SDLV100 - Vibration of a slender beam of variable rectangular section (embed-free)

Summarized:

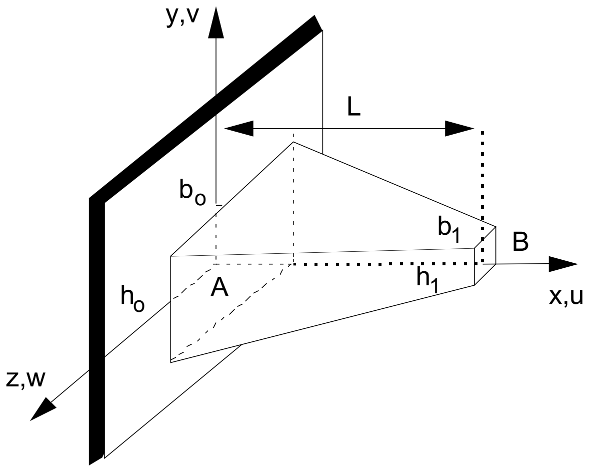
The studied structure is a free steel beam clamped with rectangular variable section modelled by voluminal elements. One is interested in his eigenfrequencies in bending. With the same problem is dealt in modelization beam in the case test SDLL09.

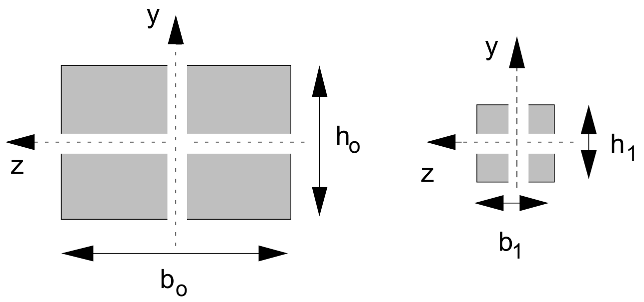
This problem makes it possible to test voluminal elements MECA_HEXA20 and MECA_PENTA15 in modal analysis. It also makes it possible to test option MASS_MECA_DIAG of computation of the diagonalized mass matrixes for the voluminal modelizations.

The reference solution is a numerical solution obtained using the computer code by finite elements the SAMCEF software for similar modelizations. The got results are also in concord with the semi-analytical results given in guide VPCS.

1 Problem of reference

1.1 Geometry





Sections rectangulaires

Length of beam:

$L=1\,m$

Rectangular section:

	Cross-section initial	**Cross-section final**
height:	$h_o=0.04\,m$	$h_1=0.01\,m$
width:	$b_o=0.04\,m$	$b_1=0.01\,m$
area:	$A_o=1.6\,10^{-3}\,m^2$	$A_1=1.10^{-4}\,m^2$
inertia:	$Iz_o=2.1333\,10^{-7}\,m^4$	$Iz_1=8.3333\,10^{-10}\,m^4$

Coordinates of the points (in meters)

	A	B
x	0.	1.
y	0.	0.
z	0.	0.

1.2 Properties of steel

$E=2.10^{11}\,Pa$

$\rho=7800\,kg/m^3$

1.3 Boundary conditions and loadings

Point: A embedded $u=v=z=0$

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is obtained using the computation software by finite elements the SAMCEF software for identical modelizations but with elementary matrixes of mass coherent.

One points out the analytical solution given in file SDLL09/89 of guide VPCS. The differential equation in bending of the beam considered, in theory of Eulerian-Bernoulli is written (Theory of Eulerian - Bernoulli):

$$\frac{\partial^2\left(E\,I_z\,\frac{\partial^2 v}{\partial x^2}\right)}{\partial x^2}=-\rho\,A\,\frac{\partial^2 v}{\partial t^2}$$

where I_z and A vary with the X-coordinate.

The eigenfrequencies are then of the form:

$$f_i=\frac{1}{2\pi}\lambda_i(\alpha,\beta)\frac{h_1}{L^2}\sqrt{\frac{E}{12\rho}}$$

with $\alpha=\frac{h_0}{h_1}=4$ and $\beta=\frac{b_0}{b_1}=4$.

For this value of α and β , the first values of the continuation (λ_i) are:

	λ_1	λ_2	λ_3	λ_4	λ_5
$\beta=4$	23.289	73.9	165.23	299.7	478.1

2.2 Results of reference

the results of reference selected are the first 5 eigenfrequencies of the modes of bending.

2.3 Uncertainty on the analytical

solution Solution in beam theory of Bernoulli, and numerical solution the SAMCEF software.

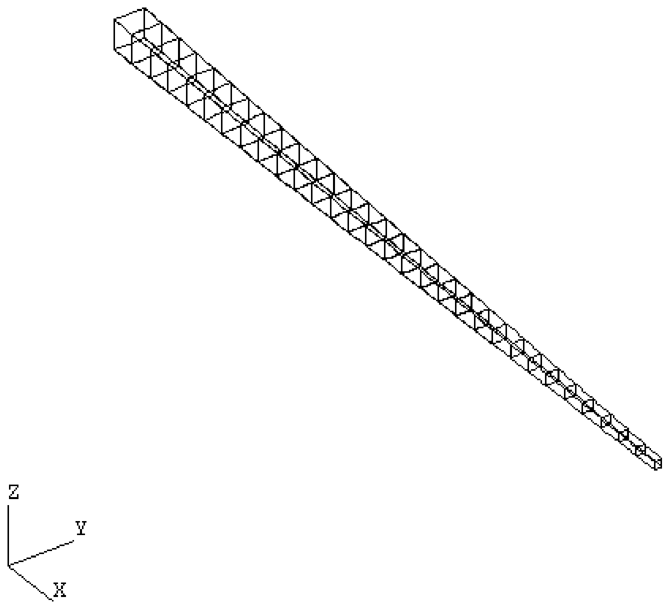
2.4 Bibliographical references

1) H.H. MABIE, C.B. ROGERS, Transverse vibrations of double-tapered cantilever beams - Newspaper of the Acoustical Society of America, n° 51, p. 1771-1774 (1972).

3 Modelization A

3.1 Characteristic of the modelization

Volume elements MECA_HEXA20



Discretization: beam AB: 30 meshes HEXA20
(1 mesh in the section)

Boundary conditions:

in all the nodes `DDL_IMPO: (TOUT: “OUI' DZ: 0.)`
at the end A (G_1 nodes group) `(GROUP_NO: G_1 DX: 0. , DY: 0)`

3.2 Characteristic of the mesh

Mesh: Many nodes: 368
Number of meshes and type: 30 HEXA20

3.3 Values tested

Identification	analytical Solution beam frequency	Reference the SAMCEF software in HZ	*Aster* in HZ	% difference the Aster-SAMCEF software
coherent matrix				
bending 1	54.18	56.84	56.85	0.0176%
bending 2	171.94	180.0	180.08	0.0444%

bending 3	384.40	401.0	401.23	0.0574%
bending 4	697.24	723.2	724.02	0.1134%
bending 5	1112.28	1145.41	1147.51	0.1833%

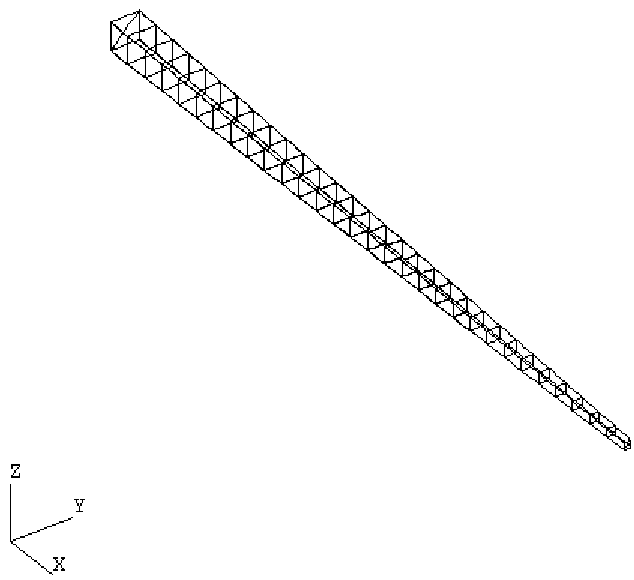
diagonal matrix

bending 1	54.18	56.84	56.78	– 0.1033%
bending 2	171.94	180.00	179.57	– 0.2419%
bending 3	384.40	401.00	399.24	– 0.4408%
bending 4	697.24	723.20	718.69	– 0.6273%
bending 5	1112.28	1145.41	1136.01	– 0.8273%

4 Modelization B

4.1 Characteristic of the modelization

Volume elements MECA_PENTA15



Discretization: beam: AB 60 meshes PENTA15 (2 meshes in the section)

Boundary conditions:

in all the nodes DDL_IMPO: (TOUT: "OUI' DZ: 0.)
at the end A (G_1 nodes group) (GROUP_NO: G_1 DX: 0. , DY: 0)

4.2 Characteristic of the mesh

Mesh: Many nodes: 368
Number of meshes and type: 60 PENTA15

4.3 Values tested

Identification	semi-analytical Solution beam	Reference the SAMCEF software	*Aster*	% difference ASTER-SAMCEF
	frequency	in HZ	in HZ	
consistent matrix				
bending 1	54.18	56.84	56.82	– 0.038%
bending 2	171.94	180.00	179.96	– 0.022%
bending 3	384.40	401.00	400.93	– 0.018%
bending 4	697.24	723.20	723.41	0.029%
bending 5	1112.28	1145.41	1146.41	0.088%

diagonal matrix

bending 1	54.18	56.84	56.76	– 0.149%
bending 2	171.94	180.00	179.51	– 0.272%
bending 3	384.40	401.00	399.25	– 0.437%
bending 4	697.24	723.20	719.	– 0.583%
bending 5	1112.28	1145.41	1140.	– 0.740%

5 Summary of the results

the differences between the results of computations *Code_Aster* and the SAMCEF software with coherent masses are lower than 0.2%.

The differences between the SAMCEF software and *computation results* Code_Aster with diagonal masses with coherent masses remain lower than 1%.

These results are in conformity so that one could wait, and validate in a reliable way computations of eigenfrequencies in *Code_Aster* by MODE_ITER_INV and operator CALC_MATR_ELEM out of coherent masses as out of diagonal masses.