SDLL401 - Inclined straight beam with 20°, subjected
to sinusoidal forces

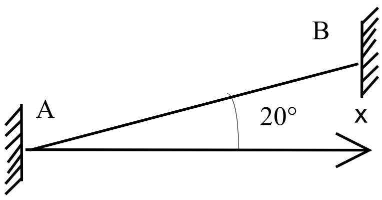
Summarized:

This test is resulting from the validation independent of version 4 of the models of beams.

It makes it possible to check the internal forces on an inclined beam, for sinusoidal loadings according to time (a modelization with elements POU_D_T, straight beam of Timoshenko).

1 Problem of reference

1.1 Geometry



Appears 1.1-1.1-a1.1-a

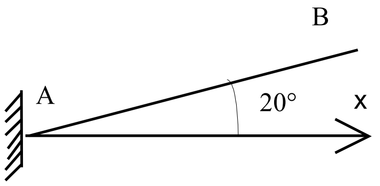


Figure 1.1-1.1-b1.1-b

Straight beam length $1\,m$.
slope $20\,°$ compared to x (trigonometrical meaning).

Characteristics of the section:

$S=\pi\times 0.01^2 m^2$

1.2 Properties of the materials

Modulus Young	$E=2.10^{11}\,Pa$
Poisson's ratio	$\nu=0,3$
Density	$\rho=7800\,kg/m^3$

1.3 Boundary conditions and loading

Boundary condition:

- For the distributed loading [fig 1.1-1]
  Nodes A and B clamped: DX , DY , DZ , DRX , DRY , DRZ blocked
- For the specific loading [fig 1.1-2]
  clamped A Node: DX , DY , DZ , DRX , DRY , DRZ blocked

Loadings:

- $f(t)=1000\times\cos(t)$ according to the direction AB
  either distributed or applied at the end B
- $M_T(t)=1000\times\cos(t)$ applied at the Reference solutions B

2 end

2.1 Method of calculating used for the reference solutions

2.1.1 Distributed loading of traction and compression

a straight beam length L working only in traction and compression is subjected to a following constant distributed loading x but varying in a sinusoidal way according to time. It is embedded at its two ends.

$$\begin{cases} \rho S \dfrac{\partial^2 u}{\partial t^2} - ES \dfrac{\partial^2 u}{\partial x^2} = f(t) \\ u(0)=0,\ u(L)=0 \end{cases}$$

To solve, one applies to the equation the transform of Fourier in time:

$$\frac{\partial^2 \hat{u}}{\partial x^2} = -\frac{\rho}{E} 4\pi^2 \omega^2 \hat{u} + \frac{1}{ES} \hat{f}(\omega)$$

$\hat{u}$: transform of Fourier of u
$\hat{f}$: transform of Fourier of f.

Thus, we have for $f(t)=F\cos(2\pi\omega_0 t)$:

$$u(x,t) = \frac{a^2\ F}{ES\ 4\pi^2\ \omega_o^2} \left\{ \left[\cos\left(\frac{2\pi\omega_o}{a}\ \mathrm{L}\right) - 1\right] \frac{\sin\left(\dfrac{2\pi\omega_o}{\mathrm{a}}\ x\right)}{\sin\left(\dfrac{2\pi\omega_o}{\mathrm{a}}\ L\right)} - \left[\cos\left(\frac{2\pi\omega_o}{a}\ \mathrm{x}\right) - 1\right] \right\} \cos\left(2\pi\omega_o t\right) .$$

with: $a^2 = \dfrac{E}{\rho}$.

The use of the constitutive law gives us the tractive effort compression:

$$N(x,t) = \frac{a\ F}{2\pi\ \omega_o} \left\{ \left[1 - \cos\left(\frac{2\pi\omega_o}{a}\ \mathrm{L}\right)\right] \frac{\cos\left(\dfrac{2\pi\omega_o}{\mathrm{a}}\ x\right)}{\sin\left(\dfrac{2\pi\omega_o}{\mathrm{a}}\ L\right)} + \sin\left(\frac{2\pi\omega_o}{a}\ \mathrm{x}\right) \right\} \cos 2\pi\omega_o t .$$

2.1.2 Specific loadings

a cantilever beam length L working only in tension compression (or torsion) is subjected to a sinusoidal force in time, (or a moment) applied at its loose lead.

2.1.2.1 Tension

$$\begin{cases} \rho S \dfrac{\partial^2 u}{\partial t^2} - \mathrm{ES} \dfrac{\partial^2 u}{\partial x^2} = 0 \\ u(0) = 0, \ \dfrac{\partial u}{\partial x}(\mathrm{L}) = \dfrac{1}{\mathrm{ES}} \ \mathrm{f(t)}. \end{cases}$$

the technique of resolution is equivalent to that of the paragraph [§ 2.1.1.1].

For $f(t)=F\cos(2\pi\,\omega_0 t)$, we have:

$$u(\mathrm{x},\ \mathrm{t}) = \frac{\mathrm{a\,F}}{\mathrm{ES}2\pi\,\omega_o} \ \frac{\sin\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\ \mathrm{x}\right)}{\cos\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\ \mathrm{L}\right)} \ \cos(2\pi\,\omega_o\ \mathrm{t})$$

avec $a^2 = \dfrac{E}{\rho}$

and $$N(\mathrm{x},\ \mathrm{t}) = F \ \frac{\cos\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\ \mathrm{x}\right)}{\cos\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\ \mathrm{L}\right)} \ \cos(2\pi\,\omega_o\ \mathrm{t})$$

2.1.2.2 Torsion

$$\begin{cases} G\, I_p \dfrac{\partial^2 \theta x}{\partial x^2} - I_{\theta_x} \dfrac{\partial^2 \theta_x}{\partial t^2} = f(t) \\ u(0) = 0, \ u(L) = 0 \end{cases}$$

$$G = \frac{E}{2(1+\nu)}$$

$$I_p = \frac{\pi\ 0{,}01^4}{2} m^4, \qquad \theta_x(x,t) = \frac{b\,F}{G\,I_p\,2\pi\omega_0} \frac{\sin\dfrac{2\pi\omega_0}{b}x}{\cos\dfrac{2\pi\omega_0}{b}L} \cos(2\pi\omega_0 t)$$

$$I_{\theta_x} = \rho\ I_p \qquad M_T(x,t) = F \frac{\cos\left(\dfrac{2\pi\omega_0}{b}x\right)}{\cos\left(\dfrac{2\pi\omega_0}{b}L\right)} \cos(2\pi\omega_0 t)$$

avec $b = \dfrac{G}{\rho}$

2.2 Results of reference

internal forces (N and MT)

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

1) Ratio n° 2314/A of the Institute Aerotechnics “Proposal and realization for new cases tests missing to the validation of beams ASTER”

3 Modelization A

3.1 Characteristic of the modelization

The model is composed of 2 elements straight beam of Timoshenko.

3.2 Characteristics of the mesh

2 elements `POU_D_T`

3.3 Quantities tested and results

3.3.1 Charges divided into tension

		analytical Results	**Results *Aster***	**Variation (%)**
normal Force for $x=0$	$t=1/3\,s$	4.7247E+02	4.7247E+02	9.12E-07
	$t=2/3\,s$	3.92944E+02	3.9294E+02	– 6.08E-07
normal Force for $x=L/2$	$t=1/3\,s$	0.0000E+00	2.1985E-12	2.20E-12*
	$t=2/3\,s$	0.0000E+00	2.5087E-12	2.51E-12*

* absolute Deviation

3.3.2 concentrated Loading

3.3.2.1 Loading in tension

normal Force for $x=0$

	analytical Results	**Results *Aster***	**Variation (%)**
$t=1/3\,s$	9.44957E+02	9.44956E+02	– 7.59E-07
$t=2/3\,s$	7.8588E+02	7.8588E+02	3.01E-06

3.3.2.2 Loading in torsion

Twisting moment for $x=0$

	analytical Results	**Results *Aster***	**Variation (%)**
$t=1/3\,s$	9.4495E+02	9.4495E+02	– 1.88E-06
$t=2/3\,s$	7.8588E+02	7.8589E+02	7.29E-06

4 Summary of the results

This test makes it possible to check that the internal forces of the beam elements in dynamics are correct. The results show a very good agreement with the analytical solution, for a mesh only made up of two elements `POU_D_T`.