SDLV122 - Extrapolation of local measurements on a complete model (3D)

Summarized:

It is about a linear test of dynamics $3D$.

The goal is to test command PROJ_MESU_MODAL in the case of a system $3D$. This command makes it possible to project experimental dynamic transient responses in a certain number of points on a modal base of a numerical modelization.

This test contains 2 modelizations:

projection (of stresses) is done on a basic concept modal of type [mode_meca],
projection (of stresses) is done on a basic concept modal of type [base_modale].

For the 2 modelizations, provided experimental measurements are identical and make it possible to test the search of the nodes in opposite and the taking into account of a local directional sense.

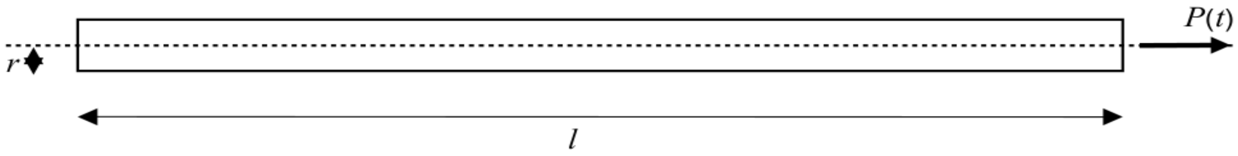
In both cases, the reference solution is obtained by a direct computation with *Code_Aster* ; projection is carried out in the successful outcome where the number of modes is equal to the number of measurements. The responses in stress obtained after projection are identical to the stresses of reference provided in data.

For the modelization A, the responses in displacements and strain obtained after projection are in perfect adequacy with the reference solutions. The values velocities and the accelerations deduced from the identified modal contributions are close to those obtained by direct computation. The weak noted variations are due to the errors of approximation generated by the determination via a linear diagram in time of the velocities and accelerations.

1 Problem of reference

1.1 Geometry

Let us consider the embed-free slender cylindrical bar described below:



Length	: $l=4m$
Radius	: $r=0.1m$
End 1	: embedded ($x=0$)
End 2	: free ($x=l$)

1.2 Properties of the materials

the characteristics of the material are the following ones:
Young modulus: $E=2.1\,10^{11}\,Pa$
Poisson's ratio: $\nu=0.3$
Density: $\rho=7800\,kg/m^3$

1.3 Boundary conditions and loading

the boundary condition is the fixed support of the end 1 of the bar. This fixed support is of standard beam to allow the effects Fish on the section.

The loading applied for the computation of response is a thrust load, constant in tension, distributed on the section of the end 2 :

$$P(t)= \begin{cases} 0 & \text{if } t<0 \\ P_0=10^6\,N & t\geq 0 \end{cases}$$

2 Reference solutions

2.1 Method of calculating used for the analytical reference solution

Solution:
An analytical solution of this problem exists. It is described in [1]:

In this case, the solution by modal superposition of this problem is written:

$$u(x,t)=\frac{P_0}{EA}x-\frac{8P_0 I}{\pi^2 EA}\sum_{s=1}^{\infty}\left[\frac{(-1)^{s-1}}{(2s-1)^2}\sin\left((2s-1)\frac{\pi x}{2I}\right)\cos\left((2s-1)\frac{\pi c}{2I}t\right)\right]$$

with: $c=\sqrt{\frac{EA}{m}}$: velocity of wave propagation in the bar

adopted Reference solution:
The analytical solution utilizing an infinite sum on the modes, it is preferable that the reference solution and the solution with projection correspond to the same configuration, with the same number of modes.
Moreover, to avoid problems involved in the discretization of the numerical mesh, the adopted reference solution is the response provided by the direct computation carried out with *Code_Aster* with command `DYNA_TRAN_MODAL`. One simulates displacement, the velocity and the stress in some structure points. The simulated responses were versed in a file with the universal format (IDEAS).

2.2 Results of reference

For the modelization A, the comparison of the results relates to displacements, velocities, accelerations, strains and forced along the axis x, of the nodes $N2$ and $N4$ at 3 different times. $N4$ corresponds to a node of measurement and $N2$ is not one. It is also checked if one obtains exactly the component of the field measured after expansion of measurement on the model numerical. One compares for that the component of the extended field following the direction of observation (displacement $N3$ according to DX, vitesseformule $N5$ according to DX and SIXX with the node $N4$).

For the modelization B, the comparison of the results relates to the stresses of the nodes $N3$ and $N4$ with 3 different times.

2.3 Uncertainty on the solution

the selected reference makes it possible to draw aside uncertainties related to the discretization of the numerical mesh. Number of modes of the base of projection is equal to the number of measurements, therefore the solution of the inversion is exact (in opposition to an approximate solution of generalized inverse problems).

If projection is done on a concept of the type [`mode_meca`], modal bases of the solution and reference solution obtained by projection are identical, the responses in displacements, strains and stresses obtained must thus be similar to the responses of reference. Some errors of approximation can appear on the velocities and accelerations which are determined by a linear diagram in time.
If projection is done on a concept of the type [`base_modale`], modal bases of the solution and reference solution obtained by projection contain the same number of modes but are different. The computation of reference not being possible on a concept of the type [`base_modale`], the comparison of the results relates only to responses corresponding to provided measurements.

2.4 Bibliography

1) Mr. GERADIN, D. RIXEN: Theory of vibrations - Application to the dynamics of structures - Edition MASSON 1993

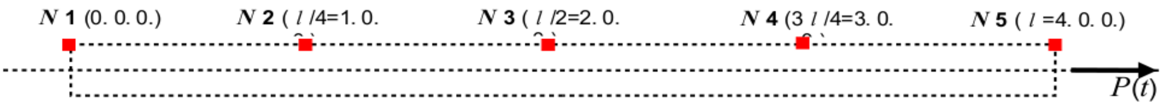
3 Modelization A

3.1 Characteristic of the modelization and the meshes

This modelization tests the reading of dataset 58 of a file to the universal format (IDEAS), containing at the same time fields of displacement, velocity and stress. One also carries out the expansion on the model numerical by exploiting mixed measurement (displacement, velocity and stress).

Numerical mesh:
The mesh numerical is realized with I-DEAS version Master degree Series 5. It comprises 2667 nodes and 3328 meshes of linear $3D$ type.

Experimental mesh :
The mesh of measurement understands only 5 positioned point elements and 5 nodes as indicates it the following figure:



3.2 Characteristics of measurements

provided experimental measurements are:

- axial stresses σ_{xx} , according to the direction x with the nodes $N3$, $N4$ and $N5$,
- axial displacement with the node $N3$,
- the axial velocity with the node $N5$.

The sampling of time is constant: initial time is $0\,s$, time step is $10^{-5}s$ and the number of times is 1001 (i.e until a final time of). $0.01\,s$

The values result from the direct computation carried out with Code_Aster . *Characteristics*

3.3 of modal base

the modes are stored in a concept of the type [mode_meca], containing the first three dynamic modes of tension. These modes are obtained by blocking transverse displacements (i.e according to and) DY of DZ the nodes of fiber of neutral and the nodes of line higher (with and $x=0.$) $4.$ $y=0.1$. Their $z=0.$ eigenfrequencies (, and $326.5\,Hz$) $980.0\,Hz$ are $1634.5\,Hz$ close to the analytically calculated eigenfrequencies of tension (, and $324.3\,Hz$) $972.9\,Hz$. Quantities $1621.5\,Hz$

3.4 tested and results Identification

	Reference		to T
		= 9 . 10-4s 2.686	10-4 DEPL_
X with the node	with T $N2$	= 17 . 10-4s 3.074	10-4 with T
(m)		= 25 . 10-4s 1.446	10-5 with T
		= 9 . 10-4s 5.793	10-4 DEPL_
X with the node	with T $N4$	= 17 . 10-4s 9.160	10-4 with T

(m)		= 25 . 10-4s 3.095	10-4 with T
		= 9 . 10-4s 6.221	10-1 VITE_
X with the node	with T $N2$	= 17 . 10-4s – 4.683	10-2 with T
(m/s)		= 25 . 10-4s – 3.542	10-1 with T
		= 9 . 10-4s 8.056	10-1 VITE_
X with the node	with T $N4$	= 17 . 10-4s – 3.556	10-1 with T
(m/s)		= 25 . 10-4s – 8.638	10-1 with T
		= 9 . 10-4s – 3.633	10+3 ACCE_X
with the node	with T $N2$	= 17 . 10-4s 6.337	10+2 with T
(m/s^2)		= 25 . 10-4s 3.801	10+3 with T
		= 9 . 10-4s 8.655	10+2 ACCE_X
with the node	with T $N4$	= 17 . 10-4s – 2.387	10+3 with T
(m/s^2)		= 25 . 10-4s – 6.355	10+2 with T
		= 9 . 10-4s 1.957	10-4 EPXX
with the node	with T $N2$	= 17 . 10-4s 3.015	10-4 with T
(m)		= 25 . 10-4s 5.422	10-5 with T
		= 9 . 10-4s 1.822	10-4 EPXX
with the node	with T $N4$	= 17 . 10-4s 2.611	10-4 with T
(m)		= 25 . 10-4s 1.681	10-4 with T
		= 9 . 10-4s 5.012	10+7 SIXX
with the node	with T $N2$	= 17 . 10-4s 7.717	10+7 with T
(Pa)		= 25 . 10-4s 1.390	10+7 with T
		= 9 . 10-4s 4.650	10+7 SIXX
with the node	with T $N4$	= 17 . 10-4s 6.671	10+7 with T
(Pa)		= 25 . 10-4s 4.293	10+7 0 0 0

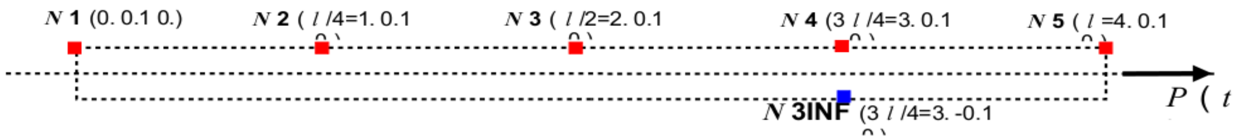
$\sum_i \left\| depl_x(NRES3)(t_i) - depl_x(N3)(t_i) \right\|$ (m)	Modelization
$\sum_i \left\| vite_x(NRES5)(t_i) - vite_x(N5)(t_i) \right\|$ (m/s)	
$\sum_i \left\| SIXX(NRES4)(t_i) - SIXX(N4)(t_i) \right\|$ (Pa)	

4 B Characteristic

4.1 of the modelization and the meshes numerical

Mesh: The mesh
numerical is identical to that used in the preceding case. It is carried out with I - DEAS version Master Series 5 and comprises 2667 nodes and 3328 meshes of linear type $3D$. A group comprising only one node is added to be used as interface. Experimental

mesh: The mesh
of measurement understands only 5 positioned point elements and 5 nodes as indicates it the following figure: Characteristic



4.2 of measurements provided

experimental measurements are: With the nodes

, and $N3$: $N4$ $N5$ The data
are the axial stresses, applied in the direction. x The sampling
of time is constant: initial time is, $0\,s$ time step is and $10^{-5}\,s$ the number of times is 1001 (i.e until a final time of). The values $0.01\,s$

result from the direct computation carried out with Code_Aster . *Characteristics*

4.3 of modal base the modes

are stored in a concept of the type [base_modale], `containing` the first two dynamic modes of tension and the static mode with the node for $N3INF$ the degree of freedom. The interface DX is of Craig-Bampton type. The base thus contains on the whole 3 modes. Note:

The number

of modes being very reduced, the solution depends on modal base. However, the modes determined for this modelization are not the same ones as those of the modal base of reference, and it is not possible to carry out direct computation with Code_Aster on a concept [base_modale]. `Only` the responses corresponding to provided measurements can thus be validated. No comparison can be realized on the other responses. Quantities

4.4 tested and results Identification

	Reference		to T =
		9. 10 – 4s 3.416 10	+7 SIXX with
`the node`	with T = $N3$	17. 10 – 4s 8.046 10	+7 with T =
(Pa)		25. 10 – 4s 4.251 10	+7 with T =

		9. 10 – 4s 4.650 10	+7 SIXX with
the node	with T = $N4$	17. 10 – 4s 6.671 10	+7 with T =
(Pa)		25. 10 – 4s 4.293 10	+7 Summary

5 of the results For

the two modelizations, the responses in stresses obtained after projection are identical to the stresses of reference obtained by computation direct with Code_Aster *and provided* in data. For

the modelization A, the responses in displacements and strain are in perfect adequacy with the reference solutions. The values velocities and the accelerations obtained after projection are close to those obtained by direct computation. The weak noted variations are due to the errors of approximation generated by the determination by a linear diagram in time of the velocities and accelerations. The cases

where the number of modes is not equal to the number of measurements are not tested (generalized inverse problems); in particular, the method of regularization of Tikhonov is not tested.