SDLL311 - Response transient dynamics of a beam in tension under Summarized imposed

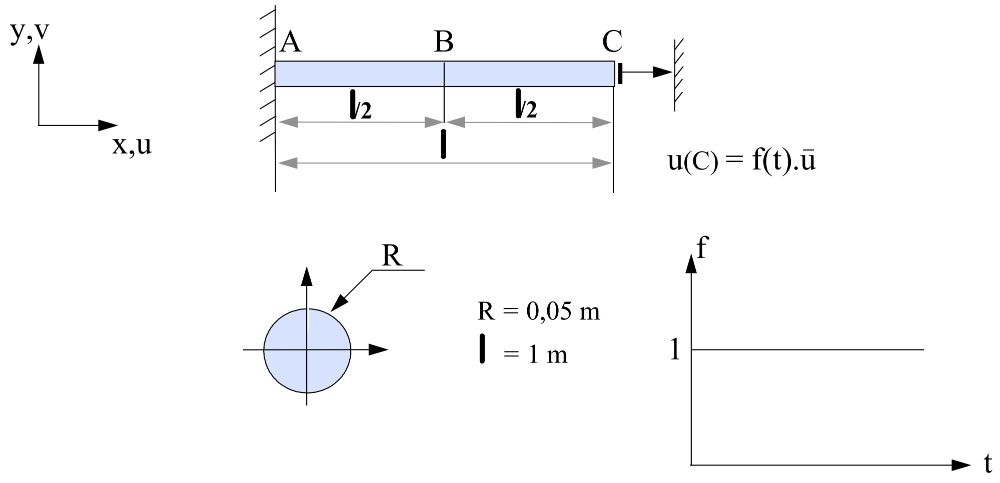
displacement:

This problem-test corresponds to a linear transient analysis of a bar requested in tension by application of a displacement imposed at an end, the other end being clamped. Function displacement of time is of type “Heaviside” imposed from initial time.

The results got in the middle of the beam for a modelization with four elements are compared with the analytical solution of the problem discretized by four elements by not taking into account the instantaneous peaks velocity and acceleration at initial time on the level of the end where displacement is imposed.

1 Problem of reference

1.1 Geometry



1.2 Material properties

$E=98696{,}044\ MPa$

$\nu=0$

$\rho=3.10^{6}\ kg/m^{3}$

Proportional damping of Rayleigh: $\boldsymbol{C}=\lambda\,\boldsymbol{K}+\mu\,\boldsymbol{M}$ $\lambda=5.10^{-4}$, $\mu=5$

1.3 Boundary conditions and loadings

Displacement imposed at the end C : $u(C)=\bar{u}\,f(t)$ with $\bar{u}=10^{-3}m$ and $f(t)$ evolution according to the time of the Heaviside type: $f(t)=1 \quad t\geq 0$.

Clamped A end.

1.4 Initial conditions

initial Displacement no one in any point.

Initial velocity null in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

the discretized problem checks:

$$\left[\begin{array}{c|c} M_{ll} & M_{ld} \\ \hline M_{ld}^T & M_{dd} \end{array}\right]\left\{\frac{\ddot{u}_l}{\ddot{u}_d}\right\}+\left[\begin{array}{c|c} C_{ll} & C_{ld} \\ \hline C_{ld}^T & C_{dd} \end{array}\right]\left\{\frac{\dot{u}_l}{\dot{u}_d}\right\}+\left[\begin{array}{c|c} K_{ll} & K_{ld} \\ \hline K_{ld}^T & K_{dd} \end{array}\right]\left\{\frac{u_l}{u_d}\right\}=\left\{\frac{0}{F_d}\right\},$$

with index l : free degree of freedom
index d : degree of freedom imposed

$F_d(t)$ external loadings applied to the nodes ends and leading to imposed displacements u_d are unknown, one thus eliminates these equations and one obtains:

$$[M_{ll}]\{u_l\}+[C_{ll}]\{u_l\}+[K_{ll}]\{u_l\}=-[M_{ld}]\{u_d\}-[C_{ld}]\{u_d\}-[K_{ld}]\{u_d\} .$$

The only non-zero terms of the second member of this system are related to the kinematical variables relative to ending node where displacement is imposed. However, with $t=0$, $\ddot{u}_{dC}$ and $\dot{u}_{dC}$ are not defined but with $t=0^-$ and $t=0^+$, $\ddot{u}_{dC}$ and $\dot{u}_{dC}$ are null. All the complexity of the problem comes from that.

To obtain a reference solution, we considered $\ddot{u}_{dC}$ and $\dot{u}_{dC}$ uniformly null what amounts considering only the elastic internal forces at the end C . This is debatable from a physical point of view but, by adopting the same assumptions at the time of the modelization of the problem, the validation of *Code_Aster* can be concluded.

One calculates the reference solution by dealing with the following problem:

$$[M_{ll}]\{\ddot{u}_l\}+[C_{ll}]\{\dot{u}_l\}+[K_{ll}]\{u_l\}=-[K_{ld}]\{u_d(t)\} \text{ with } \{u_l(0)\}=0 \text{ and } \{\dot{u}_l(0)\}=0 .$$

With this intention, one transports the problem in the modal base of the system which checks:

$$[M_{ll}]\{\ddot{u}_l\}+[K_{ll}]\{u_l\}=0 .$$

The damping being diagonal, the diagonal system is obtained:

$$[m_g]\{\ddot{X}\}+[c_g]\{\dot{X}\}+[k_g]\{X\}=\{g(t)\} \text{ où } \{g(t)\}=\{g\} \text{ pour } t\geq 0 ,$$

$$\text{with } \{X(0)\}=0 \text{ and } \{\dot{X}(0)\}=0 .$$

In modal space, one thus solves three equations (3 free degrees of freedom) differential of the second order then one returns in physical space. One obtains then the displacement of the point medium:

$$u_B(t)=\sum_{i+1} e^{-\lambda i t}(a_i\cos(\tilde{\omega}_i t)+b_i\sin(\tilde{\omega}_i t)) ,$$

with $\tilde{\omega}_i$: i ème own pseudo-pulsation of the damped system.

2.2 Results of reference

Displacement, velocity and acceleration of the point medium B of the beam.

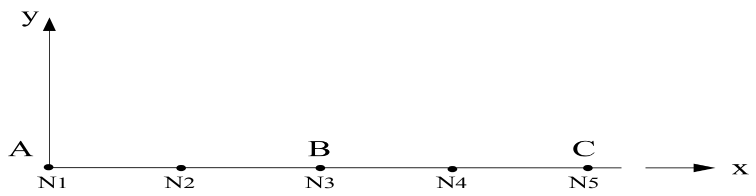
2.3 Uncertainty on the analytical

solution Solution of the problem discretized in four elements length equal by considering velocity and acceleration uniformly null to the point C where displacement is imposed.

3 Modelization A

3.1 Characteristic of the modelization

Modelization in beam element 3D : POU_D_T



Cutting:
AC = 4 meshes SEG2 length equalize

limiting Conditions:

- Clamped $N1(A)$ node
  DDL_IMPO $DX=DY=DZ=DRX=DRY=DRZ=0$
- Node $N5(C)$ in imposed displacement following x
  DDL_IMPO $DY=DZ=DRX=DRY=DRZ=0 \quad DX(t)=\bar{u}$

Resolution:
Algorithm of direct integration of Newmark
Time step: $\Delta t=10^{-5} s$
Period of observation: $0{,}03\, s$

3.2 Characteristics of the mesh

Number of node S: 5
Number of meshes and type: 4 meshes SEG2

3.3 Quantities tested and Displacement

- results at the point Time B

medium (s)	**Reference Displacement** (m)
0,0054	87,376 e-3
0,0055	87,360 e-3
0,0108	26,818 e-3
0,0109	26,800 e-3
0,0163	64,386 e-3
0,0164	64,366 e-3
0,0217	41,083 e-3
0,0218	41,084 e-3
0,0271	55,525 e-3
0,0272	55,530 e-3

4 Modelization B

4.1 Characteristic of the modelization

idem that the modelization A

4.2 Characteristic of the mesh

idem that the modelization A

4.3 Quantities tested and Displacement

- results at the point Time B

medium (s)	**Reference Displacement** (m)
0,0054	87,376 e-3
0,0055	87,360 e-3
0,0108	26,818 e-3
0,0109	26,800 e-3
0,0163	64,386 e-3
0,0164	64,366 e-3
0,0217	41,083 e-3
0,0218	41,084 e-3
0,0271	55,525 e-3
0,0272	55,530 e-3

5 Summary of the results

the results given by *Code_Aster* are in perfect agreement with the results of the analytical model, that displacement boils about it beam is imposed by `a VECTEUR ASSEMBLES` or `a CHARGE`.

Caution: the questions of Dirichlet for transient computation on physical base with `DYNA_VIBRA` are compatible only with the integration method of NEWMARK.