# SDLV121 - Plane wave of shears in an elastic column

**Abstract**

One tests the application of a loading out of transient in the form of one plane wave thanks to the elastic paraxial elements of order 0, in 3D and 2D. One applies this loading to an elastic solid mass occupying a half - space and which one models a column. This column is supposed to be infinite in its lower part and levels in her upper part on the level of surface of the free half space left. One observes the propagation of the incident wave, his reflection on the free surface of the solid mass and his absorption by the paraxial elements at the lower end of the column.

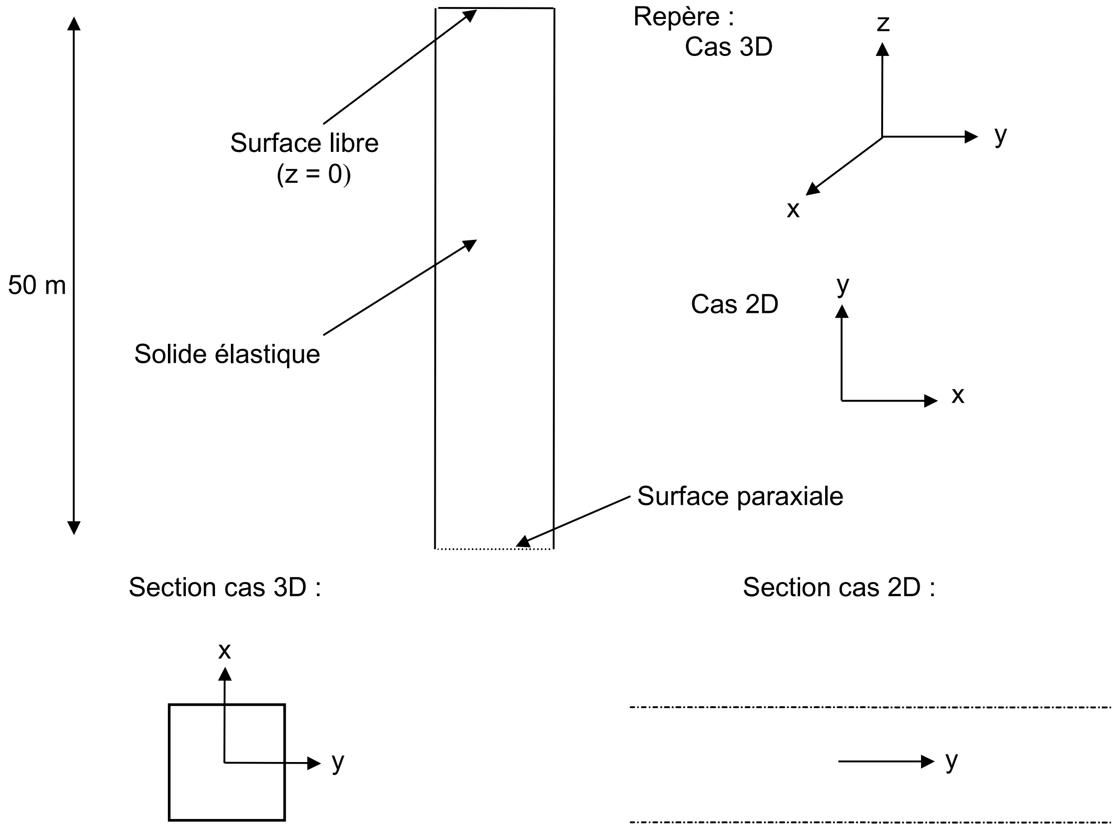
One tests successively the two direct transitory operators of *Code_Aster*, namely DYNA_VIBRA and DYNA_NON_LINE.

# 1 Problem of reference

## 1.1 Geometry

the system considered in 3D case is that of a homogeneous elastic soil occupying the half space $z<0$. The plane $z=0$ is left free. One models of this soil a vertical column, presumedly infinite in his lower part and levelling at free surface at his higher end. One places the paraxial elements on lower surface, to translate the infinite character of the column and to apply the loading by plane wave. In the case 2D, the principle is identical, with a very broad column which one models only one vertical section (see diagram).

Moreover, the direction of vibration is the y axis in 3D case. It is the axis $x$ in the case 2D.




## 1.2 Properties of the materials

elastic Solid mass: floor covering

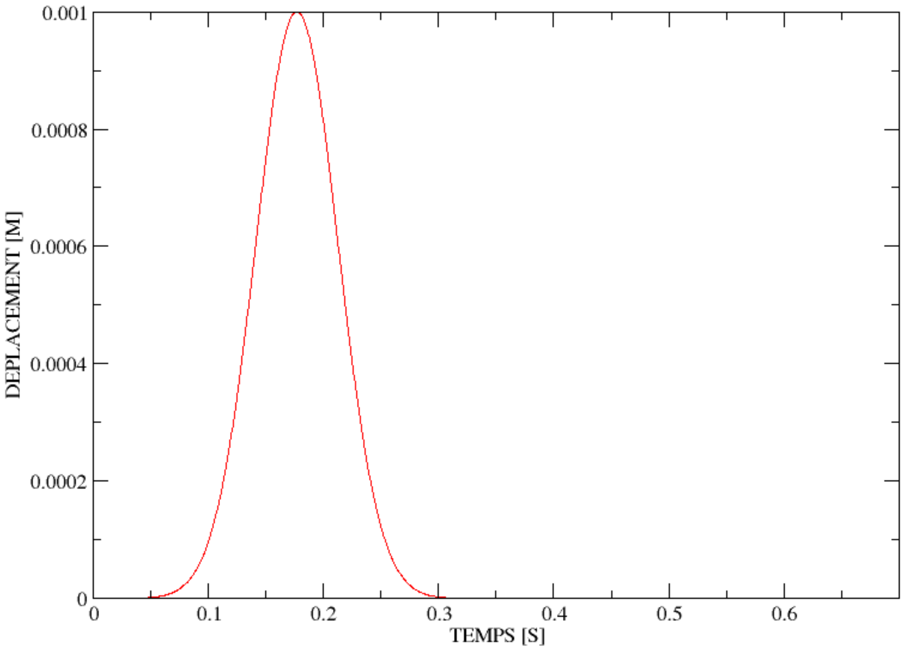
| | |
|---|---|
| Density: | $1900\,kg.m^{-3}$ |
| Young modulus: | $4{,}44.10^{8}\,Pa$ |
| Poisson's ratio: | $0{,}2$ |

## 1.3 Boundary conditions and loadings

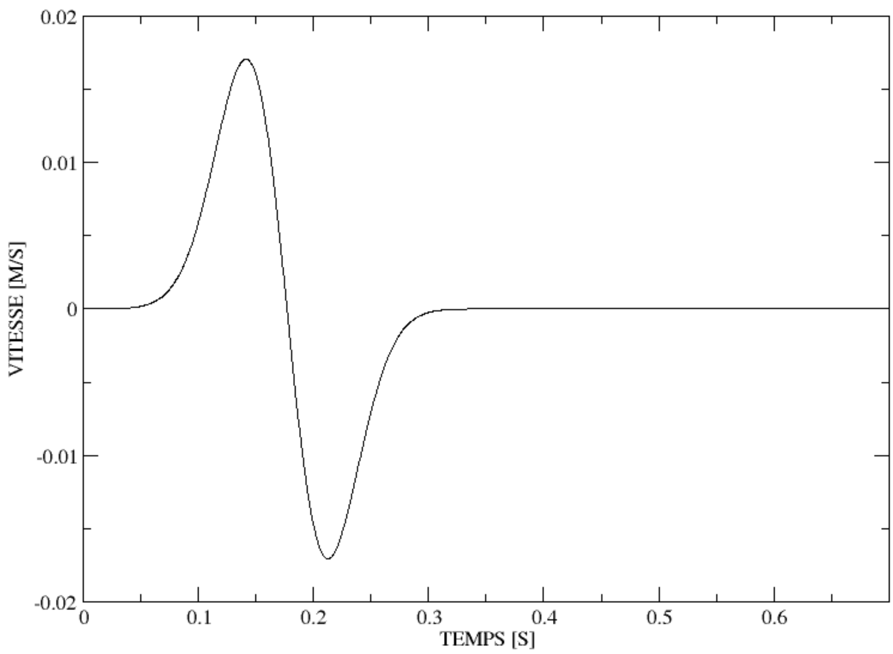
One is interested in motion 1D of the column under the exiting action of one plane wave vertical. To identify this motion, one forces all the nodes of the same horizontal section to have same displacement.
In this configuration, the loading by plane wave comprises the following characteristics:

- Direction: $(0.,0.,1.)$
- Type_d' wave: $SH$

•Signal: function velocity given below (which is used as entry with computation), and displacement corresponding:




The maximum in displacement of the wave front in the column is of $1\,mm$ with $t_m=0,177\,s$ .




## 1.4 Initial conditions

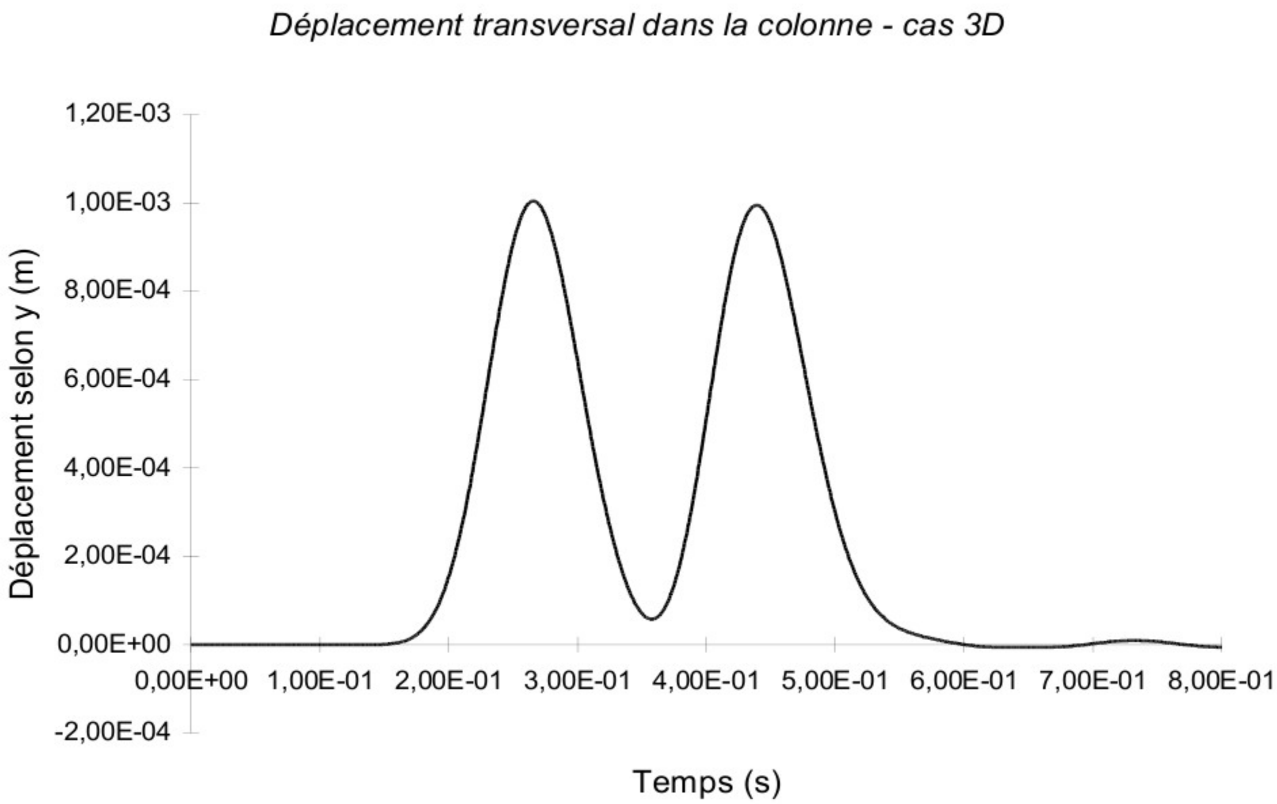
displacement is null in all the column at initial time.

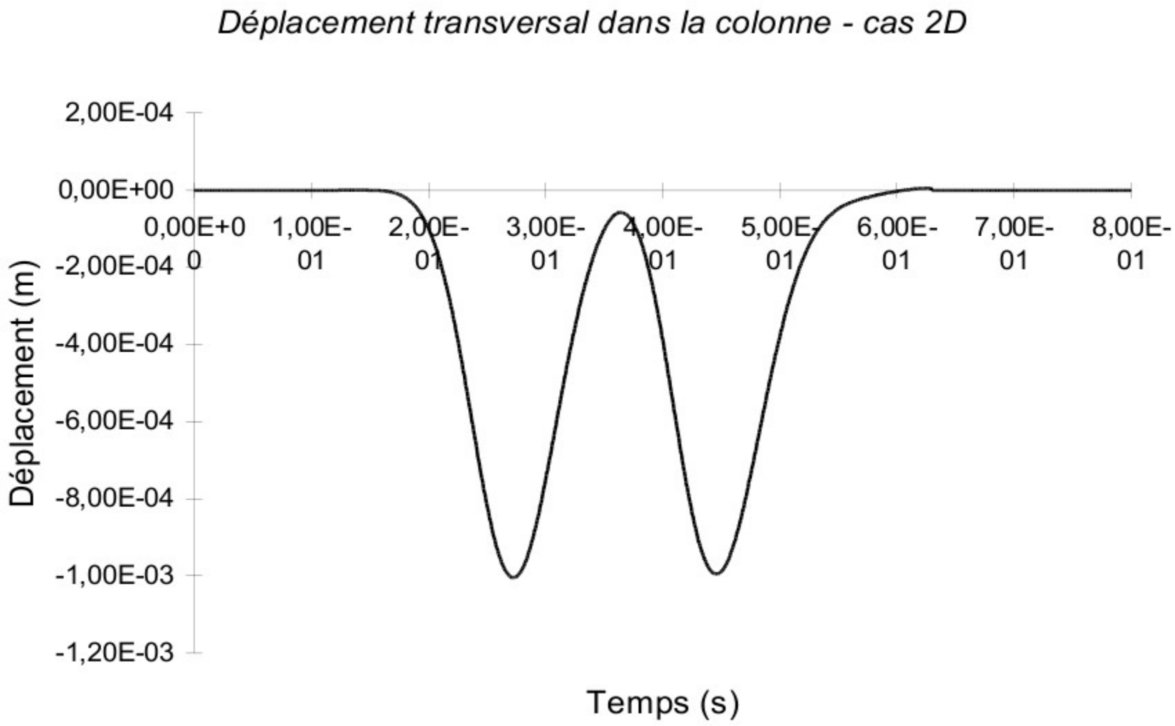
# 2 Reference solution

the propagation 1D of the signal of the incident wave in the column is known analytically [bib1]. One can for example determine the time of transition of the maximum of the incident wave with middle height, that is to say with a depth of $\frac{H}{2}=25\,m$ , and that of the maximum of the wave thought of the same point. The velocity of the waves of shears being of $C_S=281\,m.s^{-1}$ for the soil considered, the time of transition of the wave front with middle height is of $t_{\frac{H}{2}}=\frac{H}{2C_S}=0{,}089\,s$ . One can thus expect the maximum of displacement with semi - height in the column for time $t_i=t_m+t_{\frac{H}{2}}=0{,}266\,s$ . Moreover, during the transition of the reflected wave, the signal will have traversed $50\,m$ moreover, therefore one can expect it for time $t_r=t_m+t_{\frac{3H}{2}}=0{,}444\,s$ . The value of the maximum measured at these times must be of $1\,mm$ . These are these analytical values that one will test in computation.

## 2.1 Results of reference

One gives in this paragraph the results got with *Code_Aster* in this configuration. It is checked that they are satisfactory qualitatively and quantitatively.
They relate to, for 3D case, the evolution of displacement in the three directions in a point of the column located at middle height, that is to say on $25\,m$ free face in the direction $z$ . The measurement of displacement is identical in the case 2D.
Moreover, the direction of vibration is the axis of $y$ in 3D case. It is the axis $x$ in the case 2D.






It is checked first of all that displacements is null according to $x$ and $z$ in 3D case and according to $y$ in the case 2D.

It is thus checked that the width of the peaks is not deformed and is well $0{,}3\,s$ at the base.
One also observes at the times envisaged the presence of the two identical peaks due to the reflection without change of sign on free surface. Their amplitude of $1\,mm$ also finds the imposed signal.

The inversion of the sign of displacement in the case 2D is due only to the directional sense of the reference. Directional sense of the positive sign for the signal being given compared to the direction of propagation according to the meaning of Fresnel.

## 2.2 Uncertainties

It is about result of the numerical study. One finds the qualitative and quantitative forecasts. The numerical values are related to the accuracy of computation.

## 2.3 Bibliographical references

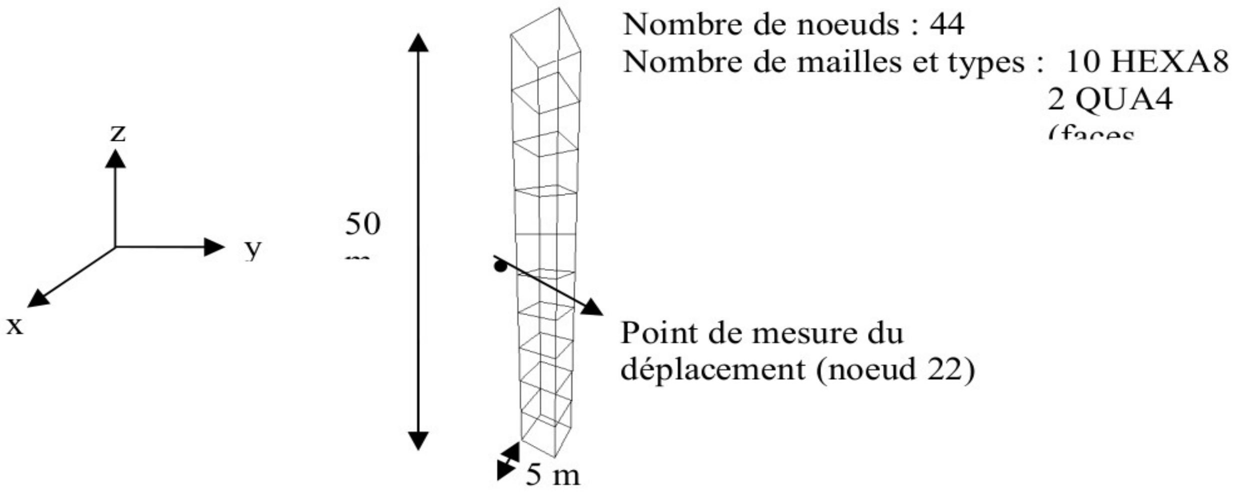
1) H. MODARESSI “numerical Modelization of the wave propagation in the elastic porous environments.” Thesis doctor-engineer, School Central of Paris (1987)

# 3 Modelization A

## 3.1 Characteristic of the modelization

the bar is modelled in 3D.

## 3.2 Characteristics of the mesh




## 3.3 Values tested

One tests the values of displacement in the three directions with the node 22 (see mesh). For the direction $y$, one tests the value of the two maximum ones and the return at rest after the transition of the wave. For the two other directions, one tests the nullity of displacement, for example at the time of the first maximum in $y$.

- DYNA_VIBRA:

| Direction | Instantformule $(s)$ | Results of reference ( displacement in $m$ ) |
|---|---|---|
| $Y$ | 2.65600E-01 | 1.E-03 |
| | 4.38400E-01 | 1.E-03 |
| | 8.00000E-01 | 0. |
| $X$ | 2.65600E-01 | 0. |
| $Z$ | 2.65600E-01 | 0. |

- DYNA_NON_LINE:

| Direction | Time $(s)$ | Computation with *Code_Aster* (displacement in $m$ ) | Results of reference ( displacement in $m$ ) | Variations reference - computation with *Code_Aster* (%) |
|---|---|---|---|---|
| $Y$ | 2.67200E-01 | 1.00396E-04 | 1.E-03 | 0.40 RELATIF |
| | 4.40000E-01 | 9.94928E-04 | 1.E-03 | 0.51 RELATIF |
| | 7.20000E-01 | 5.1E-6 | 0. | 5.1E-4 ABSOLU |
| $X$ | 2.67200E-01 | 0. | 0. | 0. ABSOLU |

| | | | | |
|---|---|---|---|---|
| $Z$ | 2.67200E-01 | 0. | 0. | 0. ABSOLU |

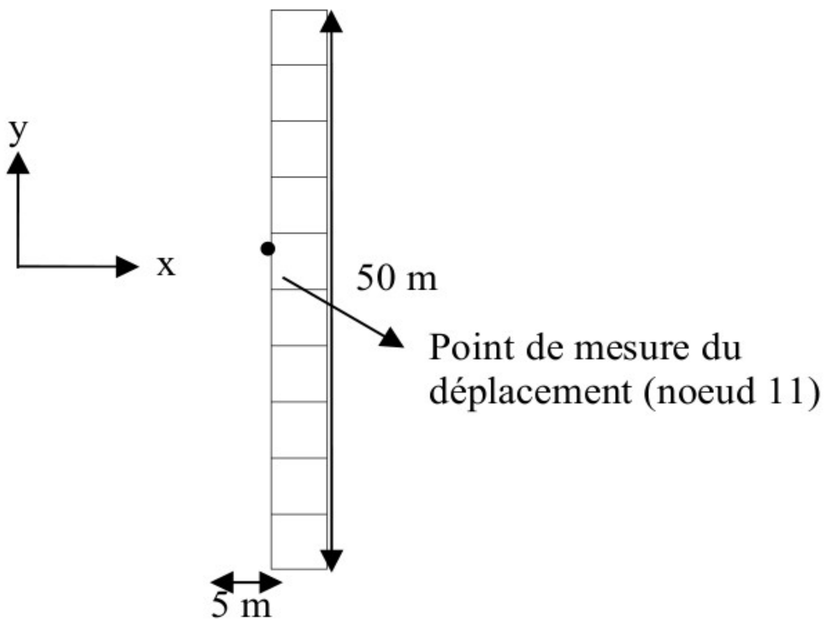
# 4 Modelization B

## 4.1 Characteristic of the modelization

the bar are modelled in D_PLAN.

## 4.2 Characteristics of the mesh

Many nodes: 22
Number of meshes and types: 10 QUA4
2 SEG2 (sides of QUA4)




## 4.3 Values tested

One test the values of displacement in the three directions with node 11 (see mesh). For the direction $x$, one tests the value of the two maximum ones and the return at rest after the transition of the wave. For the direction $y$, one tests the nullity of displacement, for example at the time of the first maximum in $y$.

- DYNA_VIBRA:

| Direction | Time $(s)$ | Results of reference ( displacement in $m$ ) |
|---|---|---|
| $X$ | 2.65600E-01 | – 1.E-03 |
| | 4.38400E-01 | – 1.E-03 |

| | | |
|---|---|---|
| | 8.00000E-01 | 0. |
| $Y$ | 2.65600E-01 | 0. |

* DYNA_NON_LINE :

| Direction | Time $(s)$ | Results of reference ( displacement in $m$ ) |
|---|---|---|
| $X$ | 2.65600E-01 | – 1.E-03 |
| | 4.38400E-01 | – 1.E-03 |
| | 8.00000E-01 | 0. |
| $Y$ | 2.65600E-01 | 0. |

# 5 Summary of the results

One finds by computation with the two modelizations quantitatively, the values of maximum of displacement equal to the maximum amplitude of the signal and the values of corresponding times and qualitatively, the return at rest after the transition of the considered wave.

The results got with operators DYNA_VIBRA and DYNA_NON_LINE are very close. The difference comes from obtaining to each time step of the state from equilibrium from the forces from the second member with operator DYNA_NON_LINE, which explains why its results are in general a little bit better even with time step the larger. This difference remains however tiny because time step used with DYNA_VIBRA is sufficiently small.