SDLL403 - Vibrations of a pendulum in rotation

Summarized

the scope of application of this test is the modal analysis of structures. The studied structure is a pendulum in rotation around an axis fixed and plunged in a gravity field. The pendulum itself is articulated around an axis perpendicular to the rotational axis and is located at a certain distance from this one.
One is interested in the first six eigenfrequencies.

The interest of this test lies in the following aspects:

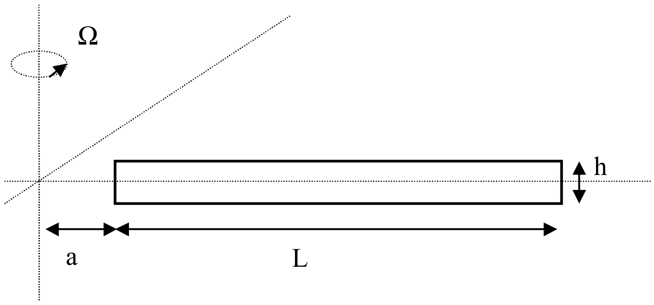
- modal analysis with taking into account of initial stresses (geometrical stiffness)
- modal analysis with taking into account of the stiffening Currently
- centrifuges important relative difference between two successive frequencies of

the spectrum, the taking into account of the centrifugal stiffening is not possible that with voluminal elements. The element used is element `HEXA20` and one employs the method of Sorensen for the computation of the eigenfrequencies.

The first eigenfrequency is compared with an analytical reference. The following frequencies are compared with numerical values obtained by a software independent of *Code_Aster* and using modelizations “beam” and “plane stress”.

1 Problem of reference

1.1 Geometry



Characteristics:

Length of the pendulum	$L=0.6\,m$
Eccentricity	$a=0.1\,m$
Height of the profile	$h=0.01\,m$
Width of the profile	$b=0.004\,m$
Section	$S=bh$
Inertia of bending	$I_Z=bh^3/12$

1.2 Properties of the materials

Modulus Young	$E=7.E10\,N/m^2$
Poisson's ratio	$\nu=0.3$
Density	$\rho=2700\,kg/m^3$

1.3 Boundary conditions and loading

the beam is articulated at the point A . The clevis pin is the axis Y . The initial stress state which allows to carry out the geometrical computation of the stiffness and centrifuges is obtained by imposing a rotational speed and gravity.

Acceleration of gravity	$g=-9.81\,m/s^2$ (parallel with the axis Z)
Rotational speed	$\Omega=10\,rad/s$

the static equilibrium position θ_0 corresponding to loading is calculated by the relation:

$$3\,g\cos\theta_0=\Omega^2(3\mathrm{a}+2\mathrm{L}\cos\theta_0)\sin\theta_0$$

One finds $\theta_0=11.269931365°$

the conditions on displacements at the point A are the following ones:

$$u=v=w=0 \quad ; \quad \phi_X=\phi_Z=0$$

One considers moreover than the section passing by A rigid rest.

1.4 Initial conditions

Without object in modal analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

- first eigenfrequency

the facts of the case are selected in such a way that the stiffness in bending and extension is large with respect to the stiffness geometrical and centrifugal. Under these conditions, the value of the first eigenfrequency is obtained analytically by considering a rigid pendulum.

While taking as degree of freedom the angle θ enters the pendulum and the axis X the equation of motion is written:

$$2L\ddot{\theta}=3\mathrm{g}\cos\theta-\Omega^2(3\mathrm{a}+2\mathrm{L}\cos\theta)\sin\theta$$

One considers here the small oscillations $\Delta\theta$ of the pendulum around a static equilibrium position θ_0 . By linearizing the equation of motion in the vicinity of this position, one obtains the equation with the small disturbances:

$$2L\Delta\ddot{\theta}+\left[3\mathrm{g}\sin\theta_0+\Omega^2(3\mathrm{a}\cos\theta_0+2\mathrm{L}\cos2\theta_0)\right]\Delta\theta=0$$

One from of deduced the pulsation from the first mode:

$$\omega=\sqrt{\frac{3\mathrm{g}}{2\mathrm{L}}\sin\theta_0+\Omega^2\left[\frac{3\mathrm{a}}{2\mathrm{L}}\cos\theta_0+\cos2\theta_0\right]}$$

This own pulsation can be still written in the form

$$\omega=\sqrt{\frac{K(\sigma)+K(\Omega^2)}{I}}$$

with

$$K(\sigma)=\frac{1}{2}\rho S L^2 g\sin\theta_0+\rho S L^2\Omega^2\left[\frac{a}{2}\cos\theta_0+\frac{L}{3}\cos^2\theta_0\right]$$ (geometrical stiffness)

$$K(\Omega^2)=-\frac{1}{3}\rho S L^3\Omega^2\sin^2\theta_0$$ (stiffness centrifuges)

$$I=\frac{1}{3}\rho S L^3$$ (inertia in rotation)

- other eigenfrequencies

the values of reference of frequencies 2 to 6 are obtained numerically by means of version 7 of software the SAMCEF software. Two different modelizations were used: 20 beam elements deformable with the shears and 20×4 elements of membrane with 8 nodes. The results got in both cases are identical if one limits oneself to the first 4 significant figures. Considering the corrections of stiffness are small with respect to the terms of linear stiffness, one can check that frequencies 2 to 6 differ little from the analytical values obtained for a nondeformable beam hurled with the shears. In fact, the maximum difference between the numerical and analytical values does not exceed 1% .

2.2 Results of reference

the first 5 critical loads are classified by order of increasing modulus.

Mode	**Eigenfrequency (Hz)**
1	1.75556
2	100.2
3	324.0
4	674.4
5	1150.
6	1748.

2.3 Uncertainty on the analytical

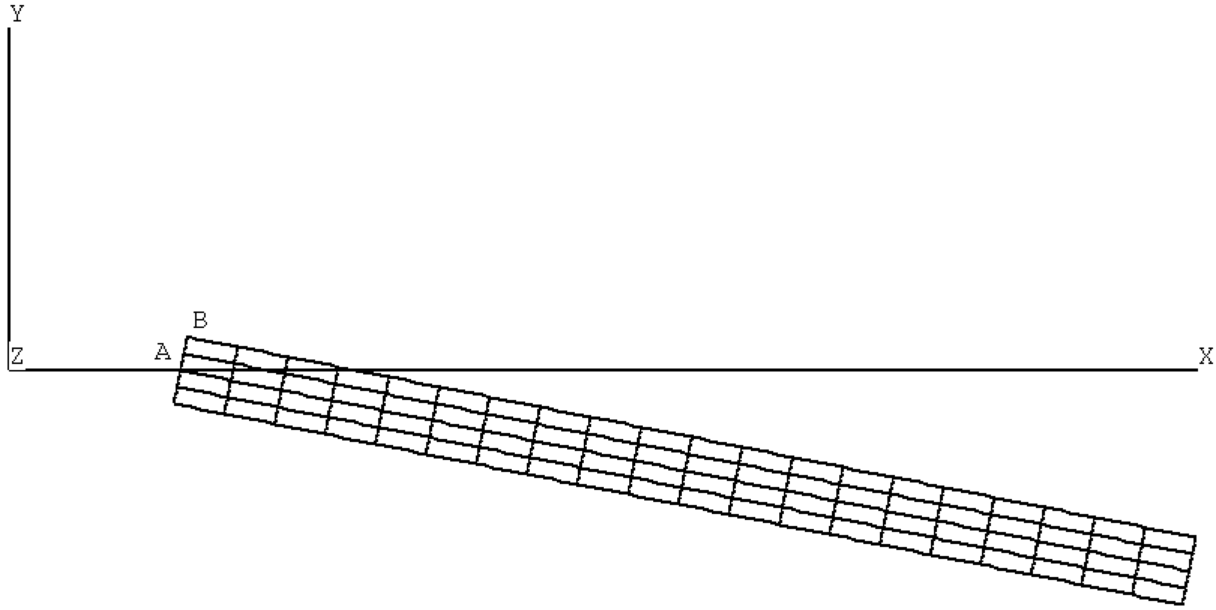
solution Solution for the first frequency. Numerical solution for the others. The estimated tolerance of the numerical results is of 1% .

2.4 Bibliographical references

Without object.

3 Modelization A

3.1 Characteristic of the modelization



the beam is with a grid by means of elements HEXA20.

Boundary conditions:

At the point A such as $X=0.1$ $Y=0$ $Z=0$:

$$DX=DY=DY=DRX=DRZ=0$$

In addition, all the nodes of the section passing by A are rigidly dependant.

3.2 Characteristics of the mesh

Many nodes: 1077
Number of meshes: 160 HEXA20
8 QUAD8

3.3 Quantities tested and Frequencies

results in Hz

Mode	Reference	*Code_Aster*	relative Variation (%)
1	1.75556	1.7871	2.121
2	100.2	100.272	0.072
3	324.0	324.65	0.200
4	674.4	677.1	0.407
5	1150.	1157.8	0.677
6	1748.	1766.7	1.072

4 Summary of the Good agreement

results with the reference solution (less $1.\%$ of error on all the modes except on the first where the error is of 2.2%).

This test could not be carried out with a beam element because the computation of the centrifugal stiffness matrix is not available for this kind of element. In the same way, as it is not available for the discrete elements, we could not use connection 3D-beam. In order to bearing this problem, all the nodes of surface containing the point A were bound by a solid connection.