SDLL148 – definition of an inter-spectrum analytical of excitation on a beam and projection on modal base

Summarized:

This case test makes it possible to validate option SPEC_CORR_CONV_3 of the operator DEFI_SPEC_TURB, who allows to define a spectrum defined by a set of analytical functions, and to project this one on a basis of modes.

1 Problem of reference

1.1 Geometry of the digital model

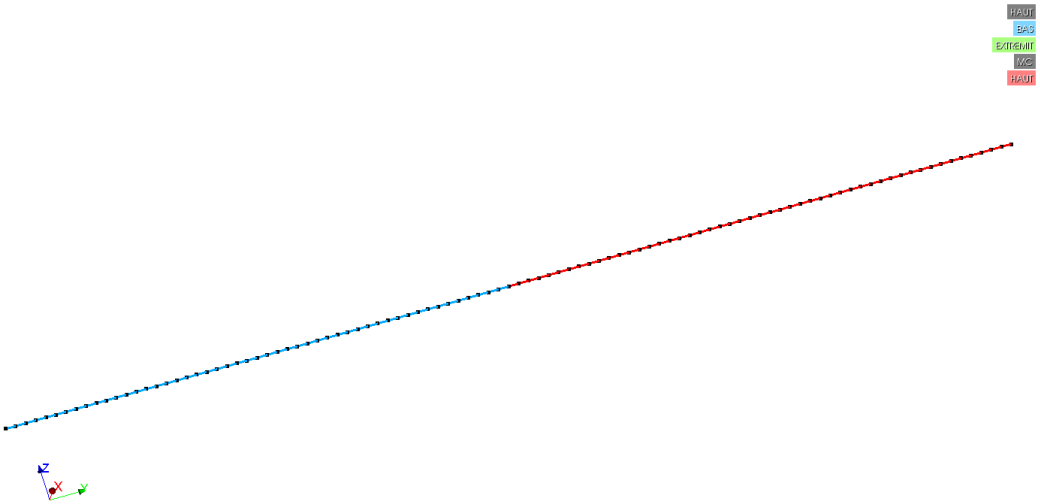

Illustration 1.1: mesh of the model.

1.2 Mesh groups and of nodes

- nodes groups and meshes MC : all the beam,
- nodes groups and of meshes BAS : low half (in blue),
- nodes groups and of meshes $HAUT$: high half (in red),
- group of the node is outside the field of definition with a right profile of the EXCLU type node: $EXTREMIT$ base beam.

1.3 Properties of the materials

- For all the elements
  - $E=2.2\times10^{11}\,Pa$ Modulus Young
  - $\nu=0.3$ Poisson's ratio
  - $\rho=8333.0\,kg.m^{-3}$ Density

1.4 Boundary conditions and loadings

- imposed Displacement:
  - group $EXTREMIT$: $DRX=DRY=DRZ=DX=DY=DZ=0.0$
  - group MC : $DZ=0.0$

1.5 Characteristic geometrical structural elements

- Groups MC : section of the type tubes ("CERCLE"):
  - radius: $R=7.94\,mm$,
  - thickness: $e=3.176\,mm$

2 Objective benchmark, validation

the purpose of the benchmark is to validate functionality SPEC_CORR_CONV_3 of the operator DEFI_SPEC_TURB , who allows to define a spectrum by means of analytical functions of the variable of space and frequency.

In the modelization A, the definite spectrum is purely theoretical, since the functions are sines. The computation of the double integral $\int_\Omega \int_\Omega \Phi_i(\underline{x_1}).S_f(\underline{x_1},\underline{x_2},\omega).\Phi_j(\underline{x_2})\,d\Omega_1\,d\Omega_2$ can thus be solved in an analytical way, by supposing that the first 2 eigen modes of the beam are also functions sine. In the modelization B, the definite spectrum is representative of a flow downstream from a grid of mixture on a fuel pin (models of Corcos).

2.1 Course of the Computation

- benchmark of the eigen modes: the first two eigen modes are calculated, they are form $\phi(y)=\sin(\pi y)$ and $\phi(y)=\sin(2\pi y)$,
- definition of the functions of the turbulent spectrum:
  - modelization A $S_{xx}=f.\sin(\pi y_1).\sin(\pi y_2)$
  - modelization B

$$S_f(\underline{x_1},\underline{x_2},\omega)=\begin{cases} S_{xx}=\exp\left(-\dfrac{|y_2-y_1|}{\lambda_{cx}(\omega)}\right).\exp\left(j\omega\dfrac{y_2-y_1}{U_c}\right)S_f(y_1,y_1,\omega) \\ S_{yy}=\exp\left(-\dfrac{|y_2-y_1|}{\lambda_{cy}(\omega)}\right).\exp\left(j\omega\dfrac{y_2-y_1}{U_c}\right)S_f(y_1,y_1,\omega) \end{cases}$$

  (while adding terms of correlation enters the forces according to x and y with the functions S_{xy} and S_{yx}).
- creation of an array containing the functions associated with the directions,
- creation of the inter-spectrum with DEFI_SPEC_TURB,
- projection of the inter-spectrum on the two eigen modes calculated with PROJ_SPEC_BASE ; projection is done on the mesh group "LOW" only.

Note:

For the modelization B, one defines an excitation in the two directions. However, the two calculated modes are in the direction X (the direction DZ is blocked). Thus the excitation according to Y will not have any influence on result of the modal excitation.

2.2 Validation of the Modelization

2.2.1 results A

For the modelization A, the validation is analytical. Indeed, the integrals with calculating are the following ones:

- auto--spectrum mode 1: $\int_0^{0.5}\int_0^{0.5}\sin^2(\pi y_1)\sin^2(\pi y_2)\,dy_1\,dy_2=\frac{1}{4}.\frac{1}{4}=0.0625$
- inter-spectrum mode 1 – mode 2:

$$\int_0^{0.5}\int_0^{0.5}-\sin^2(\pi y_1)\sin(\pi y_2)\sin(2\pi y_2)\,dy_1 dy_2=-\frac{2}{3\pi}.\frac{1}{4}=-0.05305$$

- auto--spectrum mode 2:

$$\int_0^{0.5}\int_0^{0.5}\sin(\pi y_1)\sin(2\pi y_1)\sin(\pi y_2)\sin(2\pi y_2)\,dy_1\,dy_2=-\frac{2}{3\pi}.-\frac{2}{3\pi}=0.04503$$

2.2.2 Modelization For

the modelization B B, the validation is done by non regression.

- auto--spectrum mode 1: 0.11
- inter-spectrum mode 1 – mode 2: $0.11+0.01534\mathrm{j}$
- auto--spectrum mode 2: 0.11 .