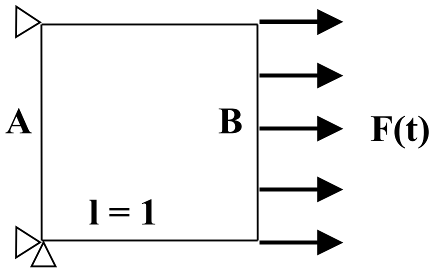
SDLS115 – Comparison with the analytical solution of a plate in tension

Summarized:

This test validates by the operator the basic operation of `modelization` DKT for a transient computation with an explicit diagram of numerical integration `DYNA_NON_LINE`. The plate is subjected to the boundary conditions corresponding to a simple tension, making it possible to find the analytically calculated response.

1 Problem of reference

1.1 Geometry



Plates square :
Length: $l=1.0\,m$
Thickness: $e=0.1\,m$

1.2 Properties of the material

Young's modulus, $E=4.388\,10^{10}\,N/m^2$
Poisson's ratio, $\nu=0.0$
Density, $\rho=2500.0\,kg/m^3$

1.3 Boundary conditions and loadings

On with dimensions one A horizontal displacement is imposed $u_x=0.0$.
One B applies the linear force to with dimensions one in the direction x , which depends on time like

$$F(t)=Q_0\,E\,K\,e\cos(Kl)\sin(\omega t)$$,

where the following parameters are used:

- Q_0 ($=10^{-4}m$) - amplitude of the loading
- E – Young's modulus defined above (in N/m^2)
- e – the thickness defined above (in m)
- l – the dimension of the plate defined above (in m)
- K ($=\frac{\pi}{81}$) the wave number of the analytical solution (in m^{-1})
- ω – frequency (time 2π), dependant on the wave number K $K=\omega/c$, c being the celerity of the waves in structure, $c=\sqrt{\frac{E}{\rho}}$

the introduced parameter setting makes it possible to apply the loading right to obtain the analytical, simply given solution by the parameters Q_0 and K , and then by other parameters of dimensions and properties structural material.

1.4 Initial conditions

At the beginning displacements are worth zero everywhere and the velocities obey the following spatial distribution,

$$v_0(x, y) = \omega Q_0 \sin(K.x)$$

2 Reference solution

2.1 Method of calculating

One deals here with a problem of structure (quasi) - unidimensional subjected to an edge force $F(t)$, where the analytical solution can be written like,

$$u(x,t)=Q_0\cos(Kx)\sin(\omega t)$$

In order to obtain this solution one must apply the force and the initial conditions specified above. The parameters are also commented on there.

2.2 Quantities and results of reference

It is displacement x to the node $N2$ and the time $t_{max}=0.0012\,s$, which must be equal to

$$u(x_{N2},t)=Q_0\cos(K\,x_{N2})\sin(\omega\,t_{max})$$

the value being calculated in the data file starting from the selected values of the parameters.

2.3 Uncertainties on the solution

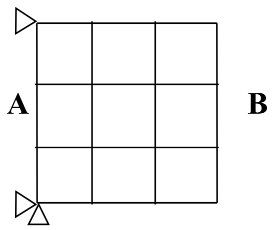
exact Solution.

2.4 Bibliographical references

S. Timoshenko, *Theory of vibrations*, 1939

3 Modelization A

3.1 Characteristic of the modelization



Modelization: **DKT**

Boundary conditions:
A – edge embedded
B – linear force

3.2 Characteristics of the mesh

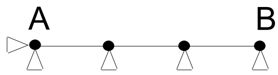
Nodes: 16
Meshes: 9 QUAD4

3.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
Displacement DX in $N2$	3.51957D-05	3.51967D-05	0.003

4 Modelization B

4.1 Characteristic of the modelization



Modelization: **BAR**

Boundary conditions:
A – node embedded
B - force

4.2 Characteristics of the mesh

Nodes: 4
Meshes: 3 SEG2

4.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
DX Displacement out of N2	3.51957D-05	3.51967D-05	1.11D-02%
Kinetic energy of the third mesh	9.10387D-02	the purpose of 0.091038669129169	3.39D-05%

5 Summary of the results

This test are principal to check if the combination of modelization DKT and operator DYNA_NON_LINE functions correctly. The difference between the solution Aster and that of the reference is very weak.