SDLS03 - Thin rectangular plate simply leaned on edges

Summarized:

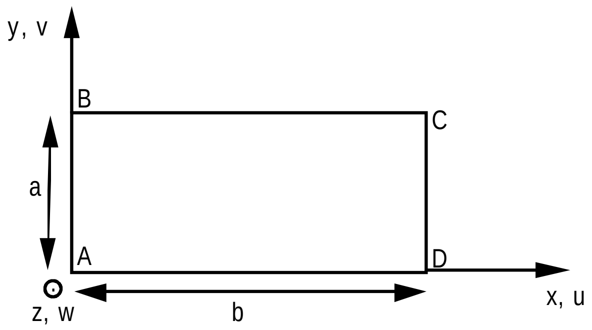
This three-dimensional problem consists in of plate type seeking the frequencies of vibration of a mechanical structure. Two different configurations make it possible to test the modes of vibration in the plan of the plate (behavior out of membrane) with elastic bearings on two opposed edges, and the modes of vibration in bending of a plate leaned on its contour.

This test of structural mechanics corresponds to a dynamic analysis of a surface model having a linear behavior. It comprises two modelizations (mesh in triangle or quadrangle).

This problem makes it possible to test the shell elements in transverse bending and membrane and the computation of the frequencies of vibration by the method of Lanczos or the method of Bathe and Wilson.

1 Problem of reference

1.1 Geometry



Is a plate whose characteristics are the following ones:

length: $a=1.5\,m$ width: $b=1\,m$ thickness: $t=0.01\,m$

The points characteristic of the plate have as coordinates:

	A	B	C	D
x	0.	0.	1.	1.
y	0.	1.5.1.5		0.
z	0.	0.	0.	0.

1.2 Properties of the materials

the parameters characterizing the properties of the material are:

$E=2.1\,10^{11}\,Pa$ $\nu=0.3$ $\rho=7800\,kg/m^3$

1.3 Boundary conditions and loadings

1.3.1 Problem of bending

the plate is out of simple bearing on all its sides: for any point P of edge one a: $w=0$.

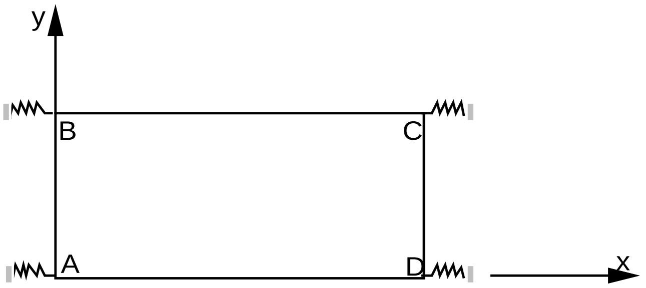
1.3.2 Problem of membrane

For all the points of the plate, one and the blocks displacement z in three degrees of rotation, i.e.:

$$w=0.\quad \theta_x=\theta_y=\theta_z=0.$$

On the sides AD and BC one blocks displacement in y : for $y=0.$ or $y=a$ one A. $v=0$

At the points A,B,C,D , one attaches springs of stiffness k . The axis of these springs is the direction x .



The numerical value of k is the following one: $k=25\,N/m$.

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Problem of bending

the reference solution of the problem of bending is that given in file SDLS03/89 of the guide VPCS which presents the method of calculating in the following way.

The formulation of M.V. BARTON for a rectangular plate, posed on his four sides leads for the modes of bending to:

$$f_{ij}=\frac{\pi}{2}\left[\left[\frac{i}{a}\right]^2+\left[\frac{j}{b}\right]^2\right]\sqrt{\frac{E^2t^4}{12\rho(1-\nu^2)}}$$

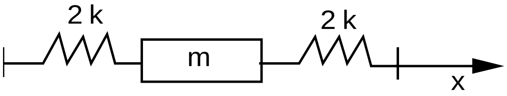
with:

i = number of half-length of wave according to y (dimension a),

j = number of half-length of wave according to x (dimension b).

2.1.2 Problem of membrane

with the problem dealt out of membrane is equivalent for the search of the first frequency of vibration to the following unidimensional problem:



where:

k is the stiffness of springs,

m is the mass of the plate.

The sought frequency is thus: $f=\frac{1}{2\pi}\sqrt{\frac{4\mathrm{k}}{m}}$

2.2 Results of reference

For the problem of bending, one calculates the first six frequencies of vibration and for computation out of membrane, one calculates only the first frequency.

2.3 Uncertainty on the solution

the solutions being analytical, it does not have there uncertainty.

2.4 Bibliographical references

1) M.V. BARTON "Vibrations of rectangular and skew cantilever punts" - Newspaper of Applied Mechanics, vol. 18, p. 129-134 (1951).

3 Modelization A

3.1 Characteristic of the modelization

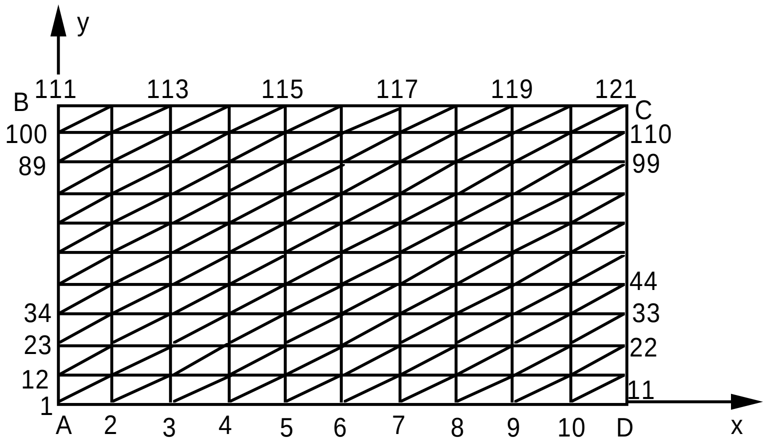
One cut out the plate in 200 meshes `TRIA3`. Two modelizations for the plate are used: `DKT` and `DST`.

For the problem of bending, the boundary conditions are the following ones:

- in all the nodes of edge: $DZ=0$

For the problem of membrane, the boundary conditions are:

- in all the nodes of mesh: $DZ=0$ $DRX=DRY=DRZ=0$,
- in all the nodes on the sides AB and BC : $DY=0$
- to the points A, B, C, D one adds discrete elements of stiffness (direction x).



3.2 Characteristics of the mesh

Many nodes: 121

Number of meshes and types: 200 `TRIA3`

the points characteristic of the mesh are the following:

Not A = $N1$ Not C = $N121$
Not B = $N111$ Not D = $N11$

3.3 Quantities tested and results

For the modes of bending:

Number of the mode				**Frequencies**
	Reference	***Aster* DKT**	**% difference**	**% tolerance**
4	35.63	35.46	– 0.477	0.5
5	68.51	67.82	– 1.003	1.1
6	109.62	108.67	– 0.867	0.9
7	123.32	121.90	– 1.150	1.2
8	142.51	139.99	– 1.761	1.8
9	197.32	191.70	– 2.846	2.9

***Aster* DST**	**% difference**	**% tolerance**
35.45	– 0.492	0.5
67.80	– 1.030	1.1
108.62	– 0.910	1.
121.84	– 1.199	1.3
139.92	– 1.815	1.9
191.57	– 2.912	3.

For the problem out of membrane:

Reference	***Aster* DKT**	**% difference**	**% tolerance**
0.14714	0.147136	– 0.002	0.1

	***Aster* DST**	**% difference**	**% tolerance**
	0.147136	– 0.001	0.1

3.4 Remarks

For the problem in bending, the modal position of the first mode found in the tape $(5.,200.)$ is the fourth, because there are three solid state modes with frequency zero:

- modes of translation u and v in the plane,
- mode of rotation around the axis z .

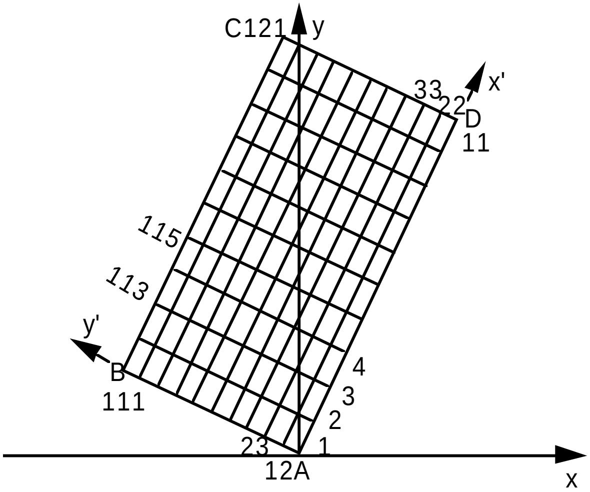
4 Modelization B

4.1 Characteristic of the modelization

One cut out the plate in 100 meshes `QUAD4`.

Three modelizations for the plate are used: `Q4G`, `DKT` (DKQ), `DST` (DSQ).

Compared to the modelization A, the plate was turned in the plane (o,x,y) of an angle of $60°$



For the problem of bending, the boundary conditions are the following ones:

- in all the nodes of edge: $DZ=0$

For the problem of membrane, the boundary conditions are:

- in all the nodes of mesh: $DZ=0$ $DRX=DRY=DRZ=0$,
- with the node A , one blocks displacement DY in the reference (A,x',y') ,
- to the points A,B,C,D one adds discrete elements of stiffness (direction x').

4.2 Characteristics of the mesh

Many nodes: 121 Number of meshes and types: 100 `QUAD4`

the points characteristic of the mesh are the following:

Not A = $N1$ Not B = $N111$ Not C = $N121$ Not D = $N11$

4.3 Quantities tested and results

For the modes of bending:

Number of the mode	**Frequencies**			
	Reference	***Aster* DKQ**	**% difference**	**% tolerance**
4	35.63	35.359	– 0.760	0.8
5	68.51	67.491	– 1.427	1.5
6	109.62	108.563	– 0.964	1.
7	123.32	121.144	– 1.765	1.8
8	142.51	138.402	– 2.882	2.9
9	197.32	188.500	– 4.470	4.5
		***Aster* DSQ**		
4	35.63	35.351	– 0.782	0.8
5	68.51	67.464	– 1.527	1.6
6	109.62	108.494	– 1.027	1.1
7	123.32	121.060	– 1.832	1.9
8	142.51	138.291	– 2.961	3.
9	197.32	188.298	– 4.572	4.6
		***Aster* Q4G**		
4	35.63	36.011	1.068	1.1
5	68.51	70.795	3.336	3.5
6	109.62	114.593	4.536	4.6
7	123.32	134.899	9.39	9.4
8	142.51	142.941	4.513	4.6
9	197.32	212.045	7.463	7.5

For the problem out of membrane:

	Reference	***Aster***	**% difference**	**% tolerance**
DKQ	0.14714	0.14713	– 0.003	0.1
DSQ	0.14714	0.14714	– 0.002	0.1
Q4G	0.14714	0.14714	0.	0.1

4.4 Remarks

For the problem in bending, the modal position of the first mode found in the tape $(5.,200.)$ is the fourth, because there are three solid state modes with frequency zero:

- modes of translation u and v in the plane,
- mode of rotation around the axis z .

5 Modelization D

5.1 Characteristic of the modelization

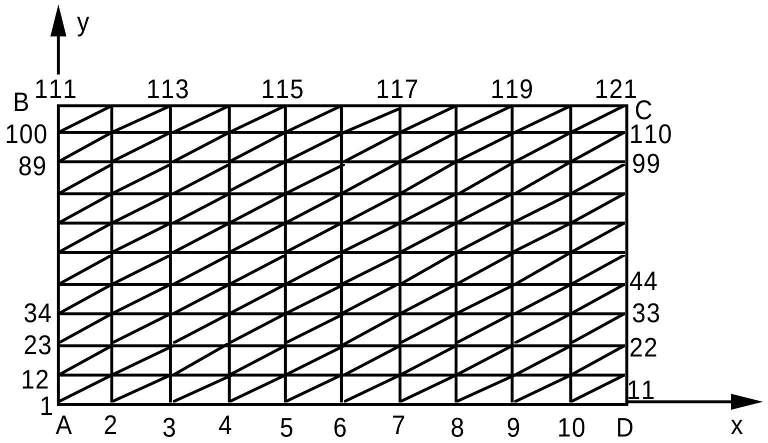
One cut out the plate in 200 meshes `TRIA3`. a modelization for the plate is used: `Q4G`.

For the problem of bending, the boundary conditions are the following ones:

- in all the nodes of edge: $DZ=0$

For the problem of membrane, the boundary conditions are:

- in all the nodes of mesh: $DZ=0 \quad DRX=DRY=DRZ=0$,
- in all the nodes on the sides AB and BC : $DY=0$
- at the points A, B, C, D one adds discrete elements of stiffness (direction x).



5.2 Characteristics of the mesh

Many nodes: 121

Number of meshes and types: 200 `TRIA3`

the points characteristic of the mesh are the following:

Not A = $N1$ Not C = $N121$
Not B = $N111$ Not D = $N11$

5.3 Quantities tested and results

For the modes of bending:

Number of Identification	the Standard	mode of Reference	Reference	% tolerance
4	Frequencies	“ANALYTIQUE”	35.63	1.5
5	Frequencies	“ANALYTIQUE”	68.51	3.5
6	Frequencies	“ANALYTIQUE”	109.62	3.0
7	Frequencies	“ANALYTIQUE”	123.32	7.0
8	Frequencies	“ANALYTIQUE”	142.51	8.5
9	Frequencies	“ANALYTIQUE”	197.32	10.0

6 Summary of the results

the accuracy, for the modes of bending, remains acceptable on the first six modes. Let us note however that the accuracy is less good than in the case of the free plate in space (test SDLS01 [V2.03.001]).

It is noticed that the processing of the solid state modes is suitable.

For the test out of membrane, the results are very satisfactory.