SDLV111 - Homogenization of a network of beams in a Summarized incompressible

fluid:

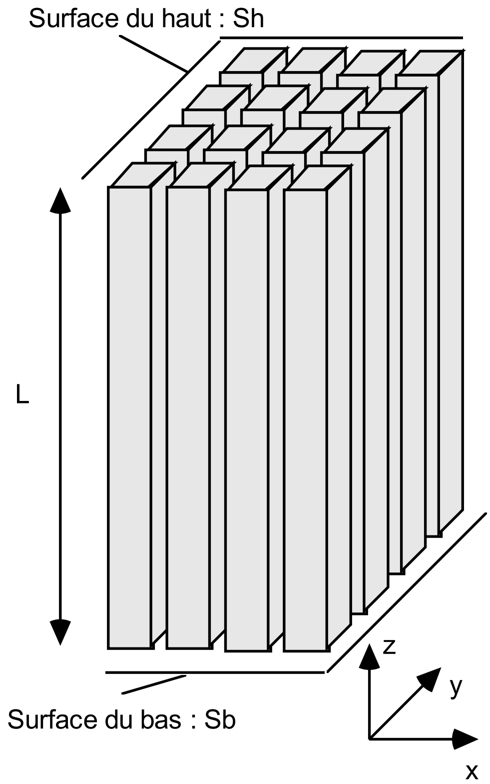
Test in modal analysis, being used to validate the elements of modelization `3D_FAISCEAU` : hexahedron with 8 nodes or hexahedron with 20 nodes. These elements represent the homogenized medium of a network of beams bathing in an incompressible fluid, initially at rest.

One tests the eigenfrequencies of the beams of the medium homogenized without or with fluid.
One calculates the mass, the position of the center of gravity as well as inertias in this point for the cases to have or without fluid.

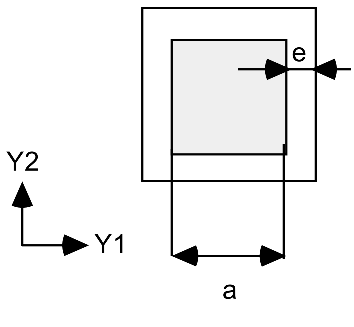
1 Problem of reference

1.1 Geometry

One considers a periodic network of 4×4 beams [fig 1.1-a]. The period of the field is εY. The figure [fig 1.1-b] represents an enlarging of $1/\varepsilon$ period. Each beam is right of square section.



Appear 1.1-a: Geometry of the heterogeneous medium - Beams without fluid



Appears 1.1-b: Cell of reference Y - Enlarging of $\frac{1}{\varepsilon}=10$

- Characteristics of the period:
  - Dimensions:

$\varepsilon Y=(0.21\,m, 0.21\,m)$
$a=1.5\,m$
$e=0.3\,m$

- Characteristics of each beam:
  - Section:
    $A=(\varepsilon\times a)^2=(0.1\times1.5)^2=0.0225\,m^2$
  - Length:
    $L=4.1\,m$
  - Main moment of inertia of bending:
    $I_x=I_y=(\varepsilon\times a)^4/12\,m^4$

1.2 Material properties

isotropic linear elastic Material:

$E=10^9\,Pa$
$NU=0.3$

Densities:

Beam:

$Rho=7641\,kg/m^3$

Fluid:

$Rho=0\,kg/m^3$ (case without fluid)
$Rho=1000\,kg/m^3$ (case with fluid)

1.3 correct Terms

the correct terms are calculated on the cell of reference Y [fig 1.1-b].

B_T= $0.79\,m^2$
B_N= $0.79\,m^2$
B_TN= $0\,m^2$
A_FLUI= $2.16\,m^2$
A_CELL= $2.25\,m^2$
COEF_ECHELLE=10

1.4 Boundary conditions and loadings

Case without fluid:

Surface bottom Sb : fixed support
All the degrees of freedom are blocked.

Surface top Sh : fixed support
All the degrees of freedom are blocked.

Case with fluid:

Surface bottom Sb : fixed support
All the degrees of freedom are blocked.

Surface top Sh : plane bearing (bilateral connection)
All rotations are blocked.
Longitudinal displacement DZ is blocked.

All the nodes of Sh have same transverse displacement DX and same normal displacement DY.

2 Reference solution

2.1 Method of calculating used for the reference solution

Case without fluid:

Let us consider the heterogeneous field described with [§1] in absence of the fluid. It is supposed that the beams respect the assumptions of modelization of a straight beam of Eulerian Bernoulli. Since the boundary conditions applied to all the beams are the same ones as for each one of them, one can bring back the search of the eigenfrequencies of the group to that of only one beam.

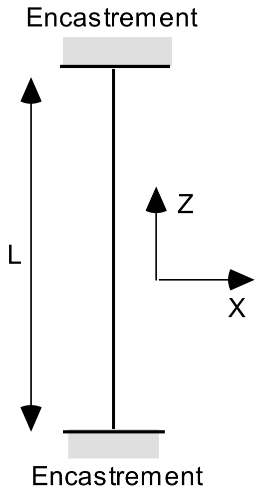
The following problem is thus studied:

That is to say an bi--embedded beam [fig 2.1-a] in the same way characteristic geometrical and material that beams of the heterogeneous medium. One notes A the area of the section, L his length, and I the main moment of inertia of bending.

By the method of stiffness dynamic one shows that such a beam admits double frequencies of the form:

$$f_i = \frac{\lambda_i^2}{2\pi L^2}\left[\frac{EI}{\rho A}\right]^{\frac{1}{2}}$$

$\lambda_i = (2i+1)\pi/2 \quad i = 1,2,\ldots$ for the second case of boundary conditions: [fig 2.1-a].



Appear 2.1-a

field contains N beams independent between them (not boundary conditions which couples displacements of two different beams), It results from it that the multiplicity of the frequencies is equal to $2N$ (2 modes of bendings by beams).

For the homogenized medium discretized by the finite elements hexahedron with 8 nodes or hexahedron with 20 nodes, the number N must be replaced by the number of straight lines parallel with the axis of the beams.

Case with fluid:

The case with fluid is more difficult to solve analytically: no result analytical was found until the drafting of this case test. The results of reference which one established thus come from a numerical resolution by finite elements of the complete heterogeneous problem. One used for this fact version 3.6.2 of *Code_Aster*.

Each beam is represented in the mesh by its average fiber modelled by `POU_D_E` (straight beam of Eulerian). For all the beams, one binds each node of average fiber to the nodes of side surface, located in the same cross-sectional area as the node in question, by `LIAISON_SOLIDE`. The fluid interface beam is modelled by `FLUI_STRU` which translates the continuity the normal velocities to the walls. The fluid, was to be perfect incompressible, one deduced his modelization from that of true fluid compressible `3D_FLUIDE` by removing the contribution of the pressure.

The boundary conditions imposed on the fields [§1.3], and especially the relation which couples the displacement of all the beams with the level of Sh , reveal two kinds of eigen modes of structure:

Modes of sets: all the beams become deformed in the same way and top surfaces it admits a non-zero displacement.

Local modes: they correspond to modes of embed-embedded beams. Surfaces top thus admits a null displacement. None of these modes can correspond to an overall mode.

The action of the fluid results in an effect of added mass and thus a lowering of the frequencies compared to the case without fluid. It also causes, in the case of the local modes, to spread out the frequency spectrum associated. In the case without fluid one saw that this spectrum was concentrated in only one frequency of vibration.

2.2 Results of reference

Value of the eigenfrequencies.
For the mass and the inertias at the center of gravity:

- In absence of fluid, the mass is determined by the product of the volume occupied by the beams and the density of these elements: $MASSE_{solide}=\rho_{poutre}\times Vol_{poutres}$ where $Vol_{poutres}$ is determined by the product amongst beam, of the area of the section and length of the beam. Starting from the data defined previously, one can calculate $MASSE_{solide}$: one obtains thus $MASSE_{poutres}=7641\times 16\times(1.5/10)^2\times 4.1=11278.116\ kg$.
- The mass being distributed uniformly in volume (because of the position of the beams in volume), the center of gravity is thus localised in its center, namely at the point of coordinates $(0.42, 0.42, 2.05)$. This result is confirmed analytically by the computation of the following integrals:

$$X_G=Y_G=\frac{1}{Vol}\int_V x\,.\,\rho\ dV \text{ and the } Z_G=\frac{1}{Vol}\int_V z\,.\,\rho\ dV$$

- inertias at the center of gravity G are calculated by: . One obtains analytically:

$$I_{xx}(G)=\int_V\left(\left(y-y_G\right)^2+\left(z-z_G\right)^2\right)\rho\,dV\ ;\ I_{yy}(G)=\int_V\left(\left(x-x_G\right)^2+\left(z-z_G\right)^2\right)\rho\,dV\ ;$$

$$I_{zz}(G)=\int_V\left(\left(y-y_G\right)^2+\left(x-x_G\right)^2\right)\rho\,dV$$

One obtains analytically:

$$I_{xx}(G)=I_{YY}(G)=\frac{8\rho}{3}\left(4\times 0.15^2\times 2.05^2+4.1\times 0.15\times\left(0.39^3-0.24^3+0.18^3-0.03^3\right)\right)=16441.61\ ;$$

$$I_{zz}(G)=\frac{16\rho\times 4.1\times 0.15}{3}\left(0.39^3-0.24^3+0.18^3-0.03^3\right)=1285.71$$

- In the presence of fluid, the total mass corresponds to the sum of the solid mass and the fluid mass. Knowing total volume, one determines the volume occupied by the fluid:

$$Vol_{fluide}=Vol-Vol_{poutres}=0.84^2\times 4.1-16\times(1.5/10)^2\times 4.1=1.41696\,m^3 \quad ;$$

$$MASSE_{fluide}=1000\times 1.41696=1416.96\,kg \quad ;$$

$$MASSE_{totale}=MASSE_{solide}+MASSE_{fluide}=12695.076\,kg$$

One from of deduced the density from the element (with fluid):

$$\rho=\frac{MASSE_{totale}}{Vol}=4388.265\,kg/m^3$$

- The center of gravity remains unchanged with or without fluid.

- Inertias at the center of gravity G are:

$$I_{xx}(G)=I_{YY}(G)=\frac{2\,\rho\times 0.84\left(4.1\times 0.42^3+0.84\times 2.05^3\right)}{3}=18530.155 \quad ;$$

$$I_{zz}(G)=\frac{2\,\rho\times 0.84\times 4.1\times 2\times 0.42^3}{3}=1492.941$$

2.3 Bibliographical references

1) Walter D. Pilkey: "Formulated for Stress, Strain and Structural Matrixes", A Wiley-Interscience Publication JOHN WILEY & SOUNDS, Inc. Edition 1994.

3 Modelization A

3.1 Characteristic of the modelization

Modelization `3D_FAISCEAU`

Boundary conditions:

Case without fluid:

```
DDL_IMPO: (  GROUP_MA: SbDX   : 0DY  : 0DZ  : 0DRX : 0 DRY: 0 DRZ: 0
             GROUP_MA: ShDX   : 0DY  : 0DZ  : 0DRX : 0 DRY: 0 DRZ: 0
             NOEUD N1: PHI: 0                                  )
```

Case with fluid:

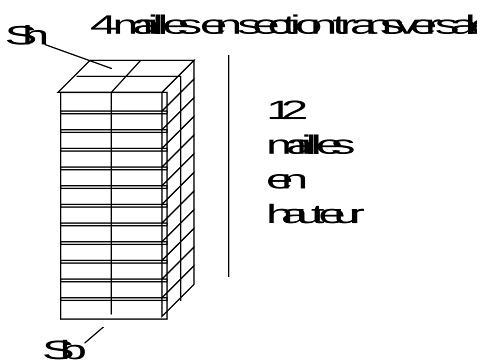
```
DDL_IMPO: (  GROUP_MA: SbDX   : 0DY  : 0DZ  : 0DRX : 0 DRY: 0 DRZ: 0
             GROUP_MA: ShDZ   : 0DRX : 0 DRY: 0 DRZ: 0
             NOEUD N1: PHI: 0                                  )
LIAISON_UNIF: (  GROUP_MA: HS
                DDL: "DX"   )
LIAISON_UNIF: (  GROUP_MA: HS
                DDL: "DY"   )
```

3.2 Characteristic of the mesh

The mesh of the homogenized medium used, for the two cases: with or without fluid, is represented by [fig 3.2-a].

It comprises 48 meshes `HEXA8`.

The mesh contains 9 straight lines parallel with average fiber of each beam.



Appear 3.2-a: mesh

3.3 Values tested

Case without fluid:

Sequence number	Quantity and unit	Reference

1,2 and 6	frequency (Hz)	3.3333
19 and 20	frequency (Hz)	9.2584

Cases with fluid:

Sequence number	Quantity and unit	Reference
1 and 2	frequency (Hz)	0.6908
19 and 20	frequency (Hz)	3.7871

Quantity and unit	Reference
Masses (kg)	12695.076
B I_{xx} Inertia G	in
18530.155 I_{zz} Inertia G	in

4 1492.941 Modelization

4.1 Characteristic of the modelization

Modelization 3D_FAISCEAU

Boundary conditions:

Case with fluid:

```
DDL_IMPO: (  GROUP_MA: SbDX   : 0DY  : 0DZ  : 0DRX : 0 DRY: 0 DRZ: 0
             GROUP_MA: ShDZ   : 0DRX : 0 DRY: 0 DRZ: 0
             NOEUD N1: PHI: 0                                    )
LIAISON_UNIF: (  GROUP_MA: HS
                DDL: "DX"   )
LIAISON_UNIF: (  GROUP_MA: HS
                DDL: "DY"   )
```

4.2 Characteristic of the mesh

The mesh of the homogenized medium used, for the two cases: with or without fluid, is represented by [fig 5.2-a].

It comprises 48 meshes HEXA20.

The mesh contains 9 straight lines parallel with average fiber of each beam.

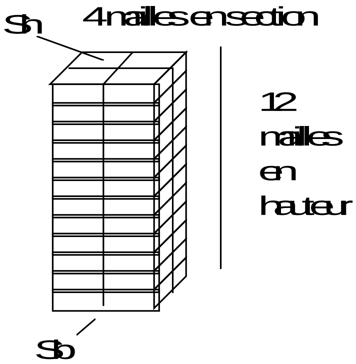


Figure 5.2-a : mesh

4.3 Values tested

Case with fluid:

Sequence number	Quantity and unit	Reference
1 and 2	frequency (Hz)	0.6908
19 and 20	frequency (Hz)	3.7871

Quantity and unit	Reference
Masses (kg)	12695.076
Inertia I_{xx} in G	18530.155
Inertia I_{zz} in G	1492.941

5 Summary of the results

the results show the good modal behavior of the elements of modelization 3D_FAISCEAU in bending, in absence of the fluid. It show also very a good agreement of the frequencies of the overall modes with Aster computation into heterogeneous, when there is fluid.

For the frequencies of the overall modes, one does not observe differences between a mesh HEXA8 and HEXA20 (this is not true for the other modes).

The numerical results of the masses and inertias obtained by *Code-Aster* are very close to the analytical results (error $<6.10^{-6}$).