SDLV123 - Computation of G elastodynamic in infinite medium for a plane crack finite length

Summarized

It acts of a problem of fracture mechanics in a medium in plane strain state in transitory elastodynamic mode. One considers a crack length $2a$ plunged in a presumedly infinite medium. One imposes a uniform pressure on the lips of the crack, which reaches a bearing in a period of time of 1 microsecond (shock). This test makes it possible to calculate rate of energy restitution G in the course of time, by taking account of the terms of inertia.

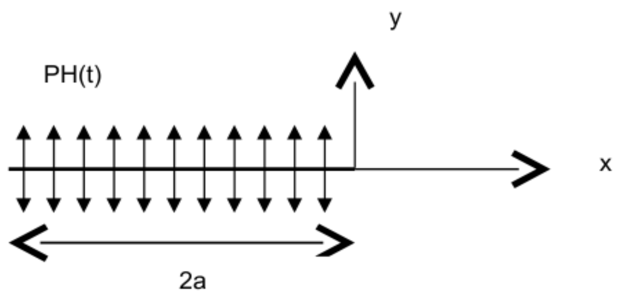
The interest of the test is the stability of G according to various contours and the comparison with an exact analytical solution until time $t=2a/V_C$, when V_C the celerity of the longitudinal waves represents.

This test contains a modelization in plane strain and a three-dimensional modelization. Boundary conditions absorbing on the borders of solid make it possible to avoid the returns of wave and thus to simulate an infinite medium.

The variations of the computation of G on various contours compared to the reference solution do not exceed 1%.

1 Problem of reference

1.1 Geometry



It acts of a plane crack of half-length $a=20\text{m}$.

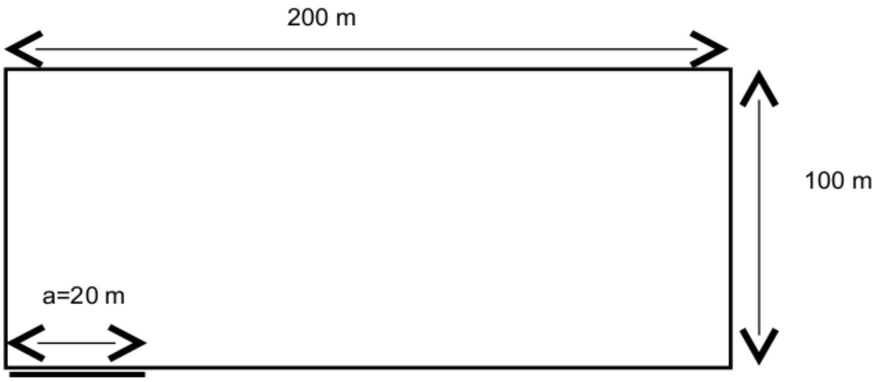
The presumedly infinite medium is modelled by a square on side of 200m . The planes xz and yz being symmetry planes, one represents only one quarter of structure and thus a half-lip of crack length $a=20\text{m}$.

1.2 Properties of the materials

Density	$\rho=7500\text{kg}/m^3$
Young Modulus:	$E=2.10^{11}\ MPa$
Poisson's ratio:	$\nu=0.3$

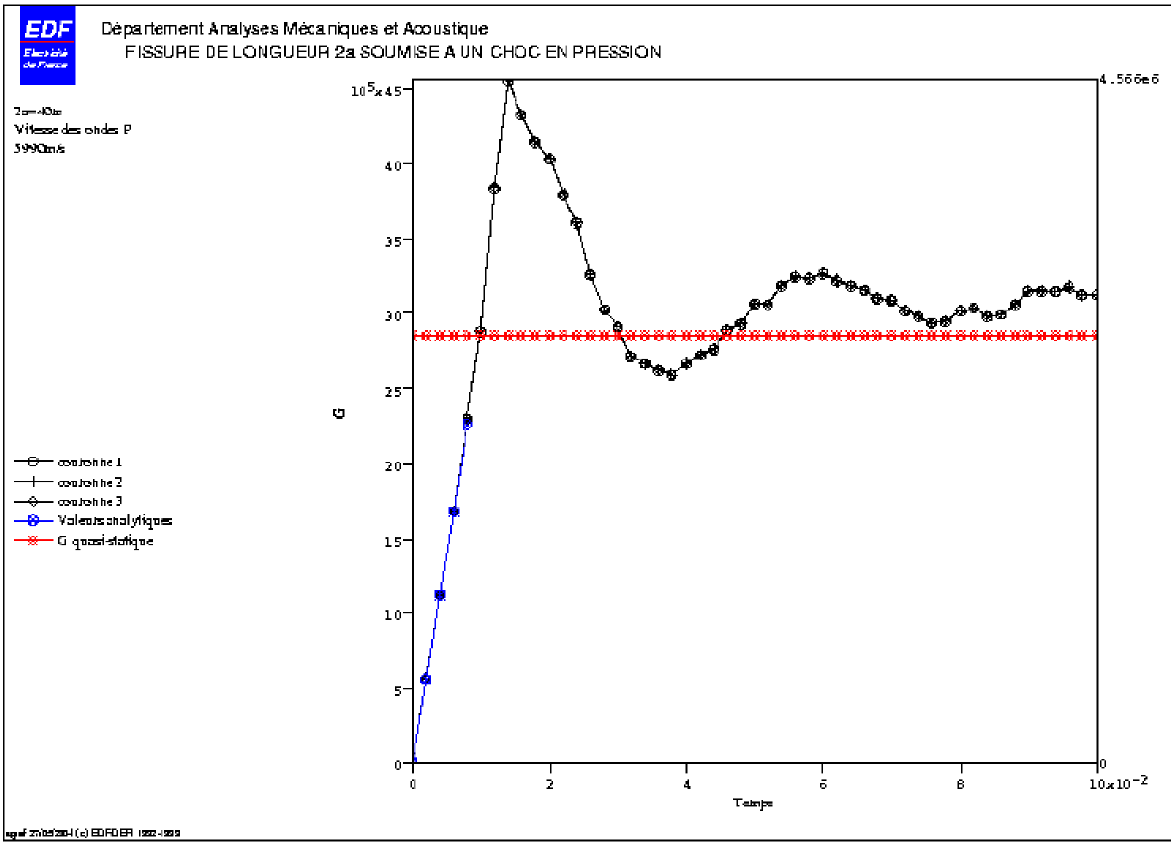
1.3 Boundary conditions and loadings

the planes xz and yz being symmetry planes, one represents only one quarter of structure (right-angled of 200m on 100m), and a crack half-lip length $a=20\text{m}$.



- Mechanics: Displacement imposed

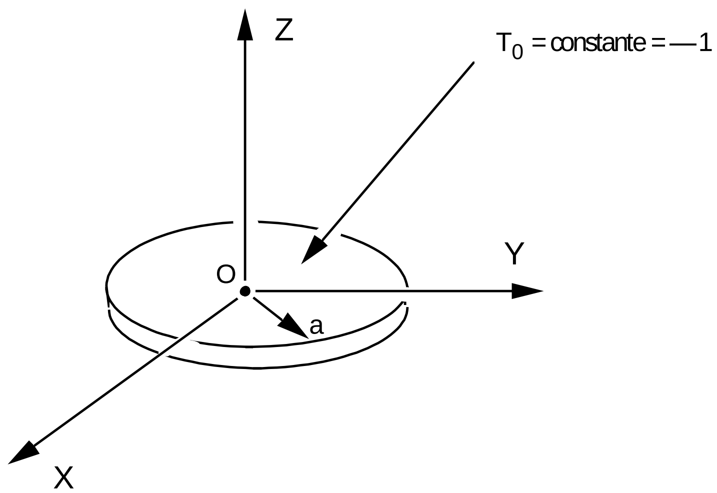
Nodes group $grnm1$	$DX=0.$
Reference solution $grnm2$	$DY=0.$


EDF
Département Analyses Mécaniques et Acoustique
FISSURE DE LONGUEUR 2a SOUMISE A UN CHOC EN PRESSION
Vitesse des ondes P
5990m/s
couronne 1
couronne 2
couronne 3
Valeurs analytiques
G quasi-statique
G
Temps
10^5x45
40
35
30
25
20
15
10
5
0
4.566e6
0
2
4
6
8
10x10^-2

2 Nodes group

2.1 Méthode de calcul used for the reference solution

the reference solution is resulting from THAU and LU [bib1] and from the book of L.B. FREUND [bib2]:



$$K_I^D(t)=2\mathrm{P}\ H(t)\frac{\sqrt{1-2\nu}}{(1-\nu)}\sqrt{\frac{c_d t}{\pi}},\ \ 0<t<2\mathrm{a}/c_d$$

The statement of the rate of refund of energy is the following one:

$$G(t)=\frac{K_I^{D^2}(t)}{E}(1-\nu^2)=4\mathrm{P}^2\ H(t)\frac{(1-2\nu)(1+\nu)}{(1-\nu)E}\frac{c_d t}{\pi},\ \ 0<t<2\mathrm{a}/c_d$$

$$G=\frac{(1-\nu^2)}{E}K_1^2 \quad \text{with} \quad K_1=\frac{\alpha E}{P(1-\nu)}T_0\sqrt{Pa}$$

is: $G=\frac{(1-\nu^2)}{P(1-\nu)^2}\alpha^2 E T_0^2 a$

2.2 Result of reference

result of reference is thus: $G=5.911510^{-7} N/mm$

t	G_{freund}
2nd-3	5.677e5
4th-3	1.135e6
6th-3	1.703e6

2.3 Bibliographical reference

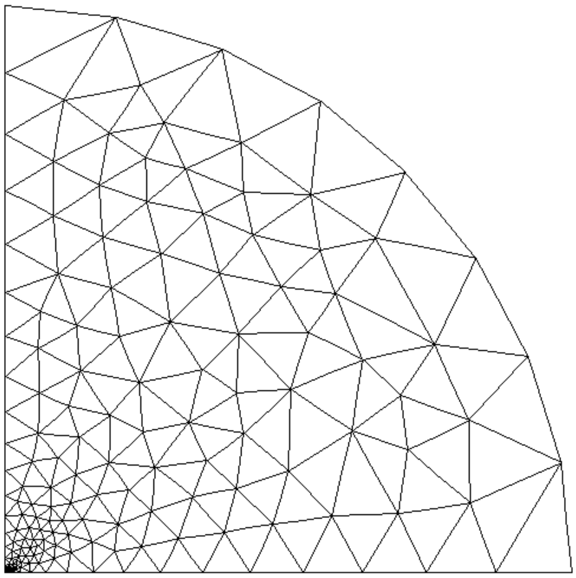
1) Transient stress intensity factors for has finite ace in year elastic solid caused by has dilatational wave, International Newspaper of Solids and Structures 7, THAU and READ (1971)

2) Dynamic Mechanics L.B FREUND Fractures.

3 Modelization A

3.1 Characteristic of the modelization

It acts of a modelization in plane strains:



3.2 Characteristics of the mesh

Many nodes: 832

Number of meshes and types: 323 TRIA6, 42 QUAD8, 59 SEG3

Crown 1:	$R_{\inf}=1.$	$R_{\sup}=4.$
Crown 2:	$R_{\inf}=0.5$	$R_{\sup}=4.5$

3.3 Results

the values tested are those of the rate of refund of energy G on various integration contours:

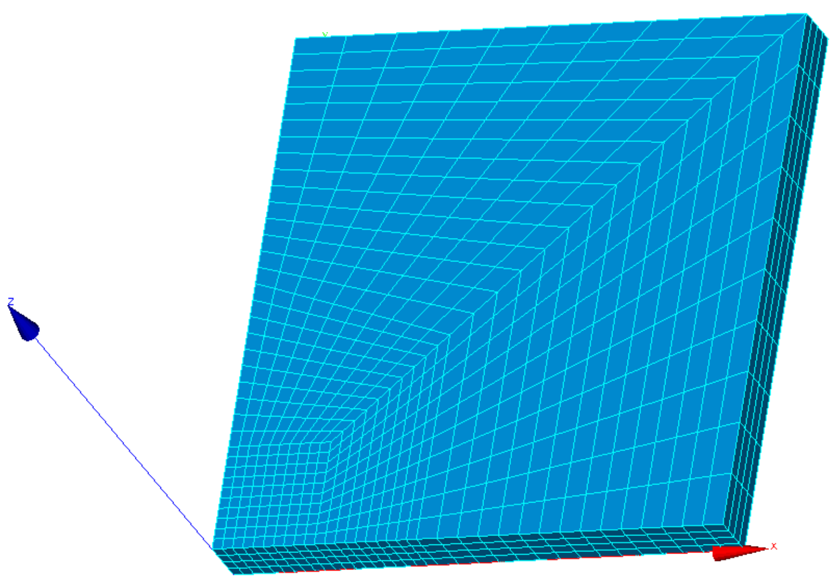
The value of reference is $G=5.945\,10^{-7} N/mm$

It is given per unit of area of extension of crack, therefore for $a=5$ and taking into account the symmetry of the mesh, it is necessary to compare result *Aster* with: $G_{Aster}=G_{réf}\,\frac{a}{2}=1.4778\,10^{-6}$, because G in *Code_Aster* to a surface of extension of 1 radian corresponds.

4 Modelization B

4.1 Characteristic of the modelization

It acts of a modelization in 3D:



4.2 Characteristics of the mesh

Many nodes: 3230

Number of meshes and types: 1560 QUAD4, 2400 HEXA8, 253 SEG2

Crown 1:	$R_{\text{inf}}=1.$	$R_{\text{sup}}=4.$
Crowns 2:	$R_{\text{inf}}=0.5$	$R_{\text{sup}}=4.5$

4.3 Results

the values tested are identical to those of modelization A.

5 Synthèse of the Invariance

results of result compared to contours. Correct thermal term.