SDLL146 - Validation of the elements “bars” in dynamics

Summarized:
The purpose of this test is to validate the computation of the mass matrix of the elements BARS . It is checked that this matrix is quite threedirectional contrary to the stiffness matrix (one-way parallel to the directional sense of the element). For that one does the following calculations:

- Computation of the internal forces and the reactions of bearing of a clamped element subjected to a field of gravity according to 3 directions (parallel with X , Y and Z) .
- Projection of the mass matrix on a basis made up of three modes (parallel with X , Y and Z).
- Computation of kinetic energy for three modes velocity (parallel with X, Y and Z).

Note: all the calculations are done with the complete mass matrix and the diagonal mass matrix except for the computation of the kinetic energy which is carried out with the complete mass matrix.

1 Problem of reference

1.1 Geometry

One considers a mesh SEG2 of with dimensions $1\,m$, directed the axis parallel to X .

1.2 Properties of the material

the material is elastic isotropic whose properties are:

$E=37\,000\,MPa$

$\nu=0.2$

$\rho=100\,kg/m^3$

1.3 Boundary conditions and loadings

the node O is clamped. In B , DY and DZ are blocked. The imposed loading is carried out with `CALC_CHAR_SEISME` and is directed according to X , then Y and Z , it corresponds to a field of gravity according to X , then Y and Z .

2 Reference solution

2.1 Méthode de calcul

2.1.1 Recalls

The modelization BAR transmits neither shears nor bending moment. So if one notes E the Young modulus of the element, A the area his section and L his length, the elemental stiffness matrix K^{elem} of a bar is the following one (with the components in the order $(DX_1\,DY_1\,DZ_1\,DX_2\,DY_2\,DZ_2)$):

$$K^{elem}=\begin{pmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \frac{-EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The profile of the mass matrix M^{elem} is different from the stiffness matrix because the mass must be taken into account in all the directions of space. Thus, if one notes ρ the density of the element, the elementary mass matrix, with $m=\rho\,A\,L$, is the following one:

$$M^{elem}=\begin{pmatrix} m/3 & 0 & 0 & m/6 & 0 & 0\\ 0 & m/3 & 0 & 0 & m/6 & 0\\ 0 & 0 & m/3 & 0 & 0 & m/6\\ m/6 & 0 & 0 & m/3 & 0 & 0\\ 0 & m/6 & 0 & 0 & m/3 & 0\\ 0 & 0 & m/6 & 0 & 0 & m/3 \end{pmatrix}$$

2.1.2 Computation of the internal forces and the reactions of bearing

Is m the mass of the element and U displacement, then if one chooses a field of gravity according to X , the external forces in each node are the following ones: $(-m/2,0\,,0)$. For an elastic behavior of stiffness K , only displacement not imposed U_{B_X} is equal to $\frac{F^{ext}_{B_X}}{K}=\frac{-m}{2K}$.

If K^{elem} the elemental stiffness matrix is noted, there is the relation $F^{int}=K^{elem}U$. One gets the following results for the internal forces: $F^{int}_{O_X}=\frac{m}{2}$ $F^{int}_{O_Y}=0$ $F^{int}_{O_Z}=0$ $F^{int}_{B_X}=\frac{-m}{2}$, $F^{int}_{B_Y}=0$ and $F^{int}_{B_Z}=0$.

By noting R^{ap} the reactions of bearing, there is the relation $R^{ap}=F^{int}-F^{ext}$. One thus has easily: $R^{ap}_{O_X}=m$ and all other null components.

Note: for the directions of loading Y and Z , there is no displacement thus the internal forces are null and the reactions of bearing are equal to the external forces.

2.1.3 The purpose of projection of the mass matrix on a modal base

This computation is checking that for a unit mode of displacement ϕ according to a given direction, one has the equality:

$$\phi^T M \phi = m$$

2.1.4 The purpose of computation of kinetic energy

This computation is checking that for a unit mode velocity ϕ according to a given direction, one has the equality:

$$\frac{1}{2}\phi^T M \phi = \frac{mv}{2} \text{ with } v=1$$

3 Modelization A

3.1 Characteristic of the modelization

One uses a modelization BARS.

3.2 Characteristics of the mesh

The mesh contains 1 element of type SEG2.

3.3 Quantities tested and results

3.3.1 Reactions of bearing and internal forces to the two nodes of the mesh.

Matrix supplements gravity according to X :

Quantity	Standard	Component	Place of reference	Value of reference (N)	Tolerance (%)
REAC_NODA	Node N001	DX	"ANALYTIQUE"	-100	1.0E-4
REAC_NODA	Node N002	DX	"ANALYTIQUE"		0.1.0E-4
FORC_NODA	Node N001	DX	"ANALYTIQUE"	-50	-50.1.0E-4
FORC_NODA	Node N002	DX	"ANALYTIQUE"	-50	-50.1.0E-4

Stamps supplements gravity according to Y :

Quantity	Standard	Component	Place of reference	Value of reference (N)	Tolerance (%)
REAC_NODA	Node N001	DY	"ANALYTIQUE"	-50	-50.1.0E-4
REAC_NODA	Node N002	DY	"ANALYTIQUE"	-50	-50.1.0E-4
FORC_NODA	Node N001	DY	"ANALYTIQUE"		0.1.0E-4
FORC_NODA	Node N002	DY	"ANALYTIQUE"		0.1.0E-4

Stamps supplements gravity according to Z :

Quantity	Standard	Component	Place of reference	Value of reference (N)	Tolerance (%)
REAC_NODA	Node N001	DZ	"ANALYTIQUE"	-50	-50.1.0E-4
REAC_NODA	Node N002	DZ	"ANALYTIQUE"	-50	-50.1.0E-4
FORC_NODA	Node N001	DZ	"ANALYTIQUE"		0.1.0E-4
FORC_NODA	Node N002	DZ	"ANALYTIQUE"		0.1.0E-4

Stamps diagonal gravity according to X :

Quantity	Standard	Component	Place of reference	Value of reference (N)	Tolerance (%)
REAC_NODA	Node N001	DX	"ANALYTIQUE"	-100	1.0E-4
REAC_NODA	Node N002	DX	"ANALYTIQUE"		0.1.0E-4
FORC_NODA	Node N001	DX	"ANALYTIQUE"	-50	-50.1.0E-4
FORC_NODA	Node N002	DX	"ANALYTIQUE"	-50	-50.1.0E-4

Stamps diagonal gravity according to Y :

Quantity	Standard	Component	Place of reference	Value of reference (N)	Tolerance (%)
REAC_NODA	Node N001	DY	"ANALYTIQUE"	-50	-50.1.0E-4
REAC_NODA	Node N002	DY	"ANALYTIQUE"	-50	-50.1.0E-4
FORC_NODA	Node N001	DY	"ANALYTIQUE"		0.1.0E-4
FORC_NODA	Node N002	DY	"ANALYTIQUE"		0.1.0E-4

Stamps diagonal gravity according to Z :

Quantity	Standard	Component	Place of reference	Value of reference (N)	Tolerance (%)
REAC_NODA	Node N001	DZ	"ANALYTIQUE"	-50	-50.1.0E-4
REAC_NODA	Node N002	DZ	"ANALYTIQUE"	-50	-50.1.0E-4
FORC_NODA	Node N001	DZ	"ANALYTIQUE"		0.1.0E-4
FORC_NODA	Node N002	DZ	"ANALYTIQUE"		0.1.0E-4

3.3.2 Projection of the mass matrix on modal base

Stamps supplements

Standard	ARRAY of reference	Value of reference (kg)	Tolerance (%)
TABX	"ANALYTIQUE"	100.0	1.0E-4
TABY	"ANALYTIQUE"	100.0	1.0E-4
TABZ	"ANALYTIQUE"	100.0	1.0E-4

diagonal Matrix

COUNTS	Standard of reference	Value of reference (kg)	Tolerance (%)
TABX	"ANALYTIQUE"	100.0	1.0E-4
TABY	"ANALYTIQUE"	100.0	1.0E-4
TABZ	"ANALYTIQUE"	100.0	1.0E-4

3.3.3 Kinetic energy

Stamps supplements

ARRAY	NOM_PARA	Type of reference	Value of reference (J)	Tolerance (%)
TABX	TOTAL	"ANALYTIQUE"	50.0	1.0E-4
TABY	TOTAL	"ANALYTIQUE"	50.0	1.0E-4
TABZ	TOTAL	"ANALYTIQUE"	50.0	1.0E-4

4 Summary of the results

the tests carried out in this documentation show that the mass of the element BAR is applied in the three directions of space.