SDLS01 - Thin square plate, free or embedded with Summarized

edge:

The scope of application of this case test relates to the dynamics of structures, and more particularly the modal computation and the harmonic computation of response.

For modal computation, it is a question of calculating the eigen modes of bending of a thin square plate in two configurations:

Plate clamped on an edge,
Plates free.

The plate is with a grid in triangular elements to which elements DKT are `affected`.

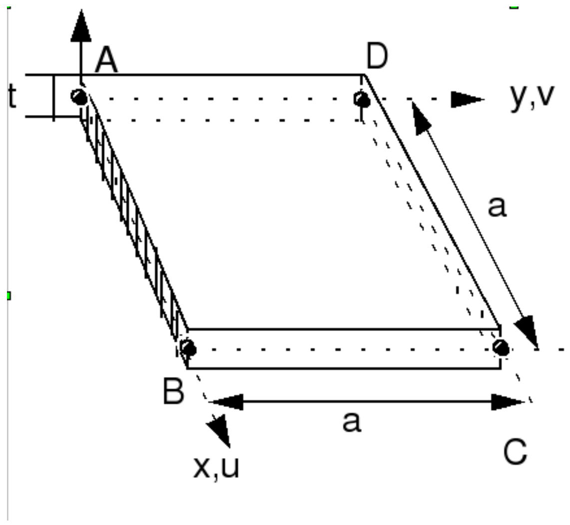
Four different modelizations are tested:

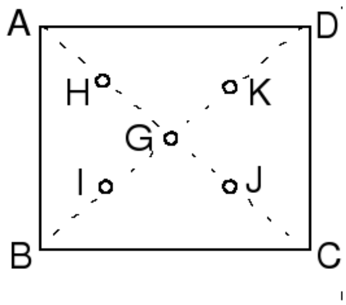
Modal computation – Edges of the plate directed according to the axes of the reference,
Modal computation – unspecified Directional sense of the plate and harmonic response for the embedded plate,
Modal computation by classical and cyclic dynamic substructuring,
Modal computation following a condensation of Guyan.

The results of reference of modal computations result from analytical computations. They validate on the one hand the tools of creation of the mass matrixes and stiffness, as well as the operators of under - classical and cyclic dynamic structuring implemented in *Code_Aster*. In addition, this case test validates modal computation following a condensation of Guyan (condensation of the mass matrix).

1 Problem of reference

1.1 Geometry





Side $a=1\text{m}$
Thickness $t=0.01\text{m}$

Coordinated of the points (in m):

	A	B	C	D	G	H	I	J	K
x	0.	1.	1.	0.	0.5	0.25	0.75	0.75	0.25
y	0.	0.	1.	1.	0.5	0.25	0.25	0.75	0.75
z	0.	0.	0.	0.	0.	0.	0.	0.	0.

1.2 Material properties

$$E=2.1\,10^{11}\,Pa \qquad \nu=0.3 \qquad \rho=7800\,kg/m^3$$

1.3 Boundary conditions and loadings

Case 1: dimensioned AB embedded
for any point P such as $y_P=0$:

$$u=v=w=0.$$
$$\theta_x=\theta_y=\theta_z=0.$$

Case 2: free plate

1.4 Initial conditions

Without object for the modal analysis

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is that given in file SDLS01/89 of the guide VPCS which presents the method of calculating in the following way:

The formulation of M.V. BARTON, for a plate of dimensioned a, led to:

$$f_i=\frac{1}{2\pi a^2}\lambda_i^2\sqrt{\frac{Et^2}{12\rho(1-\nu^2)}} \qquad i=1,2,\ldots$$

with, for a Poisson's ratio $\nu=0.3$:

1°: Plate clamped on a side

i	λ_i^2
1	3.492
2	8.525
3	21.43
4	27.33
5	31.11
6	54.44

2°: Free plate

i	λ_i^2
1 to 6	0.
7	13.49
8	19.79
9	24.43
10	35.02
11	35.02

(6 solid state modes with frequency null).

This reference solution applies to the thin plates such as: $t/a<0.1$
The coefficients λ_i are establish by development limited on the modal deformed shapes of a network of cross beams (embed-free beam and free-free beam).

2.2 Results of reference

Case 1: the first 6 eigen modes

Case 2: the first 11 Uncertainty

2.3 eigen modes on the semi-analytical

solution Solution.

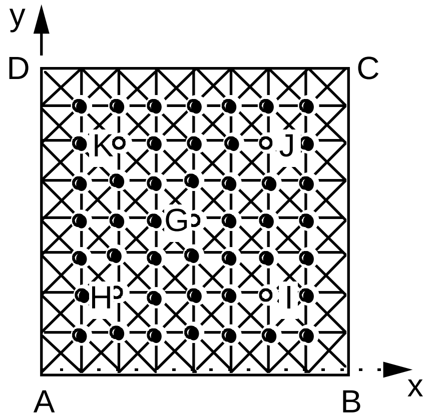
2.4 Bibliographical references

[1] Mr. V. BARTON Vibrations of rectangular and skew cantilever punts. – Newspaper of Applied Mechanics, flight 18, p. 129-134 (1951)

3 Modelization A

3.1 Characteristic of the modelization

Modelization `DKT`



Names of the Limiting nodes: $A=N1$ $B=N78$ $C=N145$ $D=N80$

$G=N65$ $H=N17$ $I=N73$ $J=N121$ $K=N71$

points Conditions:

Case 1 in all the nodes on the side AB :

```
DDL_IMPO= _F (GROUP_NO= AB DX =0., DY =0., DZ =0., DRX =0., DRY =0.,
DRZ =0.)
```

Case 2 no

3.2 Characteristics of the mesh

Many nodes: 145

Number of meshes and types: 256 `TRIA3`

3.3 Quantities tested and Frequency

results (Hz)

Eigen mode	**Reference**	***Aster***	**% difference**	**Tolerance**
	1°: Plate clamped on a side			
1	8.7266	8.6718	– 0.63	
2	21.3042	21.2904	– 0.06	
3	53.5542	53.0992	– 0.85	1. 10-2
4	68.2984	67.9269	– 0.54	
5	77.7448	77.4294	– 0.40	
6	136.0471	135.7635	– 0.21	

	Aster $epot=ecin$
1	1.4796 104
2	1.7331 104
3	4.3802 104

4	3.7367 104			
5	5.4956 104			
6	1.3483 105			
	2°: Free plate			
7	33.7119	33.6839	– 0.08	
8	49.4558	48.9362	– 1.05	
9	61.0513	60.5849	– 0.76	1.1 10-2
10	87.5160	87.0993	– 0.48	
11	87.5160	87.0993	– 0.48	

	Aster $epot = ecin$
7	2.2396 104
8	4.7270 104
9	7.2453 104
10	1.4974 105
11	1.4974 105

One calculates kinetic energy ECIN_ELEM of element DKT (connected to the pointdont A one of with dimensions is on AD) problem 1 ("plate embedded on with dimensions"):

Component	**option**	**Reference (NON_REGRESSION)**	***Aster***	**% TOTAL**
difference	ECIN_ELEM	0.011448	0.0114476	3.5 10-4
ECIN_ELEM	BENDING	2968.79	2968.7918	6.1 10-5

3.4 Remarks

```
MODE_ITER_SIMULTOPTIONS=       'BANDE'       FREQ = (8. , 140.)  CAS 1
                                           FREQ = (32. , 90.)   CAS 2
```

Contents of the file results:

1°: the first 6 eigenfrequencies, modal eigenvectors and parameters strain energy and kinetic energy of the 6 modes.

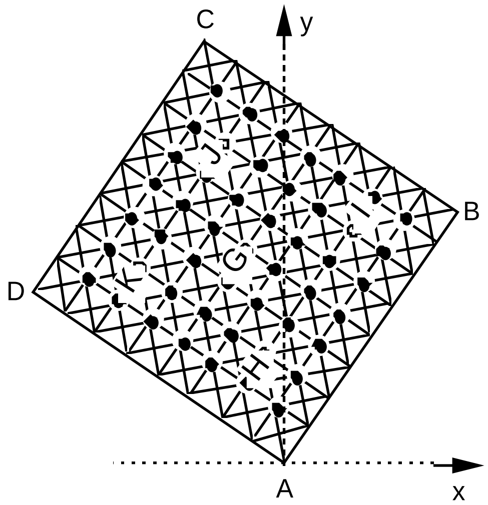
2°: 5 eigenfrequencies, modal eigenvectors and parameters ($f > 0$) strain energy and kinetics of the 5 modes.

4 Modelization B

4.1 Characteristic of the modelization B

Modelization DKT with mesh identical to modelization A.

Rotation of the plate such as the side AB is on the line $3y=4x$



Names of the nodes:	Limiting	$A=N1$	$B=N78$	$C=N145$	$D=N80$	
		$G=N65$	$H=N17$	$I=N73$	$J=N121$	$K=N71$

points Conditions:

Case 1 in all the nodes on the side AB :

```
DDL_IMPO= (GROUP_NO= AB DX =0., DY =0., DZ =0., DRX =0., DRY =0.,
DRZ =0.)
```

Case 2: no

harmonic Response:

Nodal force point C ($N145$): $Fz=-98100$

Material: AMOR_ALPHA : 0.1 AMOR_BETA : 0.1

4.2 Characteristics of the mesh

Many nodes: 145

Number of meshes and types: 256 TRIA3

4.3 Quantities tested and results

the values of the eigenfrequencies are identical to those of modelization A.

harmonic Réponse:

FREQ :	$50\,Hz$	THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE:	$N145$	NET :	$M255$

Reference	*Aster* 3.03.15	*Aster* 3.05 .16%	difference
DEPL “DZ”	2.90290E-02 5.20606E-02	2.90290E-02 5.20606E-02	0.0
DEPL “DRX”	2.52920E-02 9.44717E-02	2.52920E-02 9.44717E-02	0.0
VITE “DZ”	– 1.63553E+01 9.11973E+00	– 1.63553E+01 9.11973E+00	0.0
VITE “DRX”	– 2.96792E+01 7.94573E+00	– 2.96792E+01 7.94573E+00	0.0
ACCE “DZ”	– 2.86505E+03 – 5.13817E+03	– 2.86505E+03 – 5.13817E+03	0.0
ACCE “DRX”	– 2.49622E+03 – 9.32398E+03	– 2.49622E+03 – 9.32398E+03	0.0
“EFGE_ELNO” “MXX”	1.14053E+01 1.45539E+03	1.14053E+01 1.45539E+03	0.0
“EFGE_ELNO” “MYY”	1.10224E+01 – 1.31441E+03	1.10224E+01 – 1.31441E+03	0.0
“EFGE_ELNO” “MXY”	1.03148E+01 3.55382E+02	1.03148E+01 3.55382E+02	0.0
“EFGE_ELNO” “QX”	3.66163E+02 – 3.77331E+03	3.66163E+02 – 3.77331E+03	0.0
“EFGE_ELNO” “QY”	– 3.14676E+02 2.06813E+03	– 3.14676E+02 2.06813E+03	0.0
“SIGM_ELNO” “SIXZ”	5.49245E+04 – 5.65997E+05	5.49245E+04 – 5.65997E+05	0.0
“SIGM_ELNO” “SIYZ”	– 4.72014E+04 3.10219E+05	– 4.72014E+04 3.10219E+05	0.0

4.4 Remarks

```
MODE_ITER_SIMULTOPTIONS=      'BANDE'      FREQ = (8. , 140.)  CAS 1
                                           FREQ = (32. , 90.)   CAS 2
```

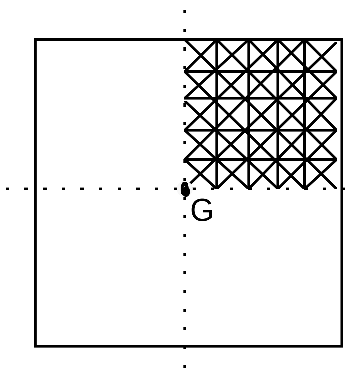
Contents of the file results:

1°: the first 6 eigenfrequencies, modal eigenvectors and parameters.

2°: the first 11 eigenfrequencies, modal eigenvectors and parameters.
3°: displacement DZ DRX with the node $N145$
forces generalized and forced mesh $M255$

5 Modelization C

5.1 Characteristic of the modelization



In the 2 cases, the plate is cut out in 4 parts of equal size. Each substructure considered is with a grid in triangles to which shell elements DKT are `affected`.

Case 1: Plate clamped on an edge
the structure is studied using the method of substructuring classical with interfaces of the type CRAIG_BAMPTON. The modal base used for each substructure is made up of 25 eigen modes and the constrained modes associated with the interfaces.
Case 2: Free plate
the structure is studied using the method of substructuring cyclic with interfaces of the type CRAIG_BAMPTON HARMONIC and taken into account of the specificity of the node of the axis (not G). The modal base used for the basic sector is made up of 25 eigen modes and the harmonic modes associated with the interfaces.

5.2 Characteristics of the mesh

Many nodes: 121
Number of meshes and types: 200 `TRIA3`

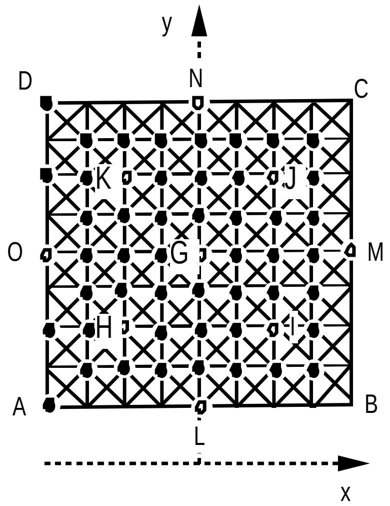
5.3 Quantities tested and Order

results of eigen mode I	the Frequency (Hz) Reference	Aster	% difference	Tolerance
	1°: Plate clamped on a side			
1	8.7266	8.6419	– 0.97	
2	21.3042	21.2253	– 0.37	
3	53.5542	52.9693	– 1.09	1.25 10-2
4	68.2984	67.5444	– 1.10	
5	77.7448	77.3966	– 0.45	
6	136.0471	134.5785	– 1.08	
	2°: Free plate			
7	33.7119	33.6808	– 0.09	
8	49.4558	48.9785	– 0.96	
9	61.0513	60.6739	– 0.62	1. 10-2
10	87.5160	87.0662	– 0.51	
11	87.5160	87.0662	– 0.51	

6 Modelization D

6.1 Characteristic of the modelization

`DKT` + substructuring of limiting



GUYAN Conditions: Free plate

Condensation of the mass matrixes and stiffness on the nodes:

$$(A, B, C, D, G, H, I, J, K, L, M, N, O) \ .$$

6.2 Characteristics of the mesh

Many nodes: 145

Number of meshes and types: 256 `TRIA3`

6.3 Quantities tested and Order

results of eigen mode I	the Frequency (Hz) Reference	*Aster*	% difference	Tolerance
		2°: Free plate		
7	33.7119	33.8758	– 0.48	
8	49.4558	49.5240	– 0.14	1.1 10-2
9	61.0513	61.6240	– 0.94	

6.4 Remarks

One seeks to calculate the first 3 non-zero eigenfrequencies of the problem of the free plate on his edges.

If one condenses the matrixes on the only nodes:

$$(A, B, C, D, G, H, I, J, K)$$

The accuracy of the frequencies is not whereas of 2% .

To get the results wanted with the expected accuracy (1%), the points should be added (L, M, N, O) .

7 Summary of the Modelizations

results A and b:

Accuracy on the eigenfrequencies $\leq 1\%$ until the sixth mode of bending.

Modelization C:

In substructuring, the quality of the results could be improved by the use of a mesh of substructure finer.

Modelization D:

To obtain an accuracy of 1% on the eigenfrequencies, it is necessary to also condense on the 4 nodes mediums of edges L M , N and O .