SDLS07 - Eigen modes of a spherical envelope mean

Abstract:

This test from guide VPCS makes it possible to validate the algorithm of search for eigenvalues MODE_ITER_SIMULT with the operators of stiffness and mass corresponding to the following modelizations:

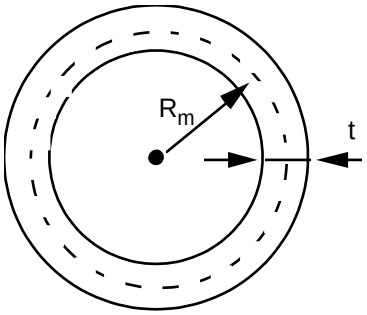
1) Three-dimensional shells: finite elements DKT (mesh of one 1/8 of sphere),
2) Finite elements 2D axisymmetric TRIA6 and QUAD8 (mesh of a section),
3) three-dimensional Shells: axisymmetric finite elements isoparametric SEG3 (linear mesh of the section),
4) three-dimensional Shells COQUE_3D : finite element MEC3QU9H (mesh 1/8 of sphere),
5) Shells three-dimensional COQUE_3D : finite element MEC3TR7H.

The got results are compared with the analytical solution (HAYEK) and reveal for the first six modes of the lower deviations than:

- 0,45% for the axisymmetric elements continuums,
- 0,20% shell elements DKT,
- 0,17% the isoparametric axisymmetric shell elements,
- 0,17% elements COQUE_3D.

1 Problem of reference

1.1 Geometry



It acts of a thin sphere, average radius $R_m = 2.5\,m$, and thickness $t = 0.1\,m$.

1.2 Properties of the materials

the material is homogeneous, isotropic, elastic linear. The elastic coefficients are: $E = 200\,000\ MPa$ and $\nu = 0.3$.

The density is constant and is worth: $\rho = 7800\,kg/m^3$.

1.3 Boundary conditions and loadings

the structure is free in space.

2 Reference solution

2.1 Method of calculating used for the reference solution

For the thin spheres ($i.t \ll R$ with i , order of the mode), the eigen modes with radial displacement and tangential establish by a theory of membrane are given by [bib1] and [bib2]:

$$f_i = \frac{\lambda_i}{2\pi R}\sqrt{\frac{E}{\rho(1-\nu^2)}}$$

with $\lambda_i = \frac{1}{\sqrt{2}}\sqrt{b \pm \sqrt{b^2 - 4(1-\nu^2)(i^2+i-2)}}$ and $b = i^2+i+1+3\nu$

the theory presented by Hayek makes it possible to introduce a correction of the effect of bending (approximation of the general theory of Wilkinson) which leads to values of λ_i function of

$a = t^2/12R^2$
$b = i(i+1)$

and solution of:

$$\lambda_i^4 - \lambda_i^2[1+3\nu - a(1-\nu) + b(1+a\nu+ab)] + ab[b^2 - 4b + 5 - \nu^2] + (1-\nu^2)[b-2(1+a)] = 0$$

2.2 Results of reference

Eigenfrequencies:

I	Eigenfrequencies
2	237.25
3	282.85
4	305.24
5	324.17
6	346.76
7	376.68
8.416	.
9	465.75
10	526.20

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

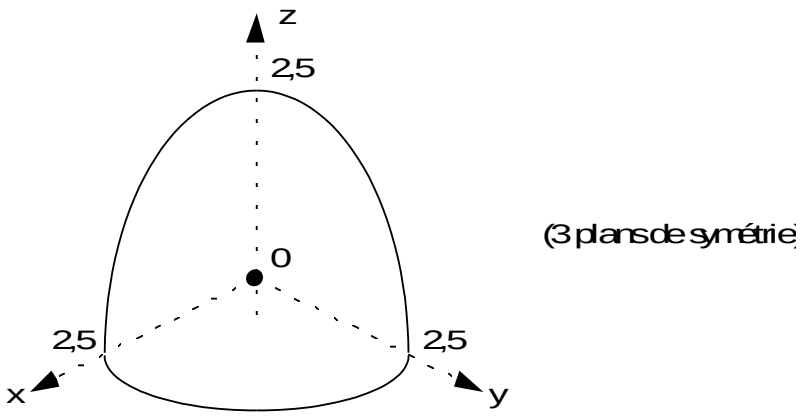
1) Card-indexes VPCS SDLS 07/89 in the Guide of Validation of the Software packages of Computation of Structures/SFM AFNOR TECHNIQUE 1990.

2) S. HAYEK: “Vibrations of has spherical Shell in acoustic medium”, Newspaper of the Acoustical Society of America, vol. 40,2,1996, p. 342-348

3 Modelization A

3.1 Characteristic of the modelization

Shells DKT



the discretized geometry is represented above. Elements DKT are plane facets with 3 nodes. The number of the nodes on the generator and the equator is: 34.

The boundary conditions applied to the three borders correspond to the conditions of symmetry (displacements and blocked rotations).

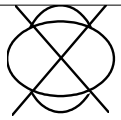
3.2 Characteristics of the mesh

Many nodes: 1128

Number of meshes and types: 2125 TRIA3

3.3 Quantities tested and results

(Frequencies in Hertz)

Value of parameter l of the reference solution	Reference	*Aster*	% difference
 2	237.25	237.24 237.24	– 0.005 – 0.003
3	282.85	not obtained [§4.2]	
4	305.24	304.97 304.99 305.08	– 0.089 – 0.080 – 0.054
5	324.17	not obtained [§4.2]	
6	346.76	346.11 346.12 346.30 346.38	– 0.186 – 0.185 – 0.133 – 0.108
7	376.68	not obtained [§4.2]	
8	416.00	414.89 414.92 415.16 415.24 415.33	– 0.266 – 0.259 – 0.201 – 0.183 – 0.161
9	465.75	not obtained [§4.2]	
10	526.20	524.34 524.43 524.71 524.94 524.97 525.12	– 0.353 – 0.337 – 0.283 – 0.240 – 0.234 –0.205

3.4 Remarks

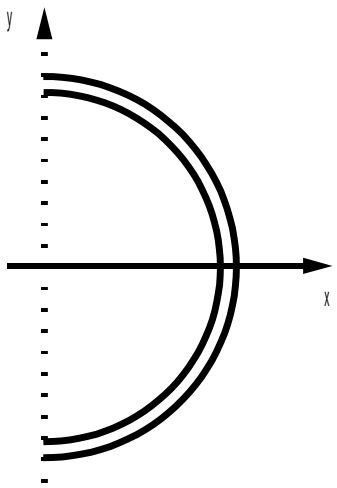
the reference solution does not give the multiplicity of the modes. One observes with computations of the orders of multiplicity which grow with the value of the frequency.

Modes 3,5,7,9 are not obtained because of boundary conditions chosen for this model, with the three symmetry planes.

4 Modelization B

4.1 Characteristic of the axisymmetric

2D modelization



No boundary condition.

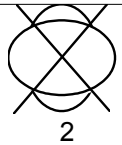
4.2 Characteristics of the mesh

Many nodes: 365

Number of meshes and types: 40 QUAD8 and 80 TRIA6

4.3 Quantities tested and Frequencies

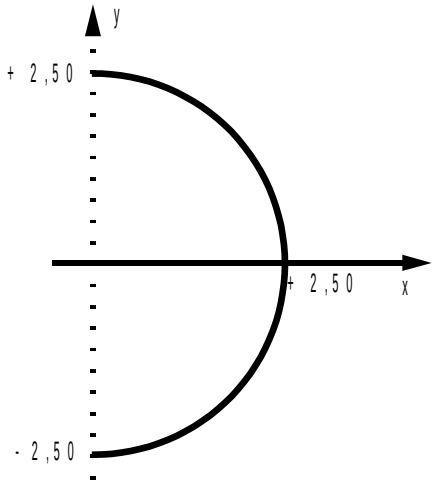
results in Hertz

Identification n ° mode	Reference	*Aster*	% difference
 2	237.25	237.24	0.036
3	282.85	282.78	– 0.023
4	305.24	304.85	– 0.125
5	324.17	323.32	– 0.262
6	346.76	345.22	– 0.443
7	376.68	374.14	– 0.674
8	416.00	412.03	– 0.955
9	465.75	459.75	– 1.286
10	526.20	517.51	– 1.651

5 Modelization C

5.1 Characteristic of the modelization

axisymmetric 1D Shells



No boundary condition.

One chooses the model Coils-Kirchhoff to describe the kinematics. With the element chosen, this kinematics is obtained by penalization: one puts a great value for coefficient A_CIS. In addition, one neglects the correction of metric.

5.2 Characteristics of the mesh

Many nodes: 81

Number of meshes and types: 40 SEG3

5.3 Quantities tested and results

(Frequencies in Hertz)

the purpose of Identification n ° mode	Reference	*Aster* ITER_SIMULT	*Aster* ITER_INV	% difference
2	237.25	237.31	237.32	0.025/0.029
3	282.85	282.77	282.78	– 0.028/-0.025
4	305.24	304.95	304.95	– 0.096
5	324.17	323.68	323.68	– 0.150
6	346.76	346.23	346.23	– 0.154

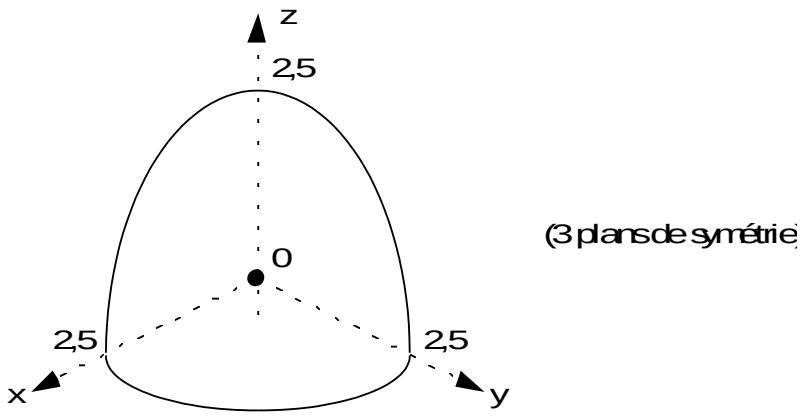
5.4 Remarks

This test with this modelization is only testing the mass matrix. A satisfactory variation being observed on the first six frequencies, one chose not to calculate the following ones.

6 Modelization D

6.1 Characteristic of the modelization

Shells 3D MEC3QU9H



the boundary conditions applied to the three borders correspond to the conditions of symmetry (displacements and blocked rotations).

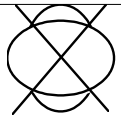
6.2 Characteristics of the mesh

Many nodes: 331

Number of meshes and types: 75 QUAD9

6.3 Quantities tested and results

(Frequencies in Hertz)

Identification n ° mode	Reference	*Aster*	% difference
 2	237.25	237.25 237.26	0 0.004
3	282.85	not obtained [§10.2]	
4	305.24	305.18 305.19 305.20	– 0.019 – 0.017 – 0.011
5	324.17	not obtained [§10.2]	
6	346.76	346.17 346.19 346.25 346.36	– 0.169 – 0.165 – 0.147 – 0.114
7	376.68	not obtained [§10.2]	
8	416.00	413.81 413.84 413.84 414.02 414.09	– 0.525 – 0.520 – 0.518 – 0.476 – 0.46
9	465.75	not obtained [§10.2]	
to 10	526.20	520.57 520.62 520.64 521.28 521.29 521.31	– 1.071 – 1.06 – 1.056 – 0.935 – 0.933 – 0.929

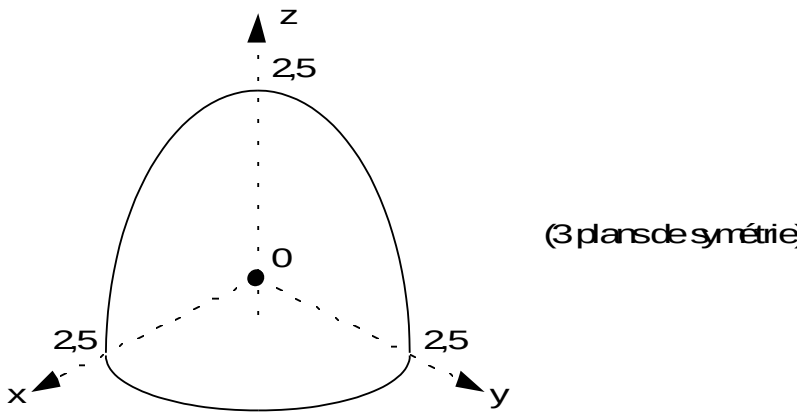
6.4 Remarks

modes 3,5,7,9 are not not obtained because of boundary conditions chosen for this model, with the three symmetry planes.

7 Modelization E

7.1 Characteristic of the modelization

Shells 3D MEC3TR7H



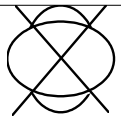
7.2 Characteristic of the mesh

Many nodes: 925

Number of meshes and types: 294 TRIA7

7.3 Quantities tested and results

(Frequencies in Hertz)

Identification n ° mode	Reference	*Aster*	% difference
 2	237.25	237.25 237.25	– 0.001 – 0.001
3	282.85	not obtained [§12.2]	
4	305.24	305.20 305.22 305.22	– 0.011 – 0.008 – 0.005
5	324.17	not obtained [§12.2]	
6	346.76	346.32 346.43 346.46 346.58	– 0.126 – 0.095 – 0.086 – 0.051
7	376.68	not obtained [§12.2]	
8	416.00	413.91 414.33 414.36 414.99 415.14	– 0.502 – 0.402 – 0.394 – 0.241 – 0.206
9	465.75	not obtained [§12.2]	
to 10	526.20	520. 521.02 521.43 522.32 523.03 523.77	– 1.176 – 0.985 – 0.907 – 0.738 – 0.602 – 0.461

7.4 Remarks

the modes 3,5,7,9 are not obtained because of boundary conditions chosen for this model, with the three symmetry planes.

8 Summary of the results

- The modelization shell DKT, here restricted to the modes having 3 symmetries compared to the planes $x=0$ $y=0$ $z=0$, provides the eigenfrequencies with an error lower than 0.4% on the first 20 modes.
- The modelization axisymmetric $2D$ continuum provides the eigenfrequencies with an error lower than 2%.
- The modelization COQUE_AXIS (quadratic isoparametric elements) provides the eigenfrequencies with an error lower than 0.2% on the first 5 modes (spatial discretization identical to the trace of the axisymmetric $2D$ mesh).
- The modelization COQUE_3D degenerated (thick shell elements MEC3QU9H, MEC3TR7H) to provide the eigenfrequencies with an error lower than 1.2% on the first 10 modes.