SDLL135 - Dynamic response of a embed-free beam-pipe

Abstract
the scope of application is the linear dynamics, and more particularly the modal analysis then the linear transient analysis.

Various modelizations:

1) finite elements of beam of Eulerian POU_D_E,
2) finite elements of beam of Timoshenko POU_D_T,
3) finite elements of pipes (TUYAU_3M and TUYAU_6M).
4) finite elements of bars

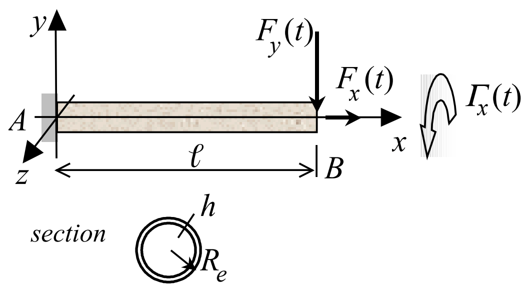
This case test is the study of a homogeneous isotropic linear beam-pipe elastic, embedded at its base, subjected to a level of force and couple at the head.

The purpose is to test the eigenfrequencies, displacements and the reactions of bearings in the transient regime (propagation of one wave acceleration), in small transformations with a diagram of temporal integration implicit. This beam-pipe is short and the effect of the transverse energy of shears is notable. To validate the taking into account of the term due to the inertia forces, one tests the nodal reaction at the charged point, this one having to be null.

The comparison is carried out compared to an analytical solution whose broad outlines are presented; intercomparison between modelizations. The comparison is also carried out for solutions obtained with other computation softwares: *Circus* (by transfer transfer functions) and *EuroPlexus* (in explicit fast dynamics). For the bars the comparison is carried out on the results got by commands DYNA_VIBRA and DYNA_NON_LINE in order to test option M_GAMMA.

1 Problem of reference

1.1 Assumptions and geometry



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considers an isotropic homogeneous beam-pipe elastic, charged at its end and embedded with the other.
This problem has an analytical solution in theory of the beams, confer for example [bib1], [bib2].

Dimensions are the following ones (example drawn from [bib3]):

1) length $l=1,00\,m$,
2) radius external of the pipe: $R_e=0.16\,m$,
3) thickness: $h=0.01\,m$

1.2 Properties of the materials

One chooses the data of the example drawn from [bib3]:

Young's modulus: $E=200000\ MPa$

Poisson's ratio: $\nu=0{,}29$

Density: $\rho=7830\ kg/m^3$

One does not introduce viscous damping into the problem.

1.3 Boundary conditions and loading

Boundary conditions

the end A has its displacements of beam blocked in x, y and z as well as rotations; on the other hand, the section can become deformed freely: the modes of ovalization-swelling of the pipe are free.

Initial conditions

the beam-pipe is initially at rest in a virgin state.

Loading

One exerts a level of thrust load $F_x=1{,}0\ N$, transverse force $F_y=1{,}0\ N$, axial couple $\Gamma_x=1{,}0\ Nm$ on the end B at time $t=0\,s$.

The period of analysis is sufficient to reach the first reflection of wave on the fixed support.

Gravity is neglected.

2 Reference solution

2.1 Method of calculating used for the reference solution

One notes $C_\ell=\sqrt{E/\rho}$ the celerity of the waves of traction and compression.

The section of the beam-pipe is: $S=\pi h\left(2R_e-h\right)$.

The inertia of bending of the beam-pipe is: $I=\pi h\left(R_e\left(R_e^2+h^2\right)-3hR_e^2/2-h^3/4\right)$.

The inertia of torsion of the beam-pipe is: $J_x=2\pi h\left(R_e\left(R_e^2+h^2\right)-3hR_e^2/2-h^3/4\right)$.

Slenderness is given by: $\eta=l\sqrt{S/I}$, l being the length of the beam.
For the dimensions given to the § 1, one a:

$$S=0.0097389\,m^2\ ;\ I=0.0001171\,m^4\ ;\ \eta=9.1192\,.$$

The vibratory behavior of structure consists of modes of "beam", and modes of "shells" (ovalization…). This beam-pipe is short: one can expect an effect of the energy of transverse shears. One builds the solution transient dynamics under excitation forced by modal recombining.

2.1.1 Elastodynamic solution of the beam in traction and compression

◊ the dynamic elastic equilibrium of free vibrations in traction and compression is written:

$$ESu_{,xx}-\rho Su_{,tt}=0$$, because the normal force is $N=ESu_{,x}$

the frequencies of j the èmes eigen modes of beam are given by [bib2] in the embed-free situation in axial extension:

$$f_j^{axial}=\frac{C_\ell}{4\ell}\left(2j-1\right),\quad \text{for } j=1,2\ldots,\quad \text{modes being: } u_j(x)=\sin\frac{\pi(2j-1)x}{2\ell}$$

With the mechanical characteristics of this problem, one finds the frequencies:

$$f_1=1263{,}497\,Hz\quad f_2=3790{,}490\,Hz\quad f_3=6317{,}484\,Hz\,,\ f_4=8844{,}477\,Hz\ldots$$

This gives the order of magnitude of the periods of harmonic pulsation of the response to the loading.

◊ now Let us treat the transitory solution in loading imposed at the end.
The transitory axial displacement of the beam, under the action of a force F_x, is given by [bib1]:

$$u(x,t)=\frac{4C_\ell}{\pi ES}\sum_{j=1,2,3\ldots}^{\infty}\frac{(-1)^{j-1}}{2j-1}\sin\left(\frac{(2j-1)\pi x}{2\ell}\right)\int_0^t F_x(\tau)\sin\left(\frac{(2j-1)\pi C_\ell(t-\tau)}{2\ell}\right)d\tau$$

For a force level F_x applied to $t=0$ in B, the wave is propagated with celerity C_ℓ and comes to be thought in A of time ℓ/C_ℓ; with the mechanical characteristics of this problem, one finds that this time is: $0{,}000197864\,s$.
One a:

$$u(x,t)=\frac{8\ell F_x}{\pi^2 ES}\sum_{j=1}^{\infty}\frac{(-1)^{j-1}}{(2j-1)^2}\sin\frac{(2j-1)\pi x}{2\ell}\left(1-\cos\frac{(2j-1)\pi C_\ell t}{2\ell}\right)$$

This series converges quickly because of term $(2j-1)^2$. In $x=\ell$, one obtains:

$$u(\ell,t)=\frac{F_x \ell}{ES}-\frac{8F_x\ell}{\pi^2 ES}\sum_{j=1,2,3\dots}^{\infty}\frac{1}{(2j-1)^2}\cos\left(\frac{(2j-1)\pi C_\ell t}{2\ell}\right)$$

Because: $\sum_{j=1,2,3\dots}^{\infty}\frac{1}{(2j-1)^2}=\frac{\pi^2}{8}$, from where $u(\ell,t)\in\left[0,\frac{2F_x\ell}{ES}\right]$

Before the first reflection of wave on the fixed support, one obtains simply: $u(\ell,t)=\frac{F_x t}{S\sqrt{E\rho}}$.

Computation of the forces

the normal force is simply: $N(x,t)=ESu_{,x}(x,t)$. From where:

$$N(x,t)=\frac{4F_x}{\pi}\sum_{j=1}^{\infty}\frac{(-1)^{j-1}}{2j-1}\cos\frac{(2j-1)\pi x}{2\ell}\left(1-\cos\frac{(2j-1)\pi C_\ell t}{2\ell}\right)$$

From where reaction in A :

$$R_x^A(t)=-N(0,t)=\frac{4F_x}{\pi}\sum_{j=1,2,3\dots}^{\infty}\frac{(-1)^{j-1}}{(2j-1)}\cos\left(\frac{(2j-1)\pi C_\ell t}{2\ell}\right)-F_x$$

The reaction A is null on the time interval $[0\,s;\ \ 0{,}000197864\,s]$ before the arrival of the wave, then is worth $-2F_x$.

2.1.2 Elastodynamic solution of the beam in bending

One is limited to motions in the average plane xOy.

◊ Let us take initially the modelization of the beams of Navier-Bernoulli-Eulerian. The inertia of rotation is not treated, cf [bib4].

The local equation of dynamic elastic equilibrium is written [bib1,2,3]: $v_{,xxxx}+\frac{\rho S}{EI_z}v_{,tt}=0$.

The equation of "dispersion" connects the pulsation ω and the wave number k : $\omega=k^2\sqrt{\frac{EI_z}{\rho S}}$.

For the boundary conditions "embed-free", there is the statement of the modes:

$$v(x,t)=e^{i\omega t}\left(c_1(\cos kx-\cosh kx)+c_2(\sin kx-\sinh kx)\right)$$

with $\cos k\ell\cosh k\ell=-1$ and the $c_2/c_1=(\sin k\ell-\sinh k\ell)/(\cos k\ell+\cosh k\ell)$
frequencies of the modes are obtained by:

$$f_n=\frac{(k\ell)_n^2}{2\pi\ell^2}\sqrt{\frac{EI_z}{\rho S}}=\frac{(k\ell)_n^2 C_\ell}{2\pi\ell^2}\sqrt{\frac{I_z}{S}}$$

with for the first modes:

n° of mode	1	2	3	4	5	$n\geq 6$
$(k\ell)_n$	1,875104069	4,694091133	7,854757438	10,99554073	14,13716839	$(2n-1)\pi/2$
c_2/c_1	-0.734095514	-1.018467319	-0.999224497	-1.000033553	-0.999998550	≈ -1

With the mechanical characteristics of this problem, one finds the frequencies:

$f_1=310.133\,Hz$ $f_2=1943.568\,Hz$ $f_3=5442.048\,Hz$ $f_4=10664.242\,Hz$, $f_5=17628.755\,Hz$

◊ Let us take then the modelization of the beams of Timoshenko with transverse shears and inertia of rotation, the latter being then treated, cf [bib4], of *Code_Aster*.

The local equation of dynamic elastic equilibrium couples the deflection and rotation and is written [bib3]:

$$\begin{cases} G S_r \left(v_{xx} - \theta_{z,x} \right) - \rho S v_{,tt} = 0 \\ E I_z \theta_{z,xx} + G S_r \left(v_{,x} - \theta_z \right) - \rho I_z \theta_{z,tt} = 0 \end{cases}$$

The equation of "dispersion" connects the pulsation ω and the wave number k :

$$k^4 + \left(\frac{\rho}{E} + \frac{\rho S}{G S_r} \right) \omega^2 k^2 - \frac{\rho S \omega^2}{E I_z} + \frac{\rho^2 S}{G E S_r} \omega^4 = 0$$

Two roots are always imaginary, the two others are complex: real below a cut-off frequency: $\frac{1}{2\pi} \sqrt{\frac{G S_r}{\rho I_z}}$, and imaginary pure beyond this cut-off frequency: the modes are then only sinusoidal (not hyperbolic terms).

With the mechanical characteristics of this problem, one finds [bib3] the frequencies (modes in sinusoids and exponential):

$f_1 = 269{,}932\ Hz$ $f_2 = 1077{,}199\ Hz$ $f_3 = 2270{,}705\ Hz$ $f_4 = 3249{,}207\ Hz$, $f_5 = 4649{,}212\ Hz$

and the first frequency of the mode in sinusoids alone: $f_5^{bis} = 4002{,}830\ Hz$.

It should be noted that in [bib3] the value of the coefficient of section reduced to the shears is selected equal to $0{,}530659727$, while in *Code_Aster*, the selected formula [bib4] gives: $0{,}510805163$. One can expect a light consequence from it on the calculated eigenfrequencies.

◊ now Let us treat the transitory solution in loading imposed at the end $x = \ell$. The solution is written in the form of a sum on the eigen modes obtained above (with the fixed support in A):

$$v(x,t) = \sum_{j=1}^{\infty} \eta_j(t) . w_j(x) \quad ; \qquad \theta_z(x,t) = \sum_{j=1}^{\infty} \eta_j(t) . \beta_j(x) \text{ (case Timoshenko)}$$

One breaks up the loading at the end $x = \ell$ on the modes. As follows:

$$F_j(t) = w_j(\ell) \; . F(t)$$

The solution checks:

$$\ddot{\eta}_j(t) + \omega_j^2 \eta_j(t) = F_j(t)$$

From where:

$$\eta_j(t) = \frac{1}{\omega_j} \int_0^t F_j(\tau) \, . \sin \omega_j (t - \tau) \, d\tau + \eta_j(0) \cos \omega_j t + \frac{1}{\omega_j} \dot{\eta}_j(0) \sin \omega_j t$$

$\eta_j(0)$ and $\dot{\eta}_j(0)$ are obtained by the initial conditions (here in the case Eulerian-Bernoulli):

$$\eta_j(0) = \int_0^l {}^t w_j(x) \, . \rho S v(x,0) \, dx \qquad \text{and} \qquad \dot{\eta}_j(0) = \int_0^l {}^t w_j(x) \, . \rho S \dot{v}(x,0) \, dx$$

2.1.3 elastodynamic Solution of the beam-pipe in torsion

◊ the dynamic elastic equilibrium of free vibrations in torsion (models free torsion) is written:

$M_{x,x} - \rho J \theta_{x,tt} = 0$, because the twisting moment is $M_x = GJ \theta_{x,x}$

the frequencies of j the èmes eigen modes of beam are given by [bib2] in the embed-free situation in torsion:

$$f_j^{tors}=\frac{C_\ell}{4\ell}\frac{2j-1}{\sqrt{2(1+\nu)}}$$, for $j=1,2\ldots$, modes being: $\theta_{xj}(x)=\sin\frac{\pi(2j-1)x}{2\ell}$

With the mechanical characteristics of this problem, one finds the frequencies:

$f_1=786{,}619Hz$ $f_2=2359{,}856Hz$ $f_3=3933{,}094Hz$, $f_4=5506{,}331Hz\ldots$

◊ the solution transient dynamics, under loading imposed at the end, presents a complete similarity with the tension; the transitory axial rotation of the beam is given by, cf [bib1]:

$$\theta_x(x,t)=\frac{4C_\ell}{\pi GJ}\sum_{j=1,2,3\ldots}^{\infty}\frac{(-1)^{j-1}}{2j-1}\sin\left(\frac{(2j-1)\pi x}{2\ell}\right)\int_0^t\Gamma_x(\tau)\sin\left(\frac{(2k-1)\pi C_\ell(t-\tau)}{2\ell}\right)d\tau$$

For a couple level Γ_x, applied to $t=0$ in B, the wave is propagated with celerity C_ℓ and comes to be thought in A of time $\ell\sqrt{2(1+\nu)}/C_\ell$; with the mechanical characteristics of this problem, one finds that this time is: $0{,}000317816\,s$.

One a:

$$\theta_x(x,t)=\frac{8\Gamma_x l}{\pi^2 GJ}\sum_{j=1,2,3\ldots}^{\infty}\frac{(-1)^{j-1}}{(2j-1)^2}\sin\left(\frac{(2j-1)\pi x}{2l}\right)\left(1-\cos\left(\frac{(2j-1)\pi C_l t}{2l}\right)\right)$$

This series converges quickly because of term $(2j-1)^2$. In $x=\ell$, one obtains:

$$\theta_x(\ell,t)=\frac{\Gamma_x\ell}{GJ}-\frac{8\Gamma_x\ell}{\pi^2 GJ}\sum_{j=1,2,3\ldots}^{\infty}\frac{1}{(2j-1)^2}\cos\left(\frac{(2j-1)\pi C_\ell t}{2\ell}\right)$$

Because $\sum_{j=1,2,3\ldots}^{\infty}\frac{1}{(2j-1)^2}=\frac{\pi^2}{8}$, from where $\theta_x(\ell,t)\in\left[0,\frac{2\Gamma_x\ell}{GJ}\right]$.

Before the first reflection of wave on the fixed support, one obtains simply: $\theta_x(\ell,t)=\frac{\Gamma_x t}{J\sqrt{G\rho}}$.

Computation of the forces

the twisting moment is simply: $M_x(x,t)=GJ\,\theta_{x,x}(x,t)$. From where:

$$M_x(x,t)=\frac{4\Gamma_x}{\pi}\sum_{j=1,2,3\ldots}^{\infty}\frac{(-1)^{j-1}}{(2j-1)}\cos\left(\frac{(2j-1)\pi x}{2\ell}\right)\left(1-\cos\left(\frac{(2j-1)\pi C_\ell t}{2\ell}\right)\right)$$

From where the couple of fixed support in A :

$$\Gamma_x^A(t)=-M_x(0,t)=\frac{4\Gamma_x}{\pi}\sum_{j=1,2,3\ldots}^{\infty}\frac{(-1)^{j-1}}{(2j-1)}\cos\left(\frac{(2j-1)\pi C_\ell t}{2\ell}\right)-\Gamma_x$$

The moment of fixed support A is null on the time interval $[0\,s;\ 0{,}000317816\,s]$ before the arrival of the wave, then is worth $-2\Gamma_x$.

2.1.4 Elastodynamic solution of the pipe in model cylindrical shell

the boundary conditions of the studied problem fix only the rigid modes of structure, on the ddls of beam. The kinematics of the cylindrical shell around that of beam is free. The parameters of configuration of the model of pipe [bib5] are thus free.

For a cylindrical shell finite length (length l, external radius R_e, thickness h average radius $R_m=R_e-h/2$), by admitting that the following membrane strains are null: $\epsilon_{\theta\theta}=\epsilon_{x\theta}\approx 0$ (bending being prevalent; but one imposes nothing on $\in_{xx}$), the eigen modes are following form [bib2] with $j=2,3,4\ldots\ n=1,2,3\ldots$:

$$\begin{cases} u_x(x,\theta,t)=-\dfrac{R_m}{j^2}U_{n,x}\cos(j\theta).\sin(\omega t+\phi) \\ u_\theta(x,\theta,t)=-\dfrac{1}{j}U_n\sin(j\theta).\sin(\omega t+\phi) \\ u_r(x,\theta,t)=U_n\cos(j\theta).\sin(\omega t+\phi) \end{cases}$$

where n indicates the number of the axial mode and j the number of the circumferential mode.
And the frequencies are [bib2]:

$$f_{jn}=\frac{C_\ell}{\sqrt{1-\nu^2}}\frac{\lambda_{jn}}{2\pi R_m}$$

where λ_{jn} is obtained by:

$$\lambda_{jn}^2=\frac{j^4\left(a_{11}a_{22}a_{33}+2a_{12}a_{23}a_{13}-a_{11}a_{23}^2-a_{22}a_{31}^2-a_{33}a_{12}^2\right)}{\left(j^4+j^2+\lambda_n^2\alpha_{n2}R_m^2/\ell^2\right).\left(a_{11}a_{22}-a_{12}^2\right)}$$

where $\lambda_n=(k\ell)_n$ is associated with the mode of beam of Eulerian, for the selected boundary conditions (cf [§ 2.1.2]), where $\alpha_{n2}=\frac{1}{\ell}\int_0^\ell U_{n,x}^{\ 2}dx$ is calculated starting from the axial mode of standard beam, according to the boundary conditions of the cylinder, and where the constants a_{11}, a_{22}, a_{33}, a_{12}, a_{23}, a_{13} are given by (while noting $\chi=h^2/\left(12R_m^{\ 2}\right)$) :

$$a_{11}=\lambda_n^2R_m^{\ 2}/\ell^2+\tfrac{1}{2}j^2\alpha_{n2}(1-\nu)(1+\chi)$$

$$a_{22}=j^2+\tfrac{1}{2}\alpha_{n2}\lambda_n^2R_m^{\ 2}(1-\nu)(1+3\chi)/\ell^2$$

$$a_{33}=1+\chi\left(\lambda_n^4R_m^{\ 4}/\ell^4+\left(j^2-1\right)^2+2\nu j^2\lambda_n^2R_m^{\ 2}\alpha_{n1}/\ell^2+2j^2\lambda_n^2R_m^{\ 2}\alpha_{n2}(1-\nu)/\ell^2\right)$$

$$a_{12}=-j\lambda_nR_m\left(\nu\ \alpha_{n1}+\tfrac{1}{2}\alpha_{n2}(1-\nu)\right)/\ell$$

$$a_{23}=j\left(1+\chi\ \lambda_n^2R_m^{\ 2}\left(\nu\ \alpha_{n1}+\tfrac{3}{2}\alpha_{n2}(1-\nu)\right)/\ell^2\right)$$

$$a_{31}=\lambda_nR_m\left(-\nu\ \alpha_{n1}+\chi\left(\tfrac{1}{2}j^2\alpha_{n2}(1-\nu)-\lambda_n^2R_m^{\ 2}/\ell^2\right)\right)/\ell$$

where $\alpha_{n1}=-\frac{1}{\ell}\int_0^\ell U_{n,xx}.U_n\ dx$.

The values are given by the table below for the embed-free case (where displacements and the rotations are completely blocked in $x=0$):

n° of mode beam	1	2	3	4	5
$(k\ell)_n$	1,875104069	4,694091133	7,854757438	10,99554073	14,13716839
α_{n1}	-0.2241	0.6033	0.7440	0.8182	0.8585
α_{n2}	1.3219	1.4712	1.2529	1.1820	1.1415

In the modelization suggested here, the fixed support is only assured on the ddls beam, in the manner of connection `COQ_POU`, to see [bib6], which ensures that only the averages and the moments of order 1 of displacements at the end $x=0$ are null. The kinematics of the elements pipes uncouples the terms generating the deformed shape from "beam" of those supplementing them to generate a kinematics of shells, using the first circumferential modes of Fourier, to see [bib5], within the meaning of the scalar product of displacements on the structure, i.e. also within the meaning of the inertia forces (mass matrix).

This is why, one gives below the two families of eigen modes completely inextensionnels $\epsilon_{\theta\theta}=\epsilon_{x\theta}=\epsilon_{xx}=0$, for a "free-free" cylinder, length ℓ, with $j=2,3,4...$:

$$\text{modes de Rayleigh}\quad \begin{cases} u_x(x,\theta,t)=0 \\ u_\theta(x,\theta,t)=-\frac{1}{j}\sin(j\theta).\sin(\omega t+\phi) \\ u_r(x,\theta,t)=\cos(j\theta).\sin(\omega t+\phi) \end{cases}$$

and

$$\text{modes de Love}\quad \begin{cases} u_x(x,\theta,t)=\frac{R_m}{j^2}\cos(j\theta).\sin(\omega t+\phi) \\ u_\theta(x,\theta,t)=-\frac{(x-\ell/2)}{j}\sin(j\theta).\sin(\omega t+\phi) \\ u_r(x,\theta,t)=(x-\ell/2)\cos(j\theta).\sin(\omega t+\phi) \end{cases}$$

These modes check well the condition in $x=0$ fixed support on average of "beam".
The frequencies of j the èmes eigen modes are respectively for the first, then for the seconds:

$$f_j^{Rayleigh}=\frac{C_\ell\sqrt{\chi}}{2\pi R_m\sqrt{1-\nu^2}}\frac{j(j^2-1)}{\sqrt{j^2+1}}$$

and

$$f_j^{Love}=\frac{C_\ell\sqrt{\chi}}{2\pi R_m\sqrt{1-\nu^2}}j(j^2-1)\sqrt{\frac{j^2\ell^2+24(1-\nu)R_m^2}{j^2\ell^2+12R_m^2}}$$

with $j=2,3,4...$. With the mechanical characteristics of this problem, one finds the frequencies:

Rayleigh: $f_2=78{,}22613\,Hz$ $f_3=221{,}2569\,Hz$ $f_4=424{,}2407\,Hz$, $f_5=686{,}0885\,Hz$ …

Coils: $f_2=179{,}7433\,Hz$ $f_3=708{,}6504\,Hz$ $f_4=1762{,}019\,Hz$, $f_5=3514{,}927\,Hz$ …

2.2 Results of reference

Eigenfrequencies, analytical reference;
Displacements, rotations in B : components DX and DRX analytical reference per series;
Reactions in A : components DX and DRX analytical reference per series.

2.3 Uncertainty on the analytical

solution Solution.

Comparison of some frequencies in model beam obtained with the software Circus [bib7].
Comparison of some displacements and transitory rotations in model beam of Eulerian obtained by S.Potapov with the EuroPlexus *software* (in fast dynamics clarifies), for which one chose a coarser mesh (100 finite elements of beam, diagram explicit of central differences and time step optimal about $\Delta t=10^{-10}s$). One does not compare the forces obtained by *EuroPlexus*, because they are sullied with oscillations coming from the explicit diagram of integration to central differences.

2.4 Bibliographical references

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3 adopts a modelization

3.1 by beam elements of

Eulerian POU_D_E. The mesh model calculated with Code_Aster `consists`

of 1001 nodes and 1000 meshes *SEG2.* Is needed a very fine mesh, because the inertia forces give solutions which are not in the base of the shape functions of the beam elements, to see [bib4]. One chooses a tight temporal discretization from

, in order to collect the initial shock as well as possible. One chooses $t=0\,s$ to solve on the interval. The diagram of temporal integration selected is: $[0\,s\,;0{,}00032\,s]$ diagram
of Newmark, in average acceleration (values
by default) and formulates. Quantities tested and Oscillatory modes results $\Delta t=10^{-7}s$ puts

3.2 for information some values

3.2.1 of eigenfrequencies

obtained with the software Circus [bib7]. Eigenfrequencies of beam of Eulerian (en) Type of

natural reference mode Code_Aster analytical Circus Hz

Bending 1	**310,133 310.132**		***310.13 analytical***	***Bending 2***
1943,568		1943.566 1943.56	analytical	
Bending 3	5442,048	5442.046	analytical	Bending 4
10664,242	10664.242	analytical	Bending 5	
17628,755	17628.756	analytical	Tension 1	
1263,497	1263.497	1263.48	analytical	
Tension 2	3790,490	3790.494	analytical	Tension 3
6317,484	6317.500	analytical	Tension 4	
8844,477	8844.522	analytical	Torsion 1	
786,619	786.619	786.62 analytical	Torsion 2	
2359,856		2359.858 analytical	Torsion	3
3933,094	3933.104	analytical	Torsion 4	
5506,331	5506.359	Transient response		
Displacements		of the end	-	

3.2.2 not (in), rotations

of the end - not Urgent (B S) Component m natural reference EuroPlexus Code_Aster B

0.00010	**DX 2.5947**	**10 – 10**	**analytical**	***2.5934***	***10 – 10 2.568***
10	–	10 0.00015 DX 3.8921	10 –	10 analytical 3.8908	10 – 10 3.865
10	–	10 0.00020 DX 5.1895	10 –	10 analytical 5.1882	10 – 10 5.163
10	–	10 0.00010 DRX 1.7329	10 –	8 analytical 1.7321	10 – 8 1.715
10 –	8	0.00020 DRX 3.4659	10 – 8	analytical 3.4651	10 – 8 3.448

10 –	8	Reactions to L	“fixed support	- not (	in), moments

with L” fixed support - not (in) Urgent A N (S) Component natural reference 0.00010 A analytical $N.m$ DX

0,00	**0.00015**	**analytical DX 0,00**	**0.00020**
analytical	DX	0.00	-2,00
0.00010	analytical	0.00	0,00
0.00020	analytical	-2.00	0,00
0.00032	DRX	0.00	analytical
Modelization		0.00	Characteristic
	of	-2.00	One

4 adopts a modelization

4.1 by beam elements of

Timoshenko POU_D_T. The mesh model calculated with Code_Aster `consists`

of 1001 nodes and 1000 meshes *SEG3.* Is needed a very fine mesh, because the inertia forces give solutions which are not in the base of the shape functions of the beam elements, to see [bib4]. It is the same mesh that in modelization A. One chooses a tight temporal discretization from

, in order to collect the initial shock as well as possible. One chooses $t=0\,s$ to solve on the interval. The diagram of temporal integration selected is: $[0\,s\,;0{,}00032\,s]$ diagram
of Newmark, in average acceleration (values
by default) and formulates. Quantities tested and Oscillatory modes results $\Delta t=10^{-7}s$ puts

4.2 for information some values

4.2.1 of eigenfrequencies

obtained with the software Circus [bib7], for which the coefficient of reduced section is obtained according to [bib8]: 0.5011, i.e. a value weaker than that selected in the reference (bib3] from where lower frequencies. To confront itself with the reference [bib3], one puts also the values of the predictions of Circus with the coefficient of reduced section equal to [bib3]. It is noted that the results are then identical $0{,}530659727$ to near; the value of the coefficient of section reduced to 10^{-4} the shears plays an important role on this example. Eigenfrequencies of beam of Timoshenko (in) Standard

of natural reference mode Code_Aster Circus coeff Hz

Batoz Circus	**coeff [bib3]**	**analytical**	***Bending 1***	***269,932*** 269.261	***268.3*** analytical Bending 2
1077,199		1059.305 analytical	Bending	3	
2270,705	2226.699	analytical	Bending 4		
3249,207	3173.253	analytical	Bending 5		
4649,212	4554.463	4557.2	4649.6 analytical		
Bending 5	(a)	4002,830	3903.0	4003.2	analytical
Tension 1 1263,497		1263.497 1263.48			analytical
Tension 2	3790,490	3790.494	analytical	Tension 3	
6317,484	6317.500	analytical	Tension 4		
8844,477	8844.522	analytical	Torsion 1		
786,619	786.619	786.62 analytical	Torsion 2		
2359,856		2359.858 analytical	Torsion	3	
3933,094	3933.104	analytical	Torsion 4		
5506,331	5506.359	Transient response			
Displacements		of the end	-		

4.2.2 not (in m), rotations

of the end - not Urgent B (S) Component natural reference 0.00010 DX 2.5947 B

10 –	**10 analytical**	**0.00015 DX**	**3.8921**

10 – 10	analytical	0.00020	DX 5.1895
10 – 10	analytical	0.00010	DRX 1.7329
10 –	8 analytical	0.00020	DRX 3.4659
10 –	8 analytical	Reactions	to
the fixed support		- not (in	), moments

with the fixed support - not (in) Urgent A N (S) Component natural reference 0.00010 A analytical $N.m$ DX

0,00	**0.00015**	**DX 0,00 analytical**	**0.00020**
DX	-2,00	0.00	analytical
0.00010	DRX	0.00	analytical
0.00020	DRX	-2.00	analytical
0.00032	DRX	0.00	analytical
Modelization		0.00	Characteristic
	of	-2.00	One

5 adopts a modelization

5.1 by elements of beam

pipe TUYAU_3M. The mesh model calculated with Code_Aster `consists`

of 1001 nodes and 500 meshes *SEG3.* Is needed a rather fine mesh, because the inertia forces give solutions which are not in the base of the shape functions of the beam elements, to see [bib4]. It is the same mesh that in modelization A. One creates two models: where the pipe is blocked in

on only the 6 degrees of freedom of beam; a second B where the pipe is dependant (by a LIAISON_DDL) on a discrete element DIS_TR (on a mesh `not` POI1) with 6 degrees of freedom , which is completely built-in `for him`. One chooses a tight temporal discretization from

, in order to collect the initial shock as well as possible. One chooses $t=0\,s$ to solve on the interval. The diagram of temporal integration selected is: $[0\,s\,;0{,}00032\,s]$ diagram
of Newmark, in average acceleration (values
by default) and formulates. Quantities tested and Oscillatory modes results $\Delta t=10^{-7}s$

5.2 of "beam"

5.2.1 and "shell

" (in), case TUYAU_3M Type of natural reference mode Bending Hz 1 269,932 `analytical`

Bending	**2 1077,199**	**analytical**
Bending	3 2270,705	analytical
Bending	4 3249,207	analytical
Bending	5 4649,212	analytical
Tension	1 1263,497	analytical
Tension	2 3790,490	analytical
Tension	3 6317,484	analytical
Tension	4 8844,477	analytical
Torsion	1 786,619	analytical
Torsion	2 2359,856	analytical
Torsion	3 3933,094	analytical
Torsion	4 5506,331	analytical
Shell	Rayleigh	2 78,22613
analytical	Shell Rayleigh	3 221,2569
analytical	Shell Rayleigh	4 424,2407
analytical	Shell Rayleigh	5 686,0885
analytical	Shell Coils	2 179,7433
analytical Shell	Coils	3 708,6504
analytical Shell	Coils	4 1762,019
analytical Shell	Coils	5 3514,927
analytical Transient response		Displacements
	of the end	-

5.2.2 not (in), rotations

of the end - not, case TUYAU_3M B m Time (S) Component natural reference B 0.00010 DX 2.5947

10 –	**10 analytical**	**0.00015 DX**	**3.8921**
10 – 10	analytical	0.00020	DX 5.1895
10 – 10	analytical	0.00005	DRX 8.6648
10 –	9 analytical	0.00010	DRX 1.7330
10 –	8 analytical	0.00020	DRX 3.4659
10 –	8 analytical	Reactions	to
the fixed support		- not (in	), moments

with the fixed support - not (in), case A TUYAU_3M N Time (S) Component natural A reference Nm 0.00010 analytical DX

0.00	**0.00015**	**analytical DX 0.00**	**0.00020**
analytical	DX		-2.00
0.00032	analytical	DRX	-2.00
Lastly,	at	the end	
free point		,	the values of

the REAC_NODA are about, and even B definitely lower , at same times 10^{-12} , which is quite close to the value zero expected. Modelization D Characteristic of the modelization One

6 adopts a modelization

6.1 by elements of beam

pipe TUYAU_6M to 3 nodes. The mesh model calculated with `Code_Aster` consists

of 1001 nodes and 500 meshes *SEG3.* Is needed a rather fine mesh, because the inertia forces give solutions which are not in the base of the shape functions of the beam elements, to see [bib4]. It is the same mesh that in modelization A. One creates two models: where the pipe is blocked in

on only the 6 degrees of freedom of beam; a second A where the pipe is dependant (by a LIAISON_DDL) on a discrete element DIS_TR (on a mesh `not` POI1) with 6 degrees of freedom , which is completely built-in `for him`. One chooses a tight temporal discretization from

, in order to collect the initial shock as well as possible. One chooses $t=0\,s$ to solve on the interval. The diagram of temporal integration selected is: $[0\,s\,;0{,}00032\,s]$ diagram
of Newmark, in average acceleration (values
by default) and formulates. Quantities tested and Oscillatory modes results $\Delta t=10^{-7}s$

6.2 of "beam"

6.2.1 and "shell

" (in), case TUYAU_6M Type of natural reference mode Bending Hz 1 269,932 `analytical`

Bending	2 1077,199	analytical
Bending	3 2270,705	analytical
Bending	4 3249,207	analytical
Bending	5 4649,212	analytical
Tension	1 1263,497	analytical
Tension	2 3790,490	analytical
Tension	3 6317,484	analytical
Tension	4 8844,477	analytical
Torsion	1 786,619	analytical
Torsion	2 2359,856	analytical
Torsion	3 3933,094	analytical
Torsion	4 5506,331	analytical
Shell	Rayleigh	2 78,22613
analytical	Shell Rayleigh	3 221,2569
analytical	Shell Rayleigh	4 424,2407
analytical	Shell Rayleigh	5 686,0885
analytical	Shell Coils	2 179,7433
analytical Shell	Coils	3 708,6504
analytical Shell	Coils	4 1762,019
analytical Shell	Coils	5 3514,927
analytical Transient response		Displacements
	of the end	-

6.2.2 not (in), rotations

of the end - not, case TUYAU_6M B m Time (S) Component natural reference B 0.00010 `DX 2.5947`

10 –	**10 analytical**	**0.00015 DX**	**3.8921**
10 – 10	analytical	0.00020	DX 5.1895
10 – 10	analytical	0.00005	DRX 8.6648
10 –	9 analytical	0.00010	DRX 1.7330
10 –	8 analytical	0.00020	DRX 3.4659
10 –	8 analytical	Reactions	to
the fixed support		- not (in	), moments

with the fixed support - not (in), case A TUYAU_6M N Time (S) Component natural A reference Nm 0.00010 analytical DX

0.00	**0.00015**	**analytical DX 0.00**	**0.00020**
analytical	DX		-2.00
0.00032	analytical	DRX	-2.00
Lastly,	at	the end	
- not	,	the values	of the REAC_NODA

are about, and even B definitely lower , at same times 10^{-12} , which is quite close to the value zero expected. Modelization E Characteristic of the modelization The modelization

7

7.1 is made by elements of bar

(BAR). The mesh model Code_Aster consists of 1001
nodes and 1000 meshes *SEG2.* Is needed a very fine mesh, because the inertia forces give solutions which are not in the base of the shape functions of the beam elements, to see [bib4]. One changed the directional sense of the mesh compared to the modelization A. One chooses a tight temporal discretization from

, in order to collect the initial shock as well as possible. One chooses $t=0\,s$ to solve on the interval. The diagram of temporal integration selected is that $[0\,s\,;0{,}00032\,s]$
of Newmark, in average acceleration (values by default) and. Quantities tested and Computation results of the oscillatory modes $\Delta t=10^{-7}s$

7.2 : Eigenfrequencies of beam

of Eulerian (en) Type of
Natural reference mode Tolerance analytical Hz Tension

1 1263.5	**1.0E-03**	**analytical**	**Tension 2**
3790.49	1.0E-03	analytical	Tension 3
6317.48	1.0E-03	analytical	Tension 4
8844.48	1.0E-03	Computation with	DYNA
_VIBRA: Displacements		of	the end

- not (in), rotations
of the end - not Urgent (B S) Component m Natural reference Tolerance 0.00010 B

DX 1.8347	**analytical**	**E-10 1.0**	**E-03 0.00010**	**DY 1.8347**
analytical	E	10 1.0	E-03 0.00015	DX 2.7521
analytical	E	10 1.0	E-03 0.00015	DY 2.7521
E –	10	analytical	1.0E-03 0.00020	analytical
DX	3.6695E-	10 1.0	E-03 0.00020	DY 3.6695
E –	10	analytical	1.0E-03 Reactions	
to the fixed support		- not	(in),	moments

with the fixed support - not (in) Urgent A N (S) Component Natural reference Tolerance A $N.m$ 0.00010

analytical DX	**0.00**	**1.0E-03 0.00010**	**DY**	**0.00 analytical**
1.0	E	- 03 0.00015	analytical DX	0.00
1.0	E	- 03 0.00015	DY 0.00	analytical
1.0	E	- 03 0.00020	analytical DX	
-1.414213		5.0%	0.00020 DY	-1.414213
analytical		5.0% Computation	with	DYNA_NON_LINE
	:	Displacement s	of	the end

- not (in), rotations

of the end - not Urgent (B S) Component m Natural Reference Tolerance 0.00010 B

DX 1.8347	analytical	E-10 1.0	E-03 0.00010	DY 1.8347
analytical	E	10 1.0	E-03 0.00015	DX 2.7521
analytical	E	10 1.0	E-03 0.00015	DY 2.7521
E –	10	analytical	1.0E-03 0.00020	analytical
DX	3.6695E-	10 1.0	E-03 0.00020	DY 3.6695
E –	10	analytical	1.0E-03 Reactions	
to the fixed support		- not	(in),	moments

with the fixed support - not (in) Urgent A N (S) Component natural reference Tolerance A $N.m$ 0.00010

analytical DX	0.00	1.0E-03 0.00010	DY	0.00 analytical
1.0	E	- 03 0.00015	analytical DX	0.00
1.0	E	- 03 0.00015	DY 0.00	analytical
1.0	E	- 03 0.00020	analytical DX	
-1.414213		5.0%	0.00020 DY	-1.414213
analytical		5.0% Summary	of	the results
	a relatively	fine		mesh are

8 necessary to get

precise results (wave propagation). It is noted that the model beam of Timoshenko is

more precise in bending, but that the value of the coefficient of section reduced to the shears plays an important role on this example. The software of frequential dynamics of the Circus pipework gives values very close to the frequencies *determined* by Code_Aster for the models of beams to Eulerian and Timoshenko *.* The fast software of dynamics EuroPlexus gives values

very close compared to *those to* Code_Aster for the beam elements of Eulerian. The model *by* finite elements of shell allows to define

the field of validity of the modelizations in beam elements and elements pipes.