# SDLS504 - Side buckling of a beam (discharge)

**Summarized:**

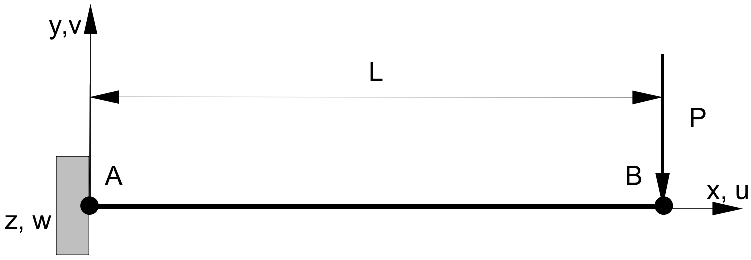
This test represents a computation of stability of a beam (IPE) comforts subjected to a bending stress at an end. One calculates the critical load leading to elastic buckling by discharge. The geometrical stiffness matrix used in the resolution of the problem to the eigenvalues is that which is due to the initial stresses.

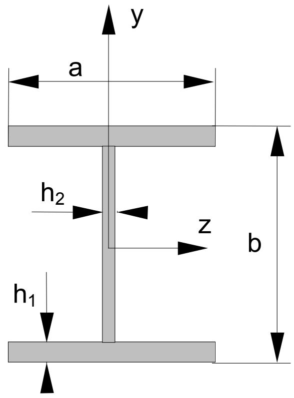
This test makes it possible to validate the modelization finite elements `COQUE_3D` with meshes `the TRIA7` and `QUAD9` in the field of the linear buckling of Eulerian.

The critical load and the associated eigen mode are compared with an analytical reference solution.

# 1 Problem of reference

## 1.1 Geometry




| | | | | |
|---|---|---|---|---|
| L | = | 2 m | | $A = 3.3645\ 10^{-3}\ m^2$ |
| a | = | 0.09m | | $I_y = 1.3792\ 10^{-6}\ m^4$ |
| b | = | 0.2 m | | $I_z = 2.1617\ 10^{-5}\ m^4$ |
| $h_1$ | = | 0.0113 m | | $J = 1.0894\ 10^{-7}\ m^4$ |
| $h_2$ | = | 0.0075 m | | |

## 1.2 Properties of the material

the properties of the material constituting the plate are:

$E = 2.10^{11}\ Pa$ Modulus
$\nu = 0.3$ Boundary conditions Young Poisson's ratio

## 1.3 and loadings

Fixed support at the point $A$
One The computation applies $P = -104\,797.82\,N$ a force to $B$ the point corresponding to the critical load given in [bib1

## 1.4 ]

Initial conditions Without

# 2 Reference solution object

## 2.1 Method of calculating used for the reference solution

of the critical load of discharge is given in detail in [bib1].

| | |
|---|---|
| $P_{cr}=\gamma_2 \frac{\sqrt{E I_y C}}{L^2}$ | critical load of discharge |
| with $C=GJ$ | torsional rigidity |
| $J=((b-2h_1)h_2^3+2a h_1^3)$ | constant of torsion [bib2] |
| $C_1 E I_y \frac{b^2}{2}$ | stiffness with warping corresponding to a beam in I |

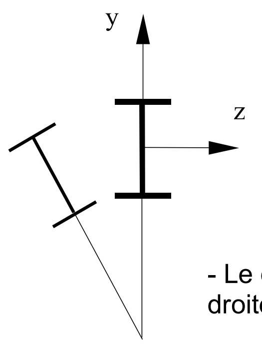
**numerical Application:**

$$C=8578.515\, N.m^2$$
$$C1=5516.8\, N.m^4$$
$$\frac{L^2 C}{C_1}=6.22$$

The value of $\gamma_2$ depends on the ratio $\frac{L^2 C}{C_1}$. In our case $\gamma_2$ is worth 8.617. This value is extracted from a table given in [bib1]. What gives us $P_{cr}=104797.82\, N$

## 2.2 Results of reference

Critical load of discharge and associated mode.




Déformée au point
d'application de la charge.

$P_{cr} = -104797.82 N$

- Le déversement peut apparaître à droite comme à gauche indifféremment

## 2.3 Uncertainties on the analytical

solution Solution

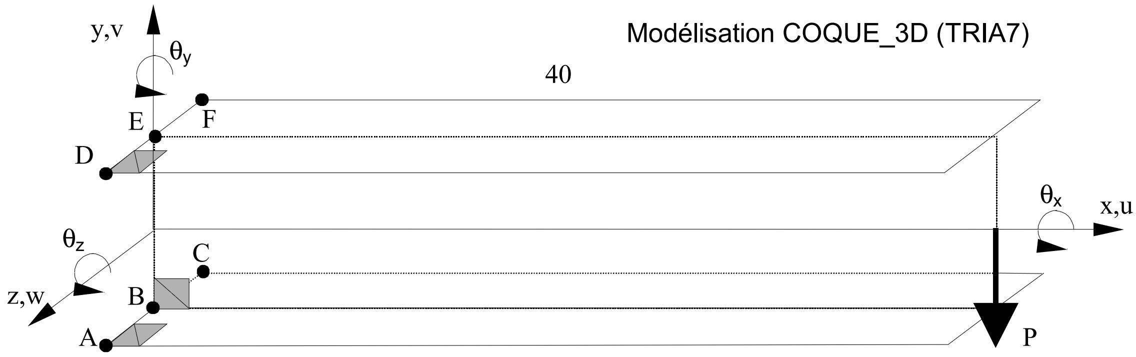
## 2.4 bibliographical References

1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, second edition, DUNOD 1966.

2) S.P. TIMOSHENKO: Strength of materials, Volume 2: DUNOD 1968.

# 3 Modelization A

## 3.1 Characteristic of the modelization




Découpage :
- cotés AB, BC, DE, EF 2 éléments
- coté EB 4 éléments

Conditions aux limites :
- cotés AB, BC, DE, EF et EB :
$u = v = w = \theta_x = \theta_y = \theta_z = 0$

## 3.2 Characteristics of the mesh

Many nodes: 3022
Number of meshes and types: 960 `TRIA7`

## 3.3 Quantities tested and results

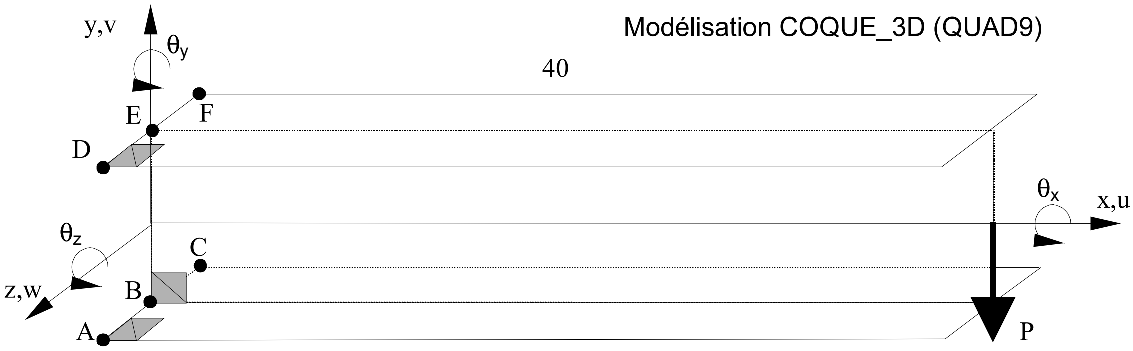
| Identification | Times | Reference | *Aster* | % difference |
|---|---|---|---|---|
| Critical load (mode 1) | | 104 797.82 N | 107 753.49 | 2.820 |
| Critical load (mode 2) | | – 104 797.82 N | – 107 878.78 | 2.940 |

## 3.4 Remarks

the two critical loads obtained are similar in amplitude, but of contrary sign. The associated modes of discharge are identical. In this case, the critical load associated with the loading applied corresponds to the second found critical load (Mode 2).

# 4 Modelization B

## 4.1 Characteristic of the modelization




Découpage :
- cotés AB, BC, DE, EF 2 éléments
- coté EB 4 éléments

Conditions aux limites :
- cotés AB, BC, DE, EF et EB :
$u = v = w = \theta_x = \theta_y = \theta_z = 0$

## 4.2 Characteristics of the mesh

Many nodes: 2106
Number of meshes and types: 480 `QUAD9`

## 4.3 Quantities tested and results

| Identification | Times | Reference | *Aster* | % difference |
|---|---|---|---|---|
| Critical load (mode 1) | | – 104 797.82 N | – 97 636.39 | – 6.834 |
| Critical load (mode 2) | | 104 797.82 N | 97 636.39 | – 6.834 |

## 4.4 Remarks

the two critical loads obtained are similar in amplitude, but of contrary sign. The associated modes of discharge are identical. In this case, the critical load associated with the loading applied corresponds to the first found critical load (Mode 1).

# 5 Summary of the results

For each modelization, one obtains two similar critical loads but of contrary sign. The associated modes of discharge are identical. The negative critical loads correspond to the loading applied while the positive critical loads correspond to the opposite loading. If one disregards sign of the loading both critical loads really exist.

The critical loads relating to the loading applied are correct. The errors obtained do not exceed:

$3\%$ for modelization COQUE_3D with meshes TRIA7,
$7\%$ for modelization COQUE_3D with meshes QUAD9.

It is noted that modelization COQUE_3D with meshes TRIA7 is more precise than modelization COQUE_3D with meshes QUAD9.

This test made it possible to test modelization COQUE_3D in linear buckling of Eulerian.