SDLS505 - Buckling of a cylindrical envelope under external pressure

Abstract:

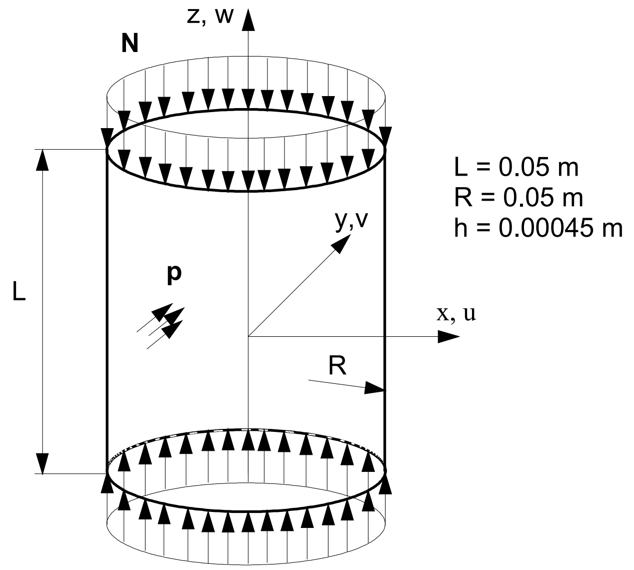
This test represents a computation of stability of a thin cylindrical envelope stopped at its ends subjected to an external pressure and an axial pressure. One calculates the critical loads leading to the elastic buckling of Eulerian. The geometrical stiffness matrix used in the resolution of the problem to the eigenvalues is that which is due to the initial stresses.

It makes it possible to validate the modelization finite elements COQUE_3D with meshes the TRIA7 and QUAD9.

The critical load and the eigen mode obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



the symmetry of the problem makes it possible to model a half rolls length $L/2$, stopped with the one of its ends, with conditions of symmetry specific to lower edge.

1.2 Properties of the material

the properties of the material constituting the plate are:

$E=2.10^{11}Pa$	Modulus Young
$\nu=0.3$	Poisson's ratio

1.3 Boundary conditions and loadings

Loading:

pressure uniformly distributed of $p_{cr}=1.523\,10^6\,Pa$ on the cylindrical part. This pressure corresponds to the value of the critical load,

force distributed on the contour of the bottom $N=0.5\times R\cdot p_{cr}=3.8075\,10^4\,N/m$.

1.4 Initial conditions

Without Reference solution

2 object

2.1 Method of calculating used for the reference solution

the critical pressure is given in [bib1] or [bib2] by the following statement:

$$P_{cr}=\frac{Eh}{R}\frac{1}{\left(n^2+\frac{b^2}{2}\right)}\left[\frac{1}{\left(\frac{n^2}{b^2}+1\right)^2}+\frac{h^2}{12R^2(1-\nu^2)}(n^2+b^2)^2\right]$$

with $b=\frac{\pi R}{L}$

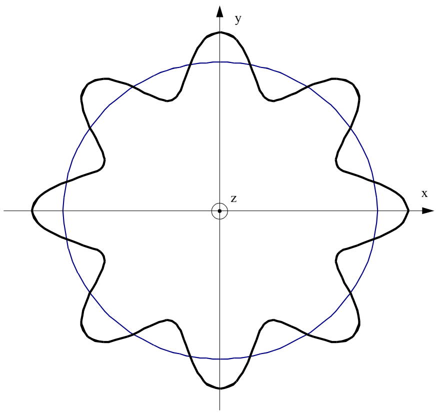
n the number of circumferential modes This

formula represents is valid if $N=0.5\,R\,p_{cr}$.

2.2 Results of reference

Pour les caractéristiques géométriques données, la pression critique est minimum pour un nombre n=8 et vaut p_{cr} = 1.523 10^6Pa.

Le nombre n est obtenu à partir d'un abaque.



2.3 Uncertainties on the analytical

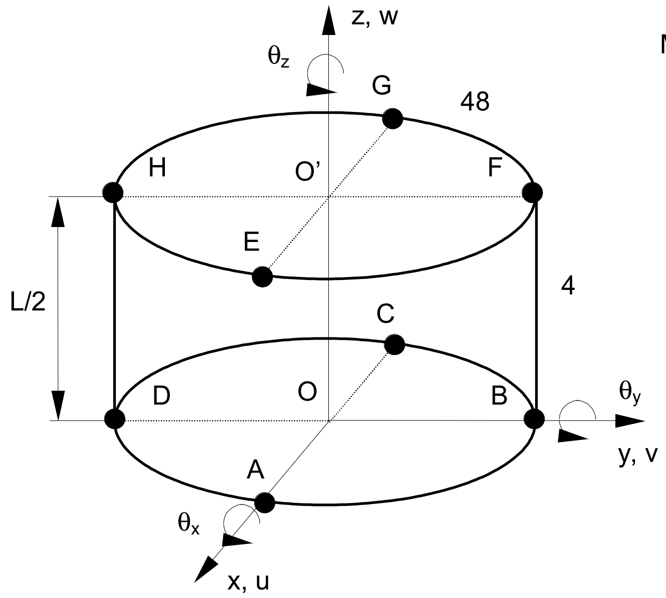
solution Solution

2.4 bibliographical References

1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, page 500, second edition, DUNOD 1966.
2) BO O. ALMROTH, D.O. BRUSH: Buckling of bars, punts and shells, page 173, Mc Graw-Hill, New York, 1975.

3 Modelization A

3.1 Characteristic of the modelization



Modélisation COQUE_3D (TRIA7)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

3.2 Characteristics of the mesh

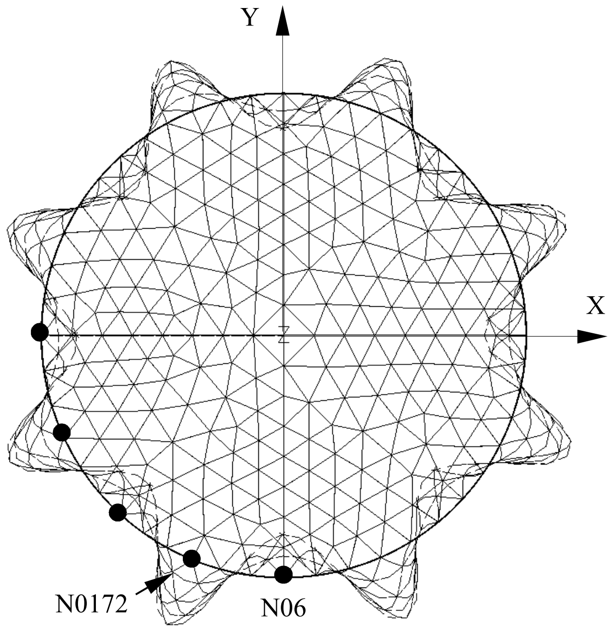
Many nodes: 2464
Number of meshes and types: 834 `TRIA7`

3.3 Quantities tested and results

Identification	Reference	*Aster*	% critical
difference Pressure ($n=8$)	– 1.523 106 Pa	– 1.6862 106 Pa	10.715
Displacement x with the node $N06$	– 0.0102.	– 0.0102.	0.
Displacement y with the node $N06$	1.	1.	0.
Displacement x to the node $N0172$	– 0.392	– 0.376	– 4.09
Displacement y with the node $N0172$	– 0.928	– 0.9498	2.352

3.4 Remarks

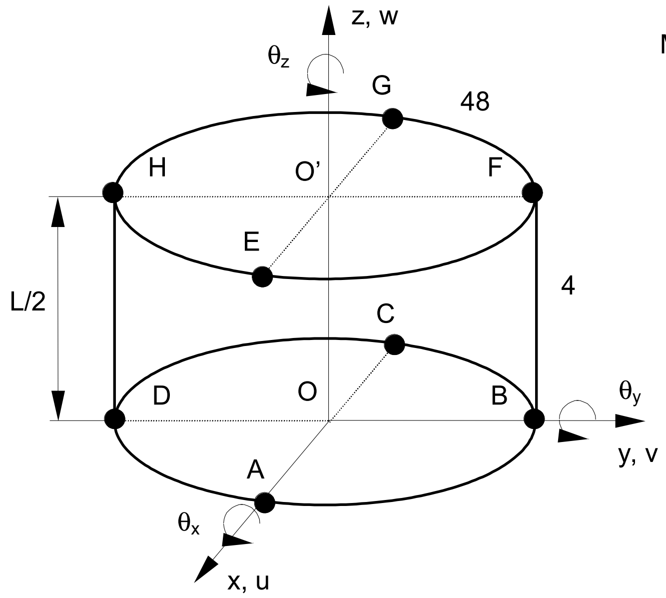
the angular position of the "bumps" is defined except for an angle, which makes difficult the checking of the deformed shape modal. To check the modal deformed shape associated with the mode with buckling, we validated the deformed shape graphically. Then we deferred in the node and command file (Operator TEST_RESU) DX DY displacements $N06$ characterizing this deformed shape, which explains the null variation obtained on displacements. The displacement of the node $N0172$ can then be immediately deduced from that of the node $N06$ of way analytical by means of the forms of the eigen modes given in [bib1] and by considering that it is in opposition of phase with the node $N06$. The value of displacements obtained of the kind is not very precise owing to the fact that the angle between the two nodes is not worth exactly $22°5$.



- Les nœuds N06 et N0172 sont situés dans le plan Z=0. L'angle entre ces deux nœuds est de 22°5 (Demi longueur d'onde) approximativement.

4 Modelization B

4.1 Characteristic of the modelization



Modélisation COQUE_3D (QUAD9)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

4.2 Characteristics of the mesh

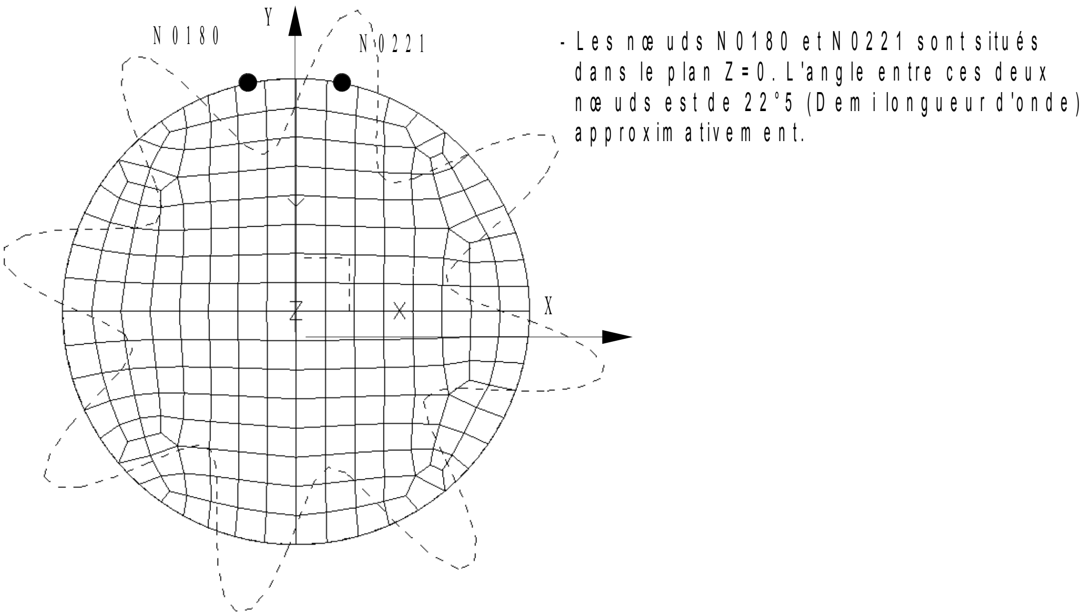
Many nodes: 1802
Number of meshes and types: 472 `QUAD9`

4.3 Quantities tested and results

Identification	Reference	*Aster*	% critical
difference Pressure ($n=8$)	– 1.523 106 Pa	1.5576 106 Pa	2.272
Displacement x of the node $N0180$	0.13596	0.1003	26.258
Displacement y of the node $N0180$	– 0.7744	– 0.66172	– 14.551
Displacement x of the node $N0221$	0.17743	0.17743	0.
Displacement y of the node $N0221$	0.765965	0.765965	0.

4.4 Remarks

the angular position of the "bumps" is defined except for an angle, which makes difficult the checking of the deformed shape modal. To check the modal deformed shape associated with the mode with buckling, we validated the deformed shape graphically. Then we deferred in the node and command file (Operator TEST_RESU) DX DY displacements $N0221$ characterizing this deformed shape, which explains the null variation obtained on displacements. The displacement of the node $N0180$ can then be immediately deduced from that of the node $N0221$ of way analytical by means of the forms of the eigen modes given in [bib1] and by considering that it is in opposition of phase with the node $N0221$. The value of displacements obtained of the kind is not very precise owing to the fact that the angle between the two nodes is not worth exactly $22°5$.



5 Summary of the results

the got results are satisfactory. Uncertainties on the critical pressure do not exceed:

10.71% for modelization COQUE_3D with meshes TRIA7,
2.3% for modelization COQUE_3D with meshes QUAD9.

The modal deformed shape obtained corresponds well to the expected circumferential mode: $n=8$ for the two modelizations.

It is noted that modelization COQUE_3D with meshes QUAD9 is more precise than modelization COQUE_3D with meshes TRIA7.

This test made it possible to test modelization COQUE_3D in linear buckling of Eulerian of a thin structure subjected to an external pressure.