SDLL141 - Eigenfrequencies of a beam alone, subjected to the gyroscopic effect.

Abstract:

This problem consists in seeking the frequencies of vibration of a beam pressed on each one of its ends, on infinitely rigid bearings. The beam is full, of section circular and subjected at a constant rotational speed. It comprises any disc.

Two modelization are studied:

- Modelization a: the beam is along the axis X,
- Modelization b: the beam is along the axis t such directing t quevector of the bisectrix (x, y) .
- Modelization C: the beam is along the axis t such as t directing vector of the bisectrix (x, y) , and it mass is distributed by discrete elements installed on each node.
- Modelization D: the beam is along the axis X. The section is circular and variable with two radius $R1$ and $R2$ identical ones.

This problem thus makes it possible to test the effect of the gyroscopic matrix which was developed for a straight beam.

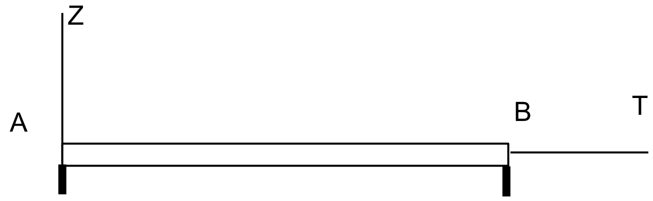
The gyroscopic effect led to the unfolding of the modes. The evolution of the eigenfrequencies according to rotational speed makes it possible to build the diagram of Campbell.

The references are based on the theory of Eulerian-Bernouilli.

The got results are in concord with those given in reference.

1 Problem of reference

1.1 Geometry



Modelizations A and D: SDLL141a and SDLL141d

$$t=x$$

Modelizations B and C: SDLL141b and SDLL141c

$$\frac{\pi}{4}=(\hat{x},t) \text{ and } t.z=0$$

For the modelization C (SDLL141c) the density of the material is taken equal to zero. The mass is installed via discrete elements installed on each node.

Length of beam:

$$L=AB=0.9\,m$$

Circular section:

Diameter: $D=0.05\,m$

Coordinates of the points (m):

		Modelizations A and D	Modelizations B and C
	A	B	B
X	0.	0.9	$0.9\cos(\pi/4)$
Y	0.	0.	$0.9\sin(\pi/4)$
Z	0.	0.	0.

Table 1.1-1 : Coordinates of the points A and B

1.2 Material properties

$E=2.10^{11}\,Pa$
$\rho=7\,800\,kg/m^3$ safe for the modelization C.

1.3 Boundary conditions and loadings

Point: A supported $u=v=w=0$
Point: B supported $u=v=w=0$

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is that presented in the work of Rene-Jean GIBERT.
By adopting the following notations:

- beam following x
- y and the z motions of bending in the plane xOz and xOy
- S : section of beam:
- I main moment of inertia of bending compared to the axes y and z
- I_x : main moment of inertia per unit of length compared to the axis Ox
- ρ , E the characteristics of the material
- Ω rotational speed of the beam

the solution singular are controls by the following system of equations:

$$EI\frac{\partial^4 Y}{\partial x^4}-\rho S\omega^2 Y+i\omega\Omega I_x\frac{\partial^2 Z}{\partial z^2}=0$$
$$EI\frac{\partial^4 Z}{\partial x^4}-\rho S\omega^2 Z-i\omega\Omega I_x\frac{\partial^2 Y}{\partial z^2}=0$$

by observing the following boundary conditions:

$$\begin{cases} Y=Z=0 \\ \dfrac{\partial^2 Y}{\partial z^2}=\dfrac{\partial^2 Z}{\partial z^2}=0 \end{cases} \text{ in } \begin{cases} x=0 \\ x=L \end{cases}$$

One obtains two families of eigen modes:

- Mode retrogresses:

$$Y_1=-i.Z_1=\sin\frac{n\pi x}{L} \text{ with } \left(\frac{\omega_1}{\omega_0}\right)=\sqrt{\lambda^2+1}-\lambda$$

- direct Mode:

$$Y_2=-i.Z_2=\sin\frac{n\pi x}{L} \text{ with } \left(\frac{\omega_2}{\omega_0}\right)=\sqrt{\lambda^2+1}+\lambda$$

while posing:

own pulsation without rotation: $\omega_0=\left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{\rho S}}$

$$\lambda=\frac{1}{2}.\frac{\Omega I_x}{\sqrt{EI\rho S}} \text{ with } I_x=\frac{\rho S D^2}{8} \text{ and } I=\frac{\pi D^4}{64}$$

2.2 Results of reference

the first 4 eigen modes of bending.

2.3 Uncertainty on the analytical

solution Solution with the assumption of beam of Eulerian.

2.4 Bibliographical references

Rene-Jean GIBERT, Vibrations of structures, n°69 of the collection R & D of EDF at EYROLLES, p. 235-237 (1988).

3 Modelization A

3.1 Characteristic of the modelization

Modelization : 18 Elements équirépartis of limiting `beam`

POU_D_E Conditions:
Node at the Node A `DDL_IMPO`: $(DX=0.0, DY=0.0, DZ=0.0)$
end at end B `DDL_IMPO`: $(DX=0.0, DY=0.0, DZ=0.0)$
`ANGL_NAUT (45. , 0,0)` for the modelizations B and C
Names of the nodes: Not A = $N1$
Not B = $N19$

For the modelization D
the elements are of circular section variables with two radius $R1$ and $R2$ identical ones.

For the modelization C

Beam following t with $\frac{\pi}{4}=(\hat{x}, t)$ and $t.z=0$

Density null
Length of an element

$$e=\frac{L}{18}=0.05\,m$$

Characteristics of the discrete elements in the base (t, v, z)

Nodes $N2$ to $N18$	Nodes $N1$ and $N19$
$m=\rho e \pi \frac{D^2}{4}=0.7657\,kg$	$m'=\rho \frac{e}{2} \pi \frac{D^2}{4}=0.3829\,kg$
$I_{tt}=m.\frac{D^2}{8}=2,393.10^{-4}\ kg.m^2$	$I'_{xx}=m'.\frac{D^2}{8}=1,196.10^{-4}\ kg.m^2$
$I_{vv}=I_{zz}=\frac{I_{tt}}{2}+m.\frac{e^2}{12}=2,791.10^{-4}\ kg.m^2$	$I'_{vv}=I'_{zz}=\frac{I'_{tt}}{2}+m'.\frac{e^2}{3}=1,395.10^{-4}\ kg.m^2$

Table 3.1-1 : Computation of the characteristics of the discrete elements

Two solutions are possible to define the characteristics in the base:
- is to carry out a basic change of the local coordinate system of the beam (t, v, z) to the total reference (x, y, z). For that, it is necessary to carry out a basic change by a rotation of axis z and value $-45°$. One obtains:

$$\overline{\overline{I}}=\begin{bmatrix} I_{xx} & I_{xy} & 0 \\ I_{xy} & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \text{ with:}$$

$$I_{xx}=\cos^2(\pi/4) I_{tt}+\sin^2(\pi/4) I_{vv}$$
$$I_{yy}=\sin^2(\pi/4) I_{tt}+\cos^2(\pi/4) I_{vv}$$
$$I_{xy}=\cos(\pi/4)\sin(\pi/4)(I_{tt}-I_{vv})$$

Characteristics of the discrete elements in the base (x, y, z)

Nodes $N2$ with $N18$	Nodes $N1$ and $N19$
$m=\rho.e.\pi\frac{D^2}{4}=0{,}7657\ kg$	$m'=\rho.\frac{e}{2}.\pi\frac{D^2}{4}=0{,}3829\ kg$
$I_{xx}=I_{yy}=\frac{1}{2}\left(m.\frac{D^2}{8}+\frac{1}{2}m.\frac{D^2}{8}+m.\frac{e^2}{12}\right)$ $=2{,}592.10^{-4}\ kg.m^2$	$I'_{xx}=I'_{yy}=\frac{1}{2}\left(m'.\frac{D^2}{8}+\frac{1}{2}m'.\frac{D^2}{8}+m'.\frac{e^2}{12}\right)$ $=1{,}296.10^{-4}\ kg.m^2$
$I_{zz}=\frac{I_{tt}}{2}+m.\frac{e^2}{12}=2{,}792.10^{-4}\ kg.m^2$	$I'_{zz}=\frac{I'_{tt}}{2}+m'.\frac{e^2}{12}=1{,}396.10^{-4}\ kg.m^2$
$I_{xy}=I_{yx}=\frac{1}{2}\left\{m.\frac{D^2}{8}-\left(\frac{1}{2}m.\frac{D^2}{8}+m.\frac{e^2}{12}\right)\right\}$ $=-1{,}994.10^{-5}\ kg.m^2$	$I'_{xy}=I'_{yx}=\frac{1}{2}\left\{m'.\frac{D^2}{8}-\left(\frac{1}{2}m'.\frac{D^2}{8}+m'.\frac{e^2}{12}\right)\right\}$ $=-9{,}971.10^{-6}\ kg.m^2$

Table 3.1-2 : Computation of the characteristics of the discrete elements

– is to declare the characteristics in the local coordinate system of the beam and to use the nautical angles to lay down the direction of the local coordinate system. It is this method which was used for the modelization C.

3.2 Characteristic of the mesh

Mesh: Many nodes: 19
Number of meshes and types: 18 SEG2

3.3 Quantities tested and Rotor

results with the stop ($\Omega=0$) (frequencies in Hz)

Reference	Modelization A		Modelization B		Modelization C		Modelization D	
frequency	ASTER	% reference	ASTER	% reference	ASTER	% reference	ASTER	% reference
122,7475	122,7475	0,000	122,7461	0,001	122,4789	-0,219	122,7475	0,000
490,9899	490,9949	0,001	490,9889	0,001	486,6954	-0,875	490,9949	0,001
1104,7273	1104,7844	0,005	1104,7710	0,004	1083,2630	-1,943	1104,7844	0,005
1963,9596	1964,2791	0,016	1964,2551	0,015	1897,3908	-3,390	1964,2791	0,016

Table 3.2-1 : Computation of the frequencies of the rotor to the stop

Computation of the eigenfrequencies using the algorithm of Sorensen
Rotor in rotation ($\Omega=10^4 rd.s^{-1}$), direct modes (frequencies in Hz)

Reference	Modelization A		Modelization B		Modelization C		Modelization D	
frequency	ASTER	% reference	ASTER	% reference	ASTER	% reference	ASTER	% reference
125,8150	125,8150	0,000	125,8135	0,001	125,5324	-0,225	125,8150	0,000
503,2598	503,2649	0,001	503,2587	0,000	498,7463	-0,897	503,2649	0,001
1132,3346	1132,3918	0,005	1132,3779	0,004	1109,7993	-1,990	1132,3918	0,005
2013,0393	2013,3589	0,016	2013,3342	0,015	1986,6704	-1,310	2013,3589	0,016

Table 3.2-2 : Computation of the frequencies of the direct modes of the rotor to the velocity $10000 rad/s$

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$), modes retrograde (frequencies in Hz)

Reference	Modelization A		Modelization B		Modelization C		Modelization D	
frequency	ASTER	% reference	ASTER	% reference	ASTER	% reference	ASTER	% reference
119,7548	119,7549	0,000	119,7534	0,001	119,4990	-0,214	119,7549	0,000
479,0191	479,0242	0,001	479,0183	0,000	474,9244	-0,855	479,0242	0,001
1077,7931	1077,8502	0,005	1077,8370	0,004	1057,3064	-1,901	1077,8502	0,005
1916,0765	1916,3958	0,017	1916,3723	0,015	1852,4597	-3,320	1916,3958	0,017

Table 3.2-3 : Computation of the frequencies of the retrograde modes of the rotor at the speed $10000 rad/s$

4 Summary of the results

Good establishment of the gyroscopic effect for the beam element. Change of axis of the beam x (direction according to which the elements were established) (modelization A) with a direction $x+y$ (modelization D) of results does not generate variations.

In analytical absence of reference for the validation of the discrete elements subjected to the gyroscopic effect, the modelization C makes it possible all the same to check the gyroscopic matrix installation of a presumedly indeformable disc. The motion of each disc is fixed by that of the nodes and thus follows the deformed shape of neutral fiber only in a discrete way. This C explains the variations noted on the modelization, all the more for the high modes characterized by a concavity of the more important modal deformed shape.