SDLS106 - Modal computation of a rectangular plate simply leaned on all its edges

Summarized:

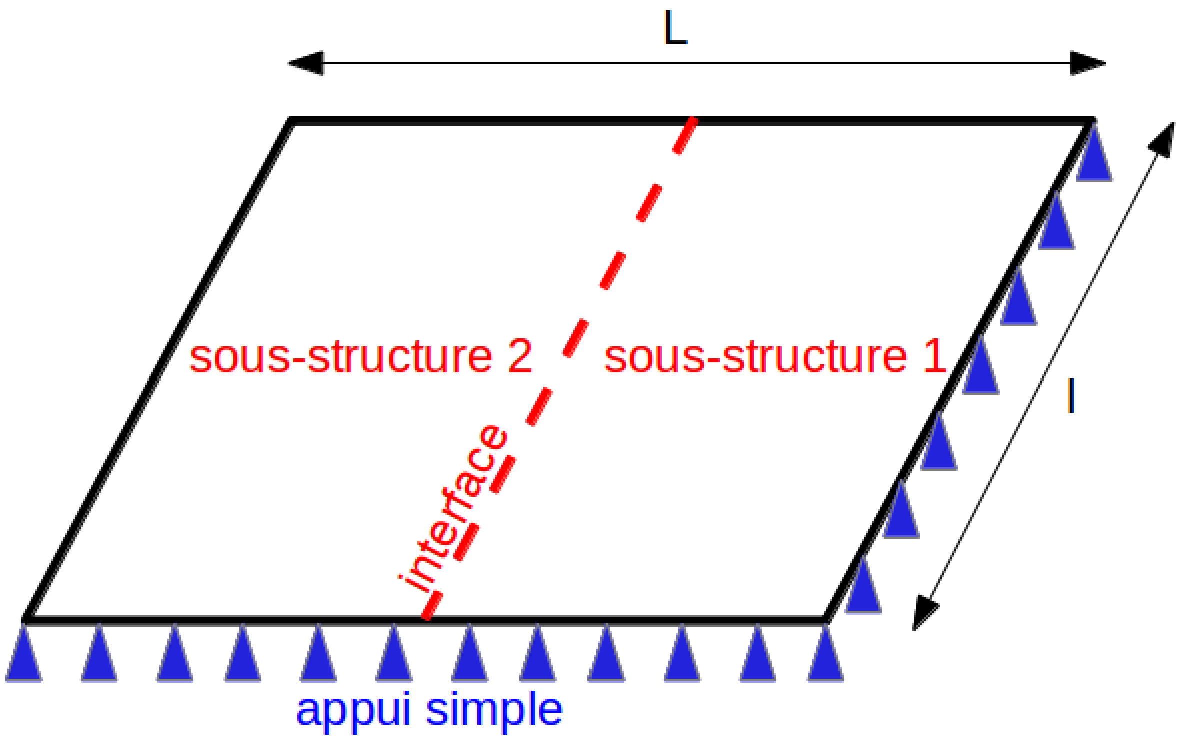
This test of the field of the modal analysis implements the computation of eigen modes of bending of a rectangular plate simply leaned on all its edges.

The computation is fact
- is under - structuring on the basis of modal base Ritz, with an interface of the type Craig-Bampton or Mac Neal;
- maybe in a direct way.

The reference solution is analytical, at the same time for the modal eigenfrequencies and the deformed.

1 Problem of reference

1.1 Geometry



$L=2\,m$

$l=1{,}5\,m$

1.2 Properties of the structure

$\rho=7800\,kg/m^3$ $E=2.10^{11}\,Pa$ $\nu=0.3$ thickness $h=1\,mm$.

1.3 Boundary conditions and loadings

the plate is out of simple bearing on its four edges.

Note: in the case of computations per substructuring (modelizations A with G), the interface of each substructure is clamped (interface of the Craig-Bampton type).

2 Reference solution

2.1 Reference solution

According to [bib1], the eigenfrequencies of vibration of a rectangular plate leaned on all its edges are given by the analytical formula:

$$f_{ij}=\frac{\lambda_{ij}^2}{2\pi L^2}\left[\frac{E h^3}{12\rho_s(1-\nu^2)}\right]^{\frac{1}{2}}$$

with

$$\lambda_{ij}^2=\pi^2\left[i^2+\left(\frac{L}{l}\right)^2 j^2\right]$$

where
i and j are the number of half-waves of the modal deformed shape along the main roads and the small axis of the plate. ρ_s is the mass per unit of area.

That is to say

$$\begin{aligned} f_{11}&=17{,}13\ Hz\\ f_{21}&=35{,}63\ Hz\\ f_{12}&=50{,}01\ Hz\\ f_{31}&=66{,}46\ Hz\\ f_{22}&=68{,}51\ Hz \end{aligned}\ .$$

The modal deformed shapes are also calculated analytically: modal displacement z_{ij} perpendicular to the plate, for the mode (i, j), according to the point of coordinates (x, y), is given by:

$$z_{ij}(x,y)=\sin\left(\frac{i\pi x}{L}\right)\sin\left(\frac{j\pi y}{l}\right)\ .$$

2.2 Case of computations by substructuring: reference solution of each substructure

Each substructure is a plate length $l=1{,}5\,m$ and width $\frac{L}{2}=1\,m$, leaned on three dimensioned and embedded on a long side, vibrating in bending.
It is shown [bib1] that the eigenfrequencies are worth:

$$f_{ij}=\frac{\lambda_{ij}^2}{2\pi l^2}\left[\frac{E h^3}{12\rho_s(1-\nu^2)}\right]^{\frac{1}{2}}$$

with $\lambda_{11}^2=42{,}53$ $\lambda_{21}^2=69{,}00$ $\lambda_{31}^2=116{,}30$, $\lambda_{12}^2=121{,}00$

which gives for the first frequencies:

$$f_{11}=47{,}26\,Hz$$
$$f_{21}=76{,}57\,Hz$$
$$f_{31}=129{,}24\,Hz$$
$$f_{12}=134{,}47\,Hz$$

2.3 Bibliographical reference

1.BLEVINS R.D: Formulated for natural frequency and shape mode. ED. Krieger 1984.

3 Modelization A

3.1 Characteristic of the modelization

For each of two substructures: 600 meshes `QUAD4`.

3.2 Quantities tested on the structure supplements

One tests the eigenfrequencies:

Identification	Reference	Tolerance
Mode n°11 frequency	$17.13\ Hz$	0.5%
Mode n°21 frequency	$35.63\ Hz$	0.5%
Mode n°12 frequency	$50.01\ Hz$	0.5%
Mode n°31 frequency	$66.46\ Hz$	0.5%
Mode n°22 frequency	$68.51\ Hz$	0.5%

One tests also the modal deformed shapes of the third and fourth modes (located by the indices $(1,2)$ and $(3,1)$ respectively), by recovering the component DZ deformed shape according to three lines: two median axes and the diagonal of the plate. Each line is discretized by 10 points.

Identification	Reference	Tolerance (relative, except contrary mention)
n°12 Mode		
Deformed along the median main roads	cf paragraph 2.1	(in absolute)
Deformed shape according to petir it median axis	"	1 %
Deformed following the diagonal	"	2 %
n°31 Mode		
Deformed along the median main roads	"	4 %
Deformed following to petir it median axis	"	1 %
Deformed following the diagonal	"	4 %

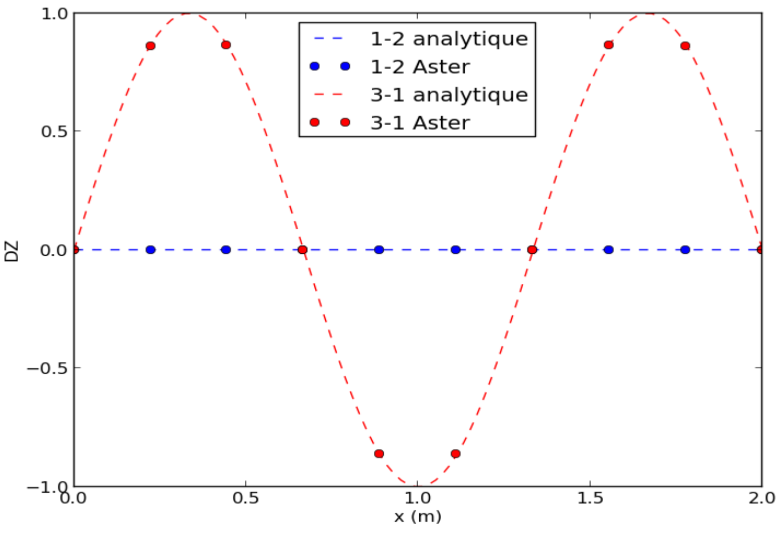


Figure 3.2-1 - modal Deformed shapes along the median main roads.

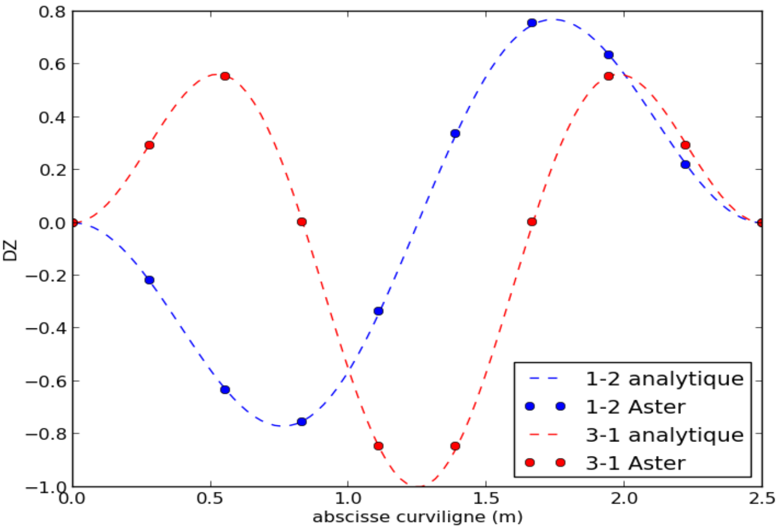


Figure 3.2-3 - modal Deformed shapes according to the diagonal.

The results are very satisfactory.

4 Modelization B

4.1 Characteristic of the modelization

Under structure 1: 600 meshes QUAD4.
Under structure 2: 509 meshes QUAD4.
For each under structure: 6 modes with interface fixes + 20 modes of coupling.

4.2 Quantities tested on the structure supplements

Identification	Reference	Tolerance
Mode n°11 frequency	$17.13\ Hz$	1.25%
Mode n°21 frequency	$35.63\ Hz$	1.25%
Mode n°12 frequency	$50.01\ Hz$	1.25%
Mode n°31 frequency	$66.46\ Hz$	1.25%
Mode n°22 frequency	$68.51\ Hz$	1.25%

5 Modelization C

5.1 Characteristic of the modelization

One releases the bearings to test the existence of the 6 modes of rigid body.

Under structure 1: 600 meshes QUAD4.
Under structure 2: 509 meshes QUAD4.
For each under structure: 6 modes with interface fixes + 20 modes of coupling.

5.2 Quantities tested on the structure supplements

Identification	Reference	Tolerance
Mode n°1 frequency	$0.0\,Hz$	$0.1\,Hz$
Mode n°2 frequency	$0.0\,Hz$	$0.1\,Hz$
Mode n°3 frequency	$0.0\,Hz$	$0.1\,Hz$
Mode n°4 frequency	$0.0\,Hz$	$0.1\,Hz$
Mode n°5 frequency	$0.0\,Hz$	$0.1\,Hz$

6 Modelization D

6.1 Characteristic of the modelization

the structure is cut out in 3 pennies structures, all leaned on external edges. The model complete presents two interfaces, one of type Craig&Bampton, the other of MacNeal type. One tests the possibility of having under structure having two different types of interfaces:

Under structure 1 and 3: 360 meshes QUAD4.
Under structure 2: 480 meshes QUAD4.
For each under structure: 6 modes with interface fixes + 20 modes of coupling.

6.2 Quantities tested on the structure supplements

Identification	Reference	Tolerance
Mode n°11 frequency	$17.13\ Hz$	1.25%
Mode n°21 frequency	$35.63\ Hz$	1.25%
Mode n°12 frequency	$50.01\ Hz$	1.25%
Mode n°31 frequency	$66.46\ Hz$	1.25%
Mode n°22 frequency	$68.51\ Hz$	1.25%

7 Modelization E

7.1 Characteristic of the modelization

Identical to the modelization B, but use of operators CREA_ELEM_SSD and ASSE_ELEM_SSD.

7.2 Quantities tested on the structure supplements

Identification	Reference	Tolerance
Mode n°11 frequency	$17.13\ Hz$	1.25%
Mode n°21 frequency	$35.63\ Hz$	1.25%
Mode n°12 frequency	$50.01\ Hz$	1.25%
Mode n°31 frequency	$66.46\ Hz$	1.25%
Mode n°22 frequency	$68.51\ Hz$	1.25%

8 Modelization F

8.1 Characteristic of the modelization

Identical to the modelization D, but use of operators CREA_ELEM_SSD and ASSE_ELEM_SSD.

8.2 Quantities tested on the structure supplements

Identification	Reference	Tolerance
Mode n°11 frequency	$17.13\,Hz$	1.25%
Mode n°21 frequency	$35.63\,Hz$	1.25%
Mode n°12 frequency	$50.01\,Hz$	1.25%
Mode n°31 frequency	$66.46\,Hz$	1.25%
Mode n°22 frequency	$68.51\,Hz$	1.25%

9 Modelization G

9.1 Characteristic of the modelization

Identical to the modelization A, but of use a method of under structuring by free modes only (MODELE_GENE with OPTION "REDUIT", NUME_DDL_GENE with method ELIMINE to ensure the compatibility of the interfaces). The results bad, like are expected by means of this kind of approach, from where the use of important tolerances.

9.2 Quantities tested on the structure supplements

Identification	Reference	Tolerance
Mode n°11 frequency	$17.13\ Hz$	100%
Mode n°21 frequency	$35.63\ Hz$	100%
Mode n°12 frequency	$50.01\ Hz$	100%
Mode n°31 frequency	$66.46\ Hz$	100%
Mode n°22 frequency	$68.51\ Hz$	100%

10 Modelization H

10.1 Characteristic of the modelization

the plate is modelled with a complete mesh (contrary to the substructuring). The mesh comprises 1200 meshes `QUAD4`, which corresponds to the double of modelization A.

10.2 Grandeurs tested on the structure supplements

One tests the eigenfrequencies:

Identification	Reference	Tolerance
Mode n°11 frequency	$17.13\,Hz$	0.5%
Mode n°21 frequency	$35.63\,Hz$	0.5%
Mode n°12 frequency	$50.01\,Hz$	0.5%
Mode n°31 frequency	$66.46\,Hz$	0.5%
Mode n°22 frequency	$68.51\,Hz$	0.5%

One tests also the modal deformed shapes of the third and fourth modes (located by the indices $(1,2)$ and $(3,1)$ respectively), by recovering the component DZ of the deformed shape according to three lines: two median axes and the diagonal of the plate. Each line is discretized by 10 points.

Identification	Reference	Tolerance (relative, except contrary mention)
n°12 Mode Deformed along the median main roads Deformed shape according to petir it median axis Deformed following the diagonal	 cf paragraph 2.1 " "	 (in absolute) 1 % 1 %
n°31 Mode Deformed along the median main roads Deformed following to petir it median axis Deformed following the diagonal	 " " "	 1 % 1 % 1 %

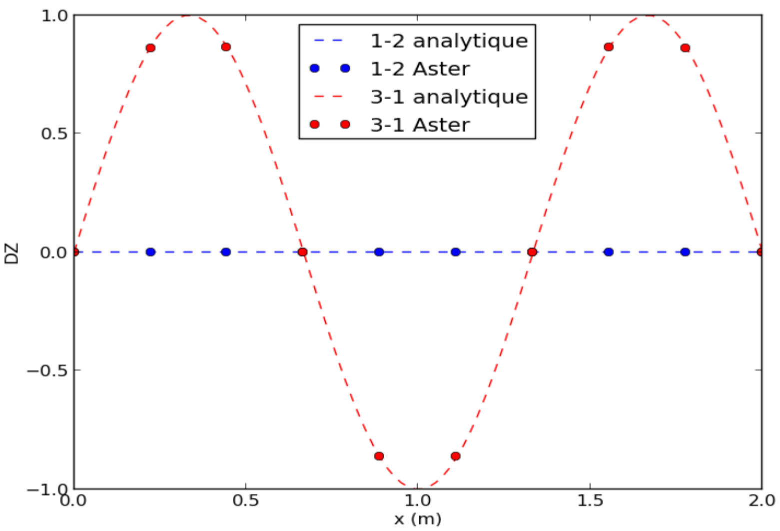


Figure 10.2-1 - modal Deformed shapes along the median main roads.

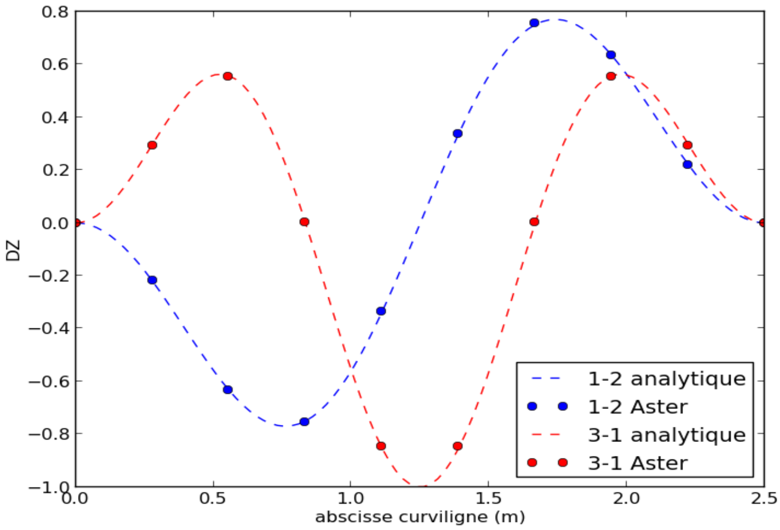


Figure 10.2-3 - modal Deformed shapes according to the diagonal.

The results are very satisfactory.

11 Summary of the results

The computation modal was validated on the eigen modes of bending of a plate simply leaned on its four edges, as well for the eigenfrequencies as the modal deformed shapes.

The results are satisfactory, that is calculated direct way or by substructuring with modal base of the type “Ritz”.