
SDLS114 - Computation of the stress intensity factors of a plate fissured by modal recombination

Abstract

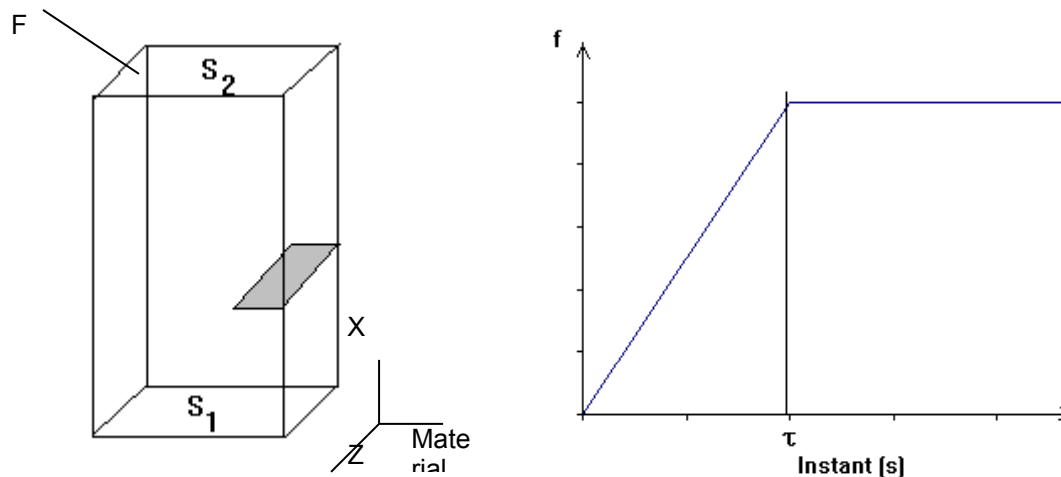
This test aims at validating the computation of the factors of intensity of a plate fissured by modal recombination. The modal factors of intensity, therefore associated with each eigen mode of vibration of structure, are calculated with operator `CALC_G` (option `CALC_K_G`). The modelizations illustrate the possibility of combining the modal factors of intensity directly in the command file by instructions python via operator `POST_K_TRANS`.

This test contains two modelizations 2D (modelization A and C) and two modelizations 3D (B and D). The reference solution is resulting from a direct temporal resolution of the transitory problem. The two modelizations A and B are for cracks with a grid. The two modelizations C and D are for cracks NON-with a grid (X-FEM).

1 Problem of reference

1.1 Geometry

One considers a plate height $h=0,1\text{ m}$, width $l=0,05\text{ m}$ and thickness $e=0.005\text{ m}$. A crack is positioned in the middle of the height of the beam, with a depth of $0,1\text{ l}$.



1.2 One

considers there the classical properties of a steel:

Young modulus:	$E = 2.10^5\text{ MPa}$
Poisson's ratio:	$\nu = 0.3$
Density	$\rho = 7800\text{ kg/m}^3$

1.3 Boundary conditions and loadings

the plate is:

- 1) embedded on surface S_1 ;
- 2) subjected to a force $F(t)$ on surface S_2 .

The evolution of the norm of $F(t)$ is traced on the figure above. One takes $\tau = 0,001\text{ s}$. The direction of the force $F(t)$ is the following one:

- 1) $F(t) = f(t) \cdot e_x$ for the modelization A;
- 2) $F(t) = (a e_x + b e_y + c e_z) f(t)$ for the modelization B, with $b = 2a$ and $c = 0.4 a$.

For the modelization A, one blocks displacements in the direction z (plane problem).

2 Reference solution

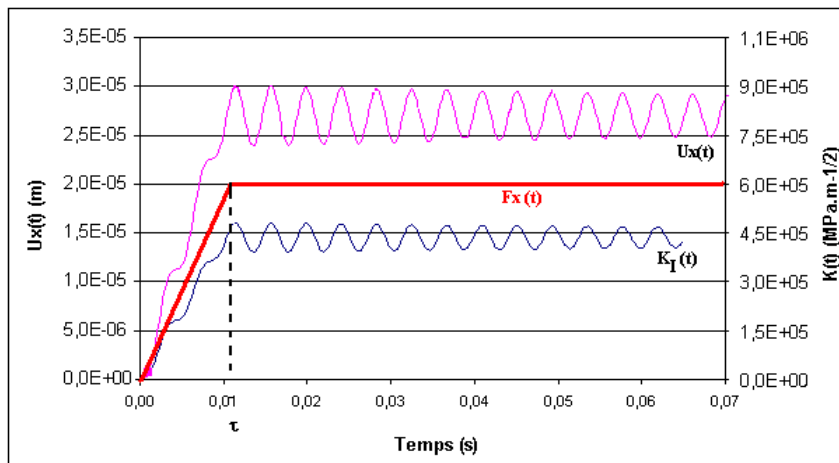
2.1 Method of calculating used

the reference solution is that obtained by a direct temporal resolution of the transitory problem. Operator `DYNA_VIBRA` is used to identify the fields of displacement, with a diagram of integration in times of Newmark.

The evolution of the stress intensity factors according to time is then calculated by interpolation of the jumps of displacements (operator `POST_K1_K2_K3`).

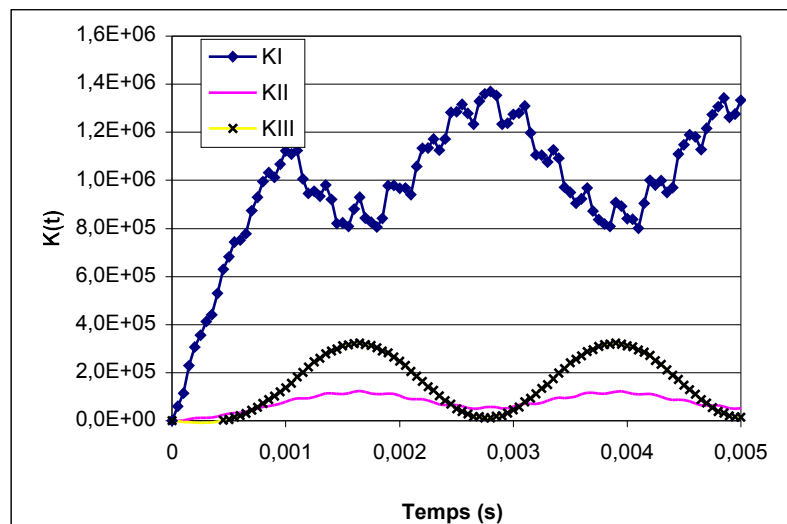
2.2 Results of reference – Modelization A

For the modelization A, the plate is requested by a force in the plane (O, x, y) and the displacements in the direction z are blocked. Result of reference, calculated by direct temporal resolution on a 2D mesh, is traced on the following figure. Horizontal displacement from the top of the plate and the factor of intensity of the stresses oscillate with a frequency corresponding to the first eigen mode of structure.



2.3 Results of reference Modelization B

the evolution of the three factors of intensity of the stresses is traced on the following figure for the node located in the middle of the crack tip. The oscillations of the factors of intensity of the stresses



Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

show the dominating contribution of the first mode of bending of the plate in the direction x and the first mode of bending in the direction z .

2.4 Results of reference – Modelization C

The modelization C represents a crack 2D NON-with a grid (X-FEM). The other characteristics are identical to modelization A. For the modelization C, the reference solution is the solution calculated by Code_Aster from a direct temporal resolution of the transitory problem. The commands used to generate this solution are provided in the file orders as comments.

2.5 Results of reference – Modelization D

The modelization D represents a crack 3D NON-with a grid (X-FEM). The other characteristics are identical to the modelization B. For the modelization D, the reference solution is the solution calculated by Code_Aster from a direct temporal resolution of the transitory problem. The commands used to generate this solution are provided in the file orders as comments.

2.6 Uncertainty on the solution

the explicit direct resolution of the transitory problem can be regarded as exact. Uncertainty on the identification of the factors of intensity of the stresses by interpolation of the jumps of displacements is about 5%.

2.7 Bibliographical references

- 1.E. CRYSTAL, S. DI DOMIZIO: Method theta in fracture mechanics: development of the bilinear form G in 3D and application to the case of the dynamics low frequency, Notes EDF HT-65/05/024/A, 2005

3 Modelization A

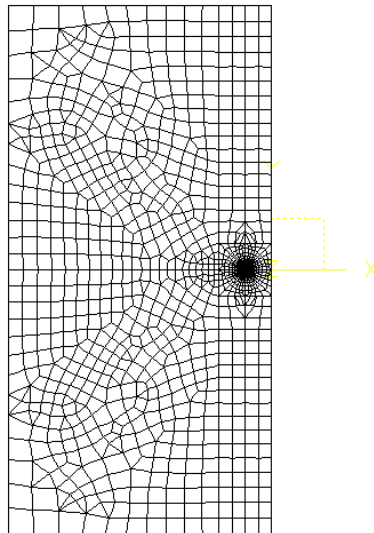
3.1 Characteristic of the modelization

It acts of a modelization 2D – plane stresses. The computation of the evolution of the factors of intensity of the stresses according to time is realized in several stages:

- 1) computation of the first 15 eigen modes of structure;
- 2) computation of the modal factors of intensity of the stresses associated with these modes by two methods;
- 3) resolution of the problem transient dynamics by projection on modal base;
- 4) recombination of K the modal ones.

3.2 Characteristics of the mesh

The mesh is composed of quadratic elements. It comprises 2000 nodes and 700 meshes and is refined around the crack tip.



3.3 Quantities tested and modal

Values results: case test of NON-regression

Number of mode	K_I (POST_K1_K2_K3)	K_I (CALC_K_G)	% difference
1	-1,921.E+10	-1,898.E+10	1,2
2	-1,166.E+11	-1,152.E+11	1,3
3	8,039.E+10	7,948.E+10	1,1
4	-1,188.E+11	-1,174.E+11	1,2
5	1,723.E+11	1,705.E+11	1,1

temporal Values $K_I(t)$: comparison with the resolution clarifies

Urgent	Reference	Aster	% difference
0,0005	24055,6	24337,0	1,2
0,001	44676,8	45159,3	1,1
0,002	90592,3	91679,4	1,2
0,003	134065,3	135633,9	1,2
0,004	181113,3	183286,7	1,2

3.4 Remark

the difference between the modal values calculated by interpolation of the jumps in displacement or by the method theta is weak and coherent with that observed on the static problems.

The value of $K_I(t)$ is calculated to leave desmodaux K_I (method CALC_K_G) and the coefficients of the resolution about modal base directly in the case test by lines of command in python:

$$K_I(t) = \sum_{i=1}^M \alpha_i(t) \cdot K_I^i$$

where the coefficients $\alpha_i(t)$ are the coefficients of modal participation, extracted result from operator DYNA_VIBRA, and the K_I^i are the modal factors of intensity of the stresses.

The accuracy obtained is satisfactory taking into account the number of elements retained in modal base. The accuracy increases quickly with the number of modes [bib1].

4 Modelization B

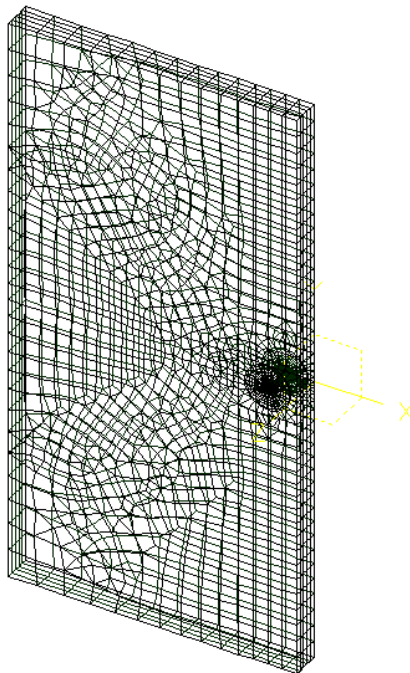
4.1 Characteristic of the modelization

It acts of a modelization 3D. The computation of the evolution of the factors of intensity of the stresses according to time is realized in several stages:

- 1) computation of the first 50 eigen modes of structure;
- 2) computation of the modal factors of intensity of the stresses associated with these modes by two methods;
- 3) resolution of the problem transient dynamics by projection on modal base;
- 4) recombination of K the modal ones.

4.2 Characteristics of the mesh

The mesh is composed of linear elements. It comprises 8200 nodes and 8900 meshes and is refined around the crack tip.



4.3 Quantities tested and results

the values indicated are those found with the node which is in the middle of the crack tip.

Modal values: case test of NON-regression

Number of mode	$K_I (\text{POST_K1_K2_K3})$	$K_I (\text{CALC_K_G})$	% difference
1	5,631E+09	4,790E+09	14,9
2	8,599E+09	7,291E+09	15,2
3	6,940E+10	5,897E+10	15,0
4	-2,702E+11	-2,897E+11	-7,2
5	-9,637E+10	-8,165E+10	15,3

temporal Values $K_I(t)$: comparison with the resolution clarifies

Urgent (s)	Reference (Pa. \sqrt{m})	Aster (Pa. \sqrt{m})	% difference
0.0005	696752,4	721825,9	3,6
0.001	1153703,3	1239061,8	7,4
0.002	997675,6	1110569,6	11,3
0.003	1305429,9	1364524,8	4,5
0.004	870347,2	1004735,2	15,4

4.4 Notices

the difference between the modal values calculated by interpolation of the jumps in displacement or by the method theta is high: that is explained by the linear mesh very little refined in the thickness of the plate.

The value of $K_I(t)$ is calculated from K_I modal (method CALC_K_G) and of the coefficients of the resolution about modal base directly in the case test by lines of command in python:

$$K_I(s, t) = \sum_{i=1}^M \alpha_i \cdot K_I^i(s)$$

where the coefficients $\alpha_i(t)$ are the coefficients of modal participation, extracted result from operator DYN_VIBRA, and the $K_I^i(s)$ are the modal factors of intensity of the stresses.

The accuracy obtained is satisfactory taking into account the number of elements retained in modal base (50) and mesh cuts it. The accuracy increases quickly with the number of modes [bib1].

5 Modelization C

5.1 Characteristic of the modelization

the crack is modelled by the method X-FEM. A modelization C_PLAN is used.

5.2 Characteristics of the mesh

The mesh contains 96000 elements of the type QUAD4.

5.3 Quantities tested and results

One tests the stress intensity factors dynamics $K_I(t)$ and $K_{II}(t)$ with $t=2.E-3 sec$.

Standard	identification of reference	Value of reference	Tolerance
$K_I(t)$	"AUTRE_ASTER"	2.41542E+06	0.2%
$K_{II}(t)$	"AUTRE_ASTER"	3.57437E+05	0.6%

6 Modelization D

6.1 Characteristic of the modelization

the crack is modelled by the method X-FEM. One uses a modelization 3D.

6.2 Characteristics of the mesh

The mesh contains 1875 elements of the type HEXA8.

6.3 Quantities tested and results

One tests the stress intensity factors dynamics $K_I(t)$, $K_{II}(t)$ and $K_{III}(t)$ with $t=4.E-3\text{ sec}$ for the crack tip in the middle of thickness.

Standard	identification of reference	Value of reference	Tolerance
$K_I(t)$	"AUTRE_ASTER"	3.5521523 E+06	2.0%
$K_{II}(t)$	"AUTRE_ASTER"	4.2648023 E+05	10.0%
$K_{III}(t)$	"AUTRE_ASTER"	-8.5924717E+05	4%

7 Summary of the results

This test makes it possible by the operator to validate the computation of the modal factors of intensity `CALC_G` (option `CALC_K_G`) and illustrates their use for the resolution of a problem of fracture mechanics in dynamics low frequency by modal recombination.

The relationship between the computing times of the explicit resolution and the resolution about modal base lie between 10 and 50 according to the type of mesh, and the accuracy of the method of modal recombination is fully satisfactory.

The results of the modelizations C and D make it possible to extend the application of the method X-FEM to compute: the modal factors of intensity of the stresses. The next stage will be for the resolution of a problem of fracture mechanics in dynamic low frequency by modal recombination