SDLV131 - Simulation of a strain gauge by the command OBSERVATION

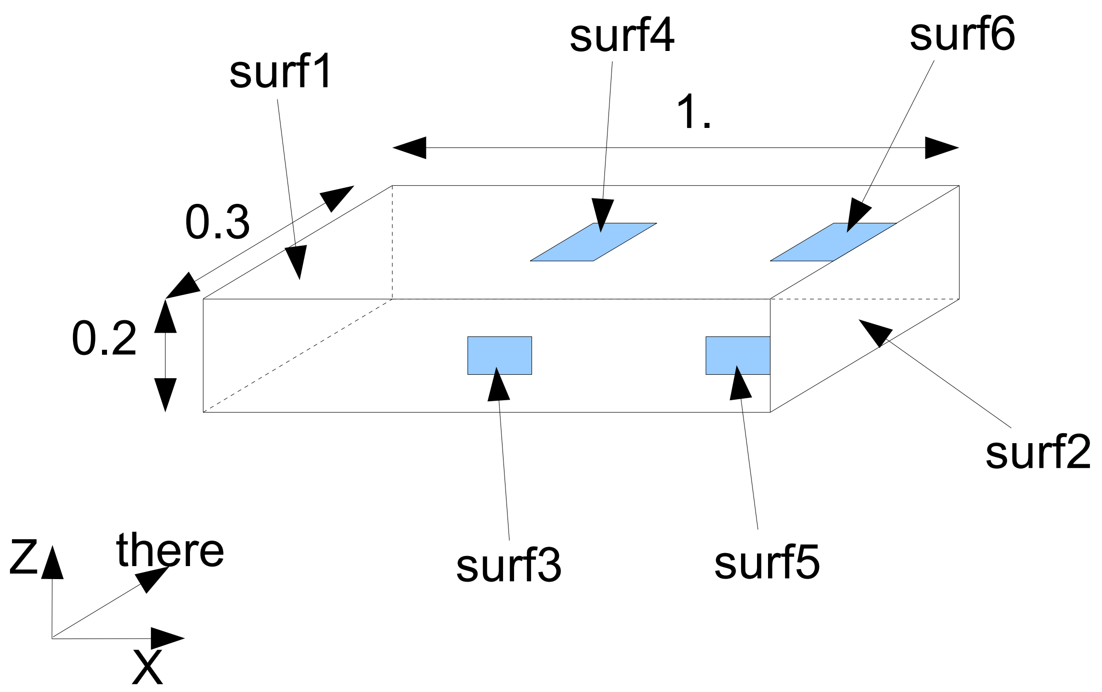
Summarized:

This test validates the operation of the computation of the mean value of a strain field on an entity given by the user. The strain field thus is estimated that a strain gauge would have measured. One carries out this computation via macro-command `OBSERVATION`. The treated case is a beam in simple tension modelled by voluminal elements.

This case test also validates the observation of mixed fields: only one call to command `OBSERVATION` for the statement of fields of different nature (`DEPL`, `QUICKLY`,…).

1 Problem of reference

1.1 Geometry



1.2 Properties of the material

Young's modulus: $E=2.1 10^{11} N/m^2$
Poisson's ratio: $\nu=0.3$
Density: $\rho=7800. kg/m^3$

1.3 Boundary conditions and loadings

One imposes a horizontal displacement $u_x=0.0$ on the face $surf1$.
One imposes a displacement $u_y=0.0$ on the nodes which are on line central upper face and the line central one of the lower face.
One imposes a displacement $u_z=0.0$ on the nodes which are on the central lines of the side sides.

One applies a surface force F_B to the face $surf2$ according to the direction x

$F_B=1000. N/m^2$,

These boundary conditions allow to obtain a behavior of the beam in simple tension.

1.4 Initial conditions

Without Reference solution

2 object

2.1 Method of calculating

the strain is estimated from the relative lengthening of the beam.

The lengthening of a beam length L following a longitudinal force F is written:

$$\Delta L=\frac{F L}{E S}$$

In our case, one applies a force per unit of area F_B at the loose lead of the beam, thus the relative lengthening of the beam puts oneself in the following form:

$$\frac{\Delta L}{L}=\epsilon_{xx}=\frac{F_B}{E}$$

For this case test, one computation results resulting from a static computation, a harmonic computation, a transient computation and of a modal computation.

For the static case, one obtains:

$$\varepsilon_{xx}=\frac{F_B}{E} \quad \text{and} \quad \varepsilon_{yy}=\varepsilon_{zz}=-\nu\,\varepsilon_{xx}$$

For the dynamic cases, the system is governed by the following equation:

$$M\frac{\partial^2 u}{\partial t^2}+K\,u=F_{ext}$$

For a beam in traction and compression, if one considers a model which contains one element, the mass matrixes M and of stiffness K put oneself in the following form:

$$M=\frac{\rho S L}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \qquad K=\frac{ES}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \qquad \text{with:} \quad u=\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

u_1 and u_2 are displacements of the nodes of the element.

In harmonic response of pulsation ω , displacements of the nodes of the element are governed by the following relation:

$$\frac{ES}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}-\omega^2\frac{\rho S L}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

By exploiting the second-row forward of this relation, and by applying the boundary conditions ($u_1=0$ and $F_2=F=F_B S$), one obtains:

$$u_2 = \frac{F_B}{\frac{E}{L} - \omega^2 \frac{\rho L}{3}} = \Delta L$$

The strain at the loose lead of the beam is written:

$$\varepsilon_{xx} = \frac{\Delta L}{L} = \frac{F_B}{E - \omega^2 \frac{\rho L^2}{3}} \quad \text{and} \quad \varepsilon_{yy} = \varepsilon_{zz} = -\nu\, \varepsilon_{xx}$$

the reference solution for the transitory solution can be obtained same way. If one applies a longitudinal force $F(t) = F\,t = F_B S\,t$ with an initial condition of beam in equilibrium (initial displacement no one and initial velocity null), one obtains:

$$u_2(t) = \frac{3 F_B}{\rho\, L \omega_0^2} \left[t - \frac{\sin(\omega_0 t)}{\omega_0} \right] = \Delta L \quad \text{with:} \quad {\omega_0}^2 = \frac{3E}{\rho L^2}$$

And the strain at the loose lead of the beam is written:

$$\varepsilon_{xx} = \frac{\Delta L}{L} = \frac{3 F_B}{\rho L^2 \omega_0^2} \left[t - \frac{\sin(\omega_0 t)}{\omega_0} \right] \quad \text{and} \quad \varepsilon_{yy} = \varepsilon_{zz} = -\nu\, \varepsilon_{xx}$$

In the case of modal computation, one carries out a test of NON-regression on the strains calculated at the point medium of the beam.

One also simulates a rotation of 90 degrees in order to check the change of reference in `OBSERVATION`.

2.2 Quantities and results of reference

One tests the value of the average strain on surfaces: $surf3$ $surf4$, $surf5$ and $surf6$. The got results are then projected to “measure” which understands only the nodes $P3$ $P4$, $P5$ and $P6$ are the model associated with surfaces $surf3$ $surf4$, $surf5$ and $surf6$.

For the validation of the static solution, one has chooses: $F_B = 1000.\, N/m^2$

For the validation of the harmonic solution, one chose: $F_B = 1000.(1+2\mathrm{j})\, N/m^2$ and: $\omega = 2\pi 200\, rd\, s^{-1}$

For the validation of the transitory solution, one chose: $F_B = 1000.\, t\, N/m^2$ and the solution is tested at time $t = 1\, s$

One also tests the values of the fields obtained by mixed observation.

2.3 Uncertainties on the analytical

solution Solution for the static case, the harmonic case and the transitory case.

One suggests a solution of NON-regression in the case of the modal strain.

2.4 Bibliographical reference

[R3.08.01] "exact" Elements of beams (right and curved).

3 Modelization A

3.1 Characteristic of the modelization

One calculates the average strain resulting from a static computation of response with `MECA_STATIQUE`.

One also calls upon operator `OBSERVATION` for the statements of fields of displacement and strain.

3.2 Characteristics of the mesh

Nodes: 1029
Meshes: 720 `HEXA8`

3.3 Quantities tested and results

One tests the value of the nodal deformation who are in the middle of the beam.

Identification	Reference	Aster	Tolerance
ε_{xx} in $P3$	4.7619047619 048D-09	4.7619047619 047D-09	0.1%
ε_{zz} in $P3$	-1.428571428 5714D-09	-1.428571428 5714D-09	0.1%
ε_{yy} in $P4$ (after rotation of $90°$)	4.7619047619 048D-09	4.7619047619 047D-09	0.1%
ε_{xx} in $P4$ (after rotation of $90°$)	-1.428571428 5714D-09	-1.428571428 5714D-09	0.1%

Tests for the mixed observation:

Identification	Reference	Aster	Tolerance
ε_{xx} in $P3$	4.7619047619 048D-09	4.7619047619 047D-09	0.1%
DX in $P5$ (m)	4.7619047619 048D-09	4.5238095238 095D-09	6.0%

4 Modelization B

4.1 Characteristic of the modelization

One calculates the average strain resulting from a harmonic computation.
One chooses an excitation frequency equalizes with $200\,Hz$.

One also calls upon operator OBSERVATION for the statements of fields of displacement and strain.

4.2 Characteristics of the mesh

Nodes: 1029
Meshes: 720 HEXA8

4.3 Quantities tested and results

One tests the value of the nodal deformation who are at the loose lead of the beam.

Identification	Reference	*Aster*	Tolerance
ε_{xx} in $P5$	4.8568623280004D-09 + i9.7137246560009D-09	4.7813760355714D-09 + i9.5627520711429D-09	2.0%
ε_{zz} in $P5$	-1.4570586984001D-09 - i2.9141173968003D-09	-1.4339303380850D-09 - i2.8678606761701D-09	2.0%
ε_{yy} in $P6$ (after rotation of $90°$)	4.8568623280004D-09 + i9.7137246560009D-09	4.7792223250189D-09 + i9.5584446500377D-09	2.0%
ε_{xx} in $P6$ (after rotation of $90°$)	-1.4570586984001D-09 - i2.9141173968003D-09	-1.4339110329539D-09 - i2.8678220659079D-09	2.0%

Tests for the mixed observation:

Identification	Reference	*Aster*	Tolerance
ε_{xx} in $P5$	4.8568623280004E-09 + i9.7137246560009E-09	4.7792696544452E-09 + i9.5585393088903E-09	2.0%
Displacement DX in $P5$ (m)	4.8568623280004E-09 + i9.7137246560009E-09	4.6188418141639E-09 + i9.2376836283278E-09	5.0%

Vitesseformule DX formulates $P5$ m/s) -1.2206626407315E-05	of it + i6.1033132036573E-06	-1.1608415609177E-05 + i5.8042078045883E-06	5.0%

5 Modelization C

5.1 Characteristic of the modelization

One calculates the average strain resulting from a transient computation on physical base. For the resolution of the system, one chooses a temporal discretization equalizes with $0.1\,s$.

One also calls upon operator `OBSERVATION` for the statements of fields of displacement and strain.

5.2 Characteristics of the mesh

Nodes: 1029
Meshes: 720 `HEXA8`

5.3 Quantities tested and results

One tests the value of the nodal deformation who are at the loose lead of the beam at time $t=1\,s$.

Identification	Reference	Aster	Tolerance
ε_{xx} in $P5$	4.7614812131879D-09	4.7619095908375D-09	2.0%
ε_{zz} in $P5$	-1.4284443639564D-09	-1.4285727587481D-09	2.0%
ε_{yy} in $P6$ (after rotation of $90°$)	4.7614812131879D-09	4.7619090586422D-09	2.0%
ε_{xx} in $P6$ (after rotation of $90°$)	-1.4284443639564D-09	-1.4285727564358D-09	2.0%

Tests for the mixed observation:

Identification	Reference	Aster	Tolerance
ε_{xx} in $P5$	4.7614812131879E-09	4.761909069047E-09	2.0%
Déplacementformule DX formulates $P5$ (m)	4.7614812131879E-09	4.5238330203209E-09	6.0%

6 Modelization D

6.1 Characteristic of the modelization

One calculates the average strain resulting from a modal computation with MODE_ITER_SIMULT.

One also calls upon operator OBSERVATION for the statements of fields of displacement and strain.

6.2 Characteristics of the mesh

Nodes: 1029
Meshes: 720 HEXA8

6.3 Quantities tested and results

One tests the mean value of the strain of the first longitudinal mode of the beam to the nodes which are in the medium and the loose lead. The values of reference are those obtained with the version 10.1 (test of NON-regression). They are given with four significant figures.

Identification	Reference	*Aster*	Tolerance
ϵ_{xx} in $P3$	1.012	1.0120571181683D+00	0.1%
ϵ_{zz} in $P3$	-3.063D-1	-3.0631275158658D-01	0.1%
ϵ_{xx} in $P5$	1.762D-1	1.7617357057031D-01	0.1%
ϵ_{zz} in $P5$	-4.646D-2	-4.6458825653459D-02	0.1%
ϵ_{yy} in $P4$ (after rotation of $90°$)	1.017	1.0168194189613D+00	0.1%
ϵ_{xx} in $P4$ (after rotation of $90°$)	-3.118D-1	-3.1181161388859D-01	0.1%
ϵ_{yy} in $P6$ (after rotation of $90°$)	1.533D-1	1.5327248667577D-01	0.1%
ϵ_{xx} in $P6$ (after rotation of $90°$)	-4.178D-2	-4.1782881091537D-02	0.1%

Tests for the mixed observation:

The purpose of identification	Reference	*Aster*	% difference
ε_{xx} in $P3$	1.01	1.0144135640 865	0.1%
Déplacementformule DX formulates $P3$ (m)	0.7570	0.7570567975 9059	0.1%

7 Summary of the results

These tests are checking the good course of computation of the mean value of the strain field using macro-command OBSERVATION. The reference solution is analytical for the static response, the harmonic response and the transient response.

This case test also validates the mixed observation of field (DEPL, QUICKLY,...) with only one call to operator OBSERVATION.

The differences between the solutions obtained with Aster and the analytical solutions are very weak. For the results resulting from a modal computation, a test of NON-regression is proposed.