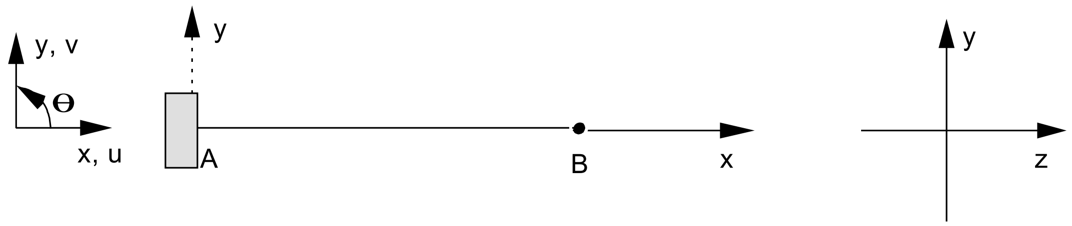
For the second modelization, two requests are tested:

- tensile force,
- tensile force and material presenting a damping.

The second modelization makes it possible to test complex loadings imposed by the command `AFFE_CHAR_MECA_C`.

# 1 Constituting problem of

## 1.1 reference




Geometry the geometrical characteristics of the beam the model mechanical are the following ones:

Length: $L=10\,m$

| Cross section | Area | $IZ=IY$ | $JX$ |
|---|---|---|---|
| | $3.439\,10^{-3}\,m^2$ | $1.377\,10^{-5}\,m^4$ | $2.754\,10^{-5}\,m^4$ |

the coordinates (in meters) of the points characteristic of the beam are:

| | $A$ | $B$ |
|---|---|---|
| $x$ | 0. | 10. |
| $y$ | 0. | 0. |

## 1.2 Material properties

the properties of the material constituting the beam are:

$E=1.658\,1010\,Pa$

$\nu=0.3$

$\rho=1.3404106\,104^{kg}/m^3$

$\alpha=\mathrm{AMOR\_ALPHA}=0.001$

$\beta=\mathrm{AMOR\_BETA}=0.$

## 1.3 Boundary conditions and loadings

the boundary condition which characterizes this problem is the fixed support of the point $A$ and is written:

$u=v=0.$

$\theta=0.$

For the loading one a:

| | | | |
|---|---|---|---|
| $Fx=3000.\,N$ | | $Fy=Fz=0.$ | (tractive effort) |
| $Fx=0.$ | $Fy=3000.\,N$ | $Fz=0.$ | (bending stress) |

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

If the beam is modelled by a beam of Eulerian-Bernoulli and only one finite element, the harmonic problem can be written in the following way:

**problem in tension:**

$$(1+i\alpha\omega)\frac{ES}{L}u(B)-\omega^2\frac{\rho S L}{6}u(B)=F_x(B)$$

from where $u(B)=\dfrac{F(B)}{\dfrac{ES}{L}-\omega^2\dfrac{\rho S L}{6}+i\alpha\omega\dfrac{ES}{L}}$

**problem in bending:**

$$\left[-\omega^2\begin{pmatrix}\frac{13L}{35} & \frac{-11L^2}{210}\\ \frac{-11L^2}{210} & \frac{L^3}{105}\end{pmatrix}+(1+i\alpha\omega)\frac{12EI_y}{L^3}\begin{pmatrix}1 & \frac{-L}{2}\\ \frac{-L}{2} & \frac{L^2}{3}\end{pmatrix}\right]\begin{pmatrix}v(B)\\ \theta(B)\end{pmatrix}=\begin{pmatrix}F_y(B)\\ 0\end{pmatrix}$$

**Note:**

*If the material does not present damping, one has then:* AMOR_ALPHA = $\alpha=0$ .

The forces at the point $B$ are calculated in the following way:

**problem in tension:**

$$N(B)=\left(\frac{ES}{L}-\omega^2\frac{\rho S L}{6}\right)u(B)$$

**problem in bending:**

$$\begin{pmatrix}VY(B)\\ MFZ(B)\end{pmatrix}=\left[-\omega^2\begin{pmatrix}\frac{13L}{35} & \frac{-11L^2}{210}\\ \frac{-11L^2}{210} & \frac{L^3}{105}\end{pmatrix}+\frac{12EI_y}{L^3}\begin{pmatrix}1 & \frac{-L}{2}\\ \frac{-L}{2} & \frac{L^2}{3}\end{pmatrix}\right]\begin{pmatrix}v(B)\\ \theta(B)\end{pmatrix}$$

One analytically solves the systems $2\times2$ to obtain the solution.

## 2.2 Results of reference

the results of reference are displacements, the velocities, accelerations and the generalized forces obtained at the point $B$ during the harmonic analysis.

## 2.3 Notice for the modelization B

For the modelization B, one wants to test in the case of the problem in tension the key word `FORCE_POUTRE` which makes it possible to apply forces distributed. To obtain the same solution as the beam subjected to nodal force in its end, the relation between the force distributed constant and the nodal force are:

$$F_x(B)=\frac{f\,L}{2}$$

With the values given to paragraph 1.3, one a: $f=600\,N/m$

## 2.4 Uncertainty on the solution

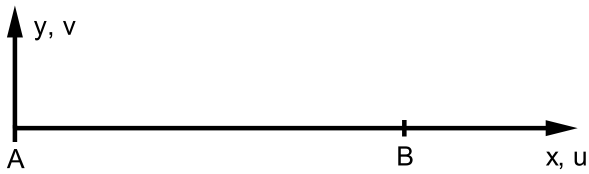
If the assumptions are checked (beam of Eulerian-Bernoulli), the solution is analytical.

## 2.5 Bibliographical references

1) Documentation of reference of *the Code_Aster* : “Exact” beam elements (right and curved) - [R3.08.01].

# 3 Modelization A

## 3.1 Characteristic of the modelization




the beam consists of only one mesh.

The modelization used for the beam is that of Eulerian-Bernoulli (POU_D_E).

The end $A$ is clamped:

DX = DY = DZ = 0.DRX = DRY = DRZ = 0.

## 3.2 Characteristics of the mesh

Many nodes: 2

Number of meshes and types: 1 mesh of the type SEG2

the points characteristic of the mesh are the following:

Not $A$ = $A$
Not $B$ = $B$

## 3.3 Quantities tested (reality-imaginary form)

**Problem 1: tension**

| Not/Quantity | | | Reference | Aster | % difference |
|---|---|---|---|---|---|
| displacement | B | DX | (5.318 10-5, 0.) | (5.318 10-5, 0.) | 0. |
| velocity | B | DX | (0. , 3.341 10-3) | (0. , 3.341 10-3) | 0. |
| acceleration | B | DX | (– 2.099 10-1, 0.) | (– 2.099 10-1, 0.) | 0. |
| force generalized | B | N | (3000. , 0.) | (3000. , 0.) | 0. |

**Problem 2: bending**

| Not/Quantity | | | Reference | Aster | % difference |
|---|---|---|---|---|---|
| displacement | B | DY | (1.828 10-2, 0.) | (1.828 10-2, 0.) | 0. |
| | | DRZ | (1.82 10-2, 0.) | (1.82 10-2, 0.) | 0. |
| velocity | B | DY | (0. , 1.1489) | (0. , 1.1489) | 0. |
| | | DRZ | (0. , 1.1438) | (0. , 1.1438) | 0. |
| acceleration | B | DY | (– 7.219 10-1) | (– 7.219 10-1, 0.) | 0. |
| | | DRZ | (– 7.186 10-1, 0.) | (– 7.186 10-1, 0.) | 0. |
| force generalized | B | VY | (3000. , 0.) | (3000. , 0.) | 0. |
| | | MFZ | (0. , 0.) | (– 1.164 10-10, 0.) | 0. |

**Problem 3: tension + damping**

| Not/Quantity | | | Reference | Aster | % diff |
|---|---|---|---|---|---|
| displacement | B | DX | (5.296 10-5, – 3.363 10-3) | (5.296 10-5, – 3.363 10-3) | 0. |
| velocity | B | DX | (2.113 10-4, 3.327 10-3) | (2.113 10-4, 3.327 10-3) | 0. |
| acceleration | B | DX | (– 2.091 10-1, 1.327 10-2) | (– 2.091 10-1, 1.327 10-2) | 0. |
| force generalized | B | N | (2.987 103, – 1.8975 102) | (2.987 103, – 1.8975 102) | 0. |

**Problem 4: bending + damping**

| Not/Quantity | | | Reference | Aster | % diff |
|---|---|---|---|---|---|
| displacement | B | DY | (1.746 10-2, – 4.469 10-3) | (1.746 10-2, – 4.469 10-3) | 0. |
| | | DRZ | (1.757 10-2, – 3.402 10-3) | (1.757 10-2, – 3.402 10-3) | 0. |
| velocity | B | DY | (2.808 10-1, 1.097) | (2.808 10-1, 1.097) | 0. |
| | | DRZ | (2.138 10-1, 1.104) | (2.138 10-1, 1.104) | 0. |
| acceleration | B | DY | (– 6.895 10-1, 1.764 10-1) | (– 6.895 10-1, 1.764 10-1) | 0. |
| | | DRZ | (– 6.94 10-1, 1.343 10-1) | (– 6.94 10-1, 1.343 10-1) | 0. |
| force generalized | B | VY | (3.021 103,1.212 102) | (3.021 103,1.212 102) | 0. |
| | | MFZ | (– 1.567 102, – 8.583 102) | (– 1.567 102, – 8.583 102) | 0. |

# 4 Modelization B

## 4.1 Characteristic of the modelization


the beam consists of only one mesh.

The modelization used for the beam is that of Eulerian-Bernoulli (POU_D_E).

The end $A$ is clamped:

DX = DY = DZ = 0.DRX = DRY = DRZ = 0.

## 4.2 Characteristics of the mesh

Many nodes: 2

Number of meshes and types: 1 mesh of the type SEG2

the points characteristic of the mesh are the following:

Not $A$ = $A$
Not $B$ = $B$

## 4.3 Quantities tested (reality-imaginary form)

**Problem 1: tension (force distributed real: imaginary part null)**

| Not/Quantity | | | Reference | *Aster* | % difference |
|---|---|---|---|---|---|
| displacement | B | DX | (5.318 10-5, 0.) | (5.318 10-5, 0.) | 0. |
| velocity | B | DX | (0. , 3.341 10-3) | (0. , 3.3414 10-3) | 0. |
| acceleration | B | DX | (– 2.099 10-1, 0.) | (– 2.0994 10-1, 0.) | 0. |
| force generalized | B | N | (3000. , 0.) | (3000. , 0.) | 0. |

**Problem 2: tension (force distributed complex: rélle part null)**

| Not/Quantity | | | Reference | *Aster* | % difference |
|---|---|---|---|---|---|
| displacement | B | DX | (0. , 5.318 10-5) | (0. , 5.318 10-5) | 0. |
| velocity | B | DX | (- 3.341 10-3 , 0.) | (- 3.3414 10-3 , 0.) | 0. |
| acceleration | B | DX | (0. , – 2.099 10-1) | (0. , – 2.0994 10-1) | 0. |
| force generalized | B | N | (0. , 3000.) | (0. , 3000.) | 0. |

**Problem 3: tension + damping (force distributed real: imaginary part null)**

| Not/Quantity | | | Reference | *Aster* | % diff |
|---|---|---|---|---|---|
| displacement | B | DX | (5.296 10-5, – 3.363 10-3) | (5.2966 10-5, – 3.3637 10-3) | 0. |
| velocity | B | DX | (2.113 10-4, 3.327 10-3) | (2.1135 10-4, 3.3279 10-3) | 0. |
| acceleration | B | DX | (– 2.091 10-1, 1.327 10-2) | (– 2.091 10-1, 1.3279 10-2) | 0. |
| force generalized | B | N | (2.9879 103, – 1.897 102) | (2.987 103, – 1.8975 102) | 0. |

**Problem 4: bending + damping (force distributed complex: real part null)**

| Not/Quantity | | | Reference | *Aster* | % diff |
|---|---|---|---|---|---|
| displacement | B | DX | (3.363 10-3 , 5.296 10-5) | (5.296 10-5, – 3.363 10-3) | 0. |
| velocity | B | DX | (- 3.327 10-3 , 2.113 10-4) | (- 3.3279 10-3 , 2.1135 10-4) | 0. |
| acceleration | B | DX | (- 1.327 10-2, -2.091 10-1) | (- 1.3279 10-2, -2.091 10-1) | 0. |
| force generalized | B | N | (1.897 102 , 2.9879 103) | (1.8975 102 , 2.98794 103) | 0. |

When the force distributed is applied as an imaginary part of the loading, the reference solution is obtained from that of real modelization A while exchanging left and imaginary part and by changing the sign of the new real parts.

# 5 Summary of the results

One finds the analytical results well.