SDLS503 - Vibrations of bending of a Summarized sandwich beam

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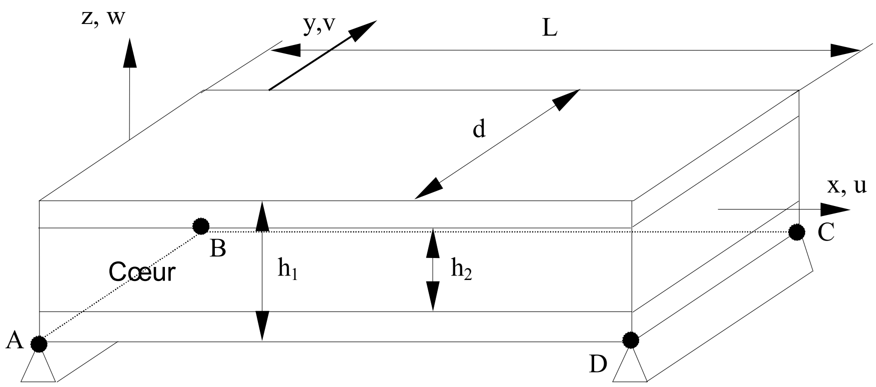
This test represents a computation in modal analysis of a sandwich beam simply supported. This test makes it possible to validate:

the modelization finite elements `DKT` with meshes `QUAD4` and `TRIA3`,
the modelization finite elements `DST` with meshes `QUAD4` and `TRIA3`,
the taking into account of the stiffness in transverse shears,
the taking into composite account.

The frequencies and the modes obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



L = 1.0 m
d = 0.1m
h_1 = 0.1m
h_2 = 0.05m

1.2 Properties of the material

Coatings:	$E_x = 4.10^{10} Pa$	$G_{xz} = 4.10^9 Pa$	$\nu_{xz} = 0.3$	$\rho_1 = 2000 \, kg/m^3$
Heart:	$E_x = 4.10^7 Pa$	$G_{xz} = 1.5.10^7 Pa$	$\nu_{xz} = 0.3$	$\rho_2 = 50 \, kg/m^3$

Shear coefficient K : $1/K = 110.8$

The Poisson's ratios are identical: $\nu_{xz} = \nu_{xy} = \nu_{yz}$

1.3 Boundary conditions and loadings

the beam rests simply on with dimensions ones AB and CD .

1.4 Initial conditions

Without Reference solution

2 object

2.1 Method of calculating used for the reference solution

The computation is carried out starting from the relations of dynamic equilibrium and behavior [bib2] pointed out Ci - afterwards:

$$\frac{\partial M_x}{\partial X}+T_y=\langle \rho I\rangle \frac{\partial^2 \theta}{\partial t^2} \qquad \frac{\partial T_y}{\partial X}=\langle \rho S\rangle \frac{\partial^2 v}{\partial t^2}$$

$$M_z=\langle E I\rangle \frac{\partial \theta_z}{\partial X} \qquad T_y=K\langle G S\rangle \frac{\partial v}{\partial X}-\theta_z$$

These relations make it possible to write the equation of the motion of dynamic bending transverse $v(x,t)$. One obtains the equation with the eigenfrequencies after having associated the boundary conditions.
The equation with the eigenfrequencies is written:

$$\sin(X_2)=0 \quad \text{with} \quad X_2=\left[\bar{\omega}^2\frac{(1+a)}{2}+\sqrt{\bar{\omega}^2\left(\bar{\omega}^2\left(\frac{1-a}{2}\right)^2+\frac{1}{r^2}\right)}\right]^{1/2}$$

and

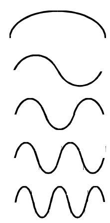
$$\bar{\omega}^2=\frac{\langle \rho I\rangle \omega^2 l^2}{\langle E I\rangle} \;;\; \bar{r}^2=\frac{\langle \rho I\rangle}{\langle \rho S\rangle l^2} \;;\; a=\frac{\langle \rho S\rangle\langle E I\rangle}{K\langle \rho I\rangle\langle G S\rangle}$$

The solutions of the equation to the eigenfrequencies are written then: $X_2=n\pi \quad (n=1,2,3,\ldots)$

2.2 Results of reference

the first 5 frequencies and associated eigen modes of bending.

Frequency mode 1:	$64.476\,Hz$
Frequency mode 2:	$131.918\,Hz$
Frequency mode 3:	$198.734\,Hz$
Frequency mode 4:	$265.383\,Hz$
Frequency mode 5:	$331.963\,Hz$



2.3 Uncertainties on the solution

the reference solution is calculated in the frame of the assumptions of the theory of the beams [bib2]: $\sigma_y=\sigma_z=0$.

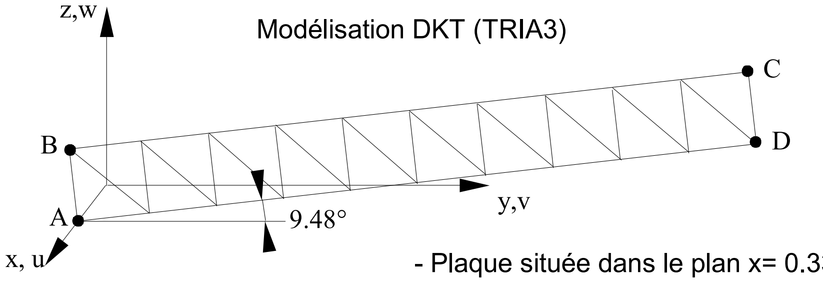
2.4 Bibliographical references

1) VPCS: Software package of composite structural analysis; Examples of validation. Review of the composites and advanced materials, Volume 5 - number except series 1995 - Hermes Edition.

2) CIEAUX J.M.: Dynamic bending of the composite beams with orthotropic phases; Validity of the quasi-static field, thesis of the university Paul Sabatier Toulouse III, 1988.

3 Modelization A

3.1 Characteristic of the modelization



3.2 Characteristics of the mesh

Many nodes: 22
Number of meshes and type: 20 TRIA3

3.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64.476	277.449	330.
Frequency mode 2	131.918	1105.83	738.
Frequency mode 3	198.734	2473.80	1.14E3
Frequency mode 4	265.383	4363.97	1.54E3
Frequency mode 5	331.963	6753.904	1.93E3

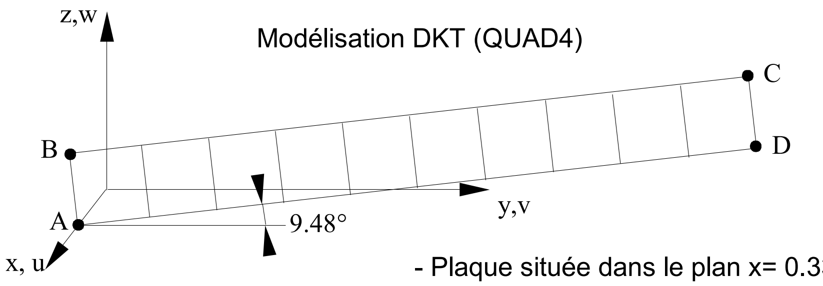
3.4 Remarks

In the table of results, we deferred the frequencies whose modes are identical to the modes of reference.

the effects of the transverse shears are neglected in the modelization DKT,
the Aster results are much higher than the results of reference,
appearance of a mode of membrane between modes 2 and 3 and modes 5 and 6 of reference.

4 Modelization B

4.1 Characteristic of the modelization



- Plaque située dans le plan x= 0.33
- Conditions aux limites : Cotés AB et CD : u=0

4.2 Characteristics of the mesh

Many nodes: 22
Number of meshes and type: 10 QUAD4

4.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64.476	277.788	331
Frequency mode 2	131.918	1111.225	742.
Frequency mode 3	198.734	2500.930	1.16E3
Frequency mode 4	265.383	4449.073	1.52E3
Frequency mode 5	331.963	6960.324	2.00E3

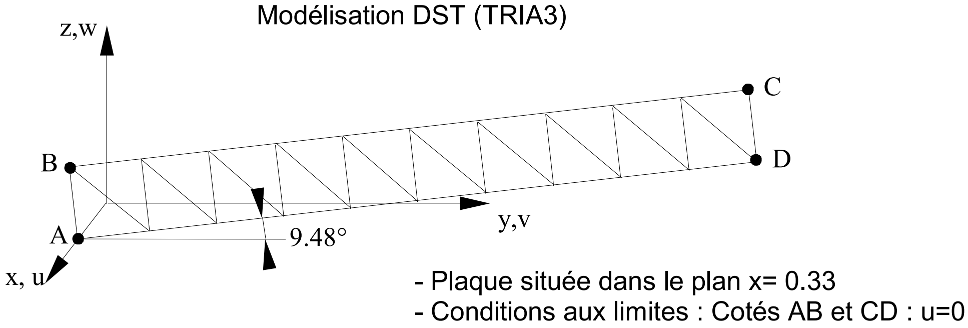
4.4 Remarks

In the table of results, we deferred the frequencies whose modes are identical to the modes of reference.

the effects of the transverse shears are neglected in the modelization DKT,
the Aster results are much higher than the results of reference,
appearance of a mode of membrane between modes 2 and 3 and modes 5 and 6 of reference.

5 Modelization C

5.1 Characteristic of the modelization



5.2 Characteristics of the mesh

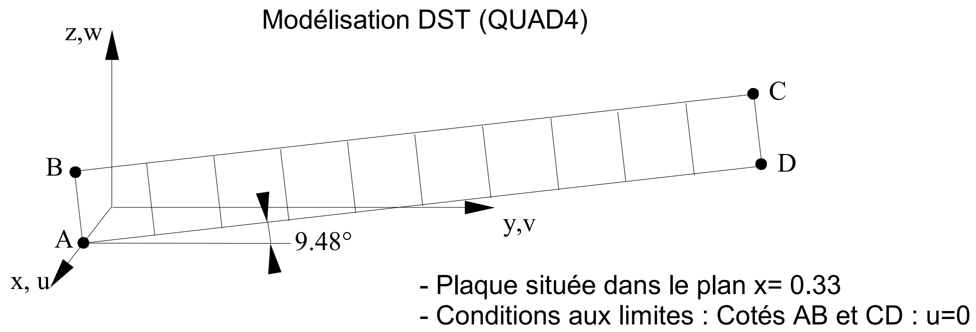
Many nodes: 22
Number of meshes and type: 20 `TRIA3`

5.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64.476	64.573	0.150
Frequency mode 2	131.918	133.987	1.568
Frequency mode 3	198.734	206.046	3.679
Frequency mode 4	265.383	282.875	6.591
Frequency mode 5	331.963	365.919	10.229

6 Modelization D

6.1 Characteristic of the modelization



6.2 Characteristics of the mesh

Many nodes: 22
Number of meshes and type: 10 `QUAD4`

6.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64.476	64.595	0.184
Frequency mode 2	131.918	131.495	-0.320
Frequency mode 3	198.734	196.861	-0.942
Frequency mode 4	265.383	260.247	-1.935
Frequency mode 5	331.963	320.409	-3.480

7 Summary of results

The modelization DKT is not adapted to model this benchmark, the errors are very important. Formulation DKT does not take into account the transverse shears contrary to modelization DST. For this kind of example, where the structure makes up of a composite and relatively thick ($h/L=0.1$), it is preferable to use modelization DST.

The results got with DST are:

satisfactory for the first 3 frequencies with mesh TRIA3 and for the first 5 frequencies for mesh QUAD4 with a better accuracy for the mesh QUAD4,
the error of 10% for 4th and 5th frequency with mesh TRIA3 is significant. A finer mesh should make it possible to improve the results by having a better representation of the last modes.