
SDLL129 - Diagram of Campbell of a beam with 3 discs and 2 of the bearings with variable characteristics according to rotational speed

Summarized:

This test makes it possible to validate the computation of the modes in rotation of a system of rotating shafts with the macro `CALC_MODE_ROTATION` and of the diagram of Campbell with the macro `IMPR_DIAG_CAMPBELL`, if the characteristics in stiffness and damping depend on rotational speed.

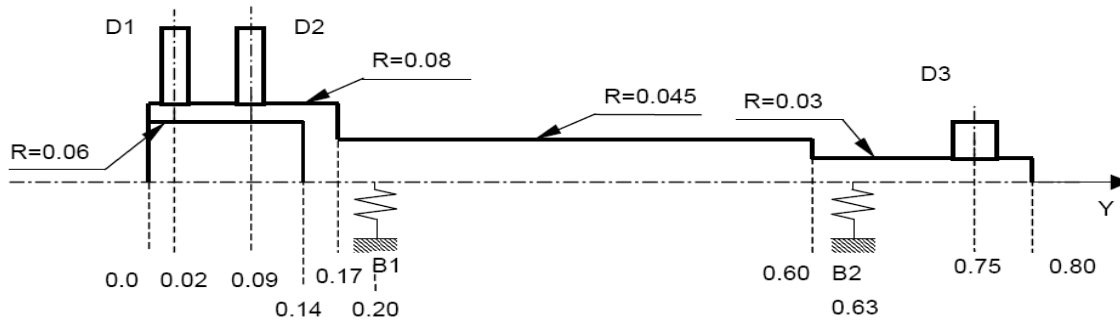
In this test, there is a model of rotor with three discs, supported by two hydrodynamic bearings, whose stiffness matrixes and of damping are asymmetric and depend on rotational speed. This example as well as the corresponding results of reference are drawn from the handbook of qualification of ROTORINSA, [bib2], software finite elements intended to envisage the dynamic behavior of rotors in bending.

A good agreement is observed between the results of *Code_Aster* and the reference solution.

1 Problem of reference

1.1 Geometry

a model of rotor supported by 2 bearings (nodes $B1$ and $B2$ on the figure below), whose stiffness matrixes and of damping are asymmetric. It is composed of 3 discs, and 4 sections of shaft.



Appear 1.1-a: Model rotor with 3 discs and 2 asymmetrical bearings

1.2 Properties of the material

the geometrical characteristics and material are listed in the following table.

Material		$E = 2.10^{11} \text{ N/m}^2$	$\rho = 7800 \text{ kg/m}^3$	$\nu = 0.3$
Disc	D1	$M = 20 \text{ kg}$	$I_D = 200.10^{-3} \text{ kg m}^2$	$I_P = 400.10^{-3} \text{ kg m}^2$
	D2	$M = 17 \text{ kg}$	$I_D = 170.10^{-3} \text{ kg m}^2$	$I_P = 340.10^{-3} \text{ kg m}^2$
	D3	$M = 10 \text{ kg}$	$I_D = 15.10^{-3} \text{ kg m}^2$	$I_P = 30.10^{-3} \text{ kg m}^2$

the characteristics of the bearings are indicated in the tables which follow.

$$\Omega = 0 \text{ tr/min}$$

Bearin g P1	$K_{yy} = 90.10^6 \text{ N/m}$	$K_{zz} = 50.10^7 \text{ N/m}$
	$K_{yz} = 8.10^4 \text{ N/m}$	$K_{zy} = -9.10^4 \text{ N/m}$
	$c_{yy} = 15.10^4 \text{ Ns/m}$	$c_{zz} = 45.10^4 \text{ Ns/m}$
	$c_{yz} = -1.10^2 \text{ Ns/m}$	$c_{zy} = 1.10^2 \text{ Ns/m}$

Bearin g P2	$K_{yy} = 60.10^6 \text{ N/m}$	$K_{zz} = 15.10^7 \text{ N/m}$
	$K_{yz} = 8.10^4 \text{ N/m}$	$K_{zy} = -8.10^4 \text{ N/m}$
	$c_{yy} = 12.10^4 \text{ Ns/m}$	$c_{zz} = 19.10^4 \text{ Ns/m}$
	$c_{yz} = -1.10^2 \text{ Ns/m}$	$c_{zy} = 1.10^2 \text{ Ns/m}$

$\Omega = 5000 \text{ tr/min}$		
Bearin P1 g	$K_{yy} = 90.10^6 \text{ N/m}$	$K_{zz} = 50.10^7 \text{ N/m}$
	$K_{yz} = 9.10^4 \text{ N/m}$	$K_{zy} = -9.10^4 \text{ N/m}$
	$c_{yy} = 15.10^4 \text{ Ns/m}$	$c_{zz} = 45.10^4 \text{ Ns/m}$
	$c_{yz} = -1.10^2 \text{ Ns/m}$	$c_{zy} = 1.10^2 \text{ Ns/m}$
Bearin P2 g	$K_{yy} = 60.10^6 \text{ N/m}$	$K_{zz} = 15.10^7 \text{ N/m}$
	$K_{yz} = 8.10^4 \text{ N/m}$	$K_{zy} = -8.10^4 \text{ N/m}$
	$c_{yy} = 12.10^4 \text{ Ns/m}$	$c_{zz} = 19.10^4 \text{ Ns/m}$
	$c_{yz} = -1.10^2 \text{ Ns/m}$	$c_{zy} = 1.10^2 \text{ Ns/m}$
$\Omega = 6500 \text{ tr/min}$		
Bearin P1 g	$K_{yy} = 100.10^6 \text{ N/m}$	$K_{zz} = 40.10^7 \text{ N/m}$
	$K_{yz} = 15.10^4 \text{ N/m}$	$K_{zy} = -15.10^4 \text{ N/m}$
	$c_{yy} = 13.10^4 \text{ Ns/m}$	$c_{zz} = 33.10^4 \text{ Ns/m}$
	$c_{yz} = -1.10^2 \text{ Ns/m}$	$c_{zy} = 1.10^2 \text{ Ns/m}$
Bearin P2 g	$K_{yy} = 70.10^6 \text{ N/m}$	$K_{zz} = 14.10^7 \text{ N/m}$
	$K_{yz} = 13.10^4 \text{ N/m}$	$K_{zy} = -13.10^4 \text{ N/m}$
	$c_{yy} = 10.10^4 \text{ Ns/m}$	$c_{zz} = 15.10^4 \text{ Ns/m}$
	$c_{yz} = -1.10^2 \text{ Ns/m}$	$c_{zy} = 1.10^2 \text{ Ns/m}$
$\Omega = 8000 \text{ tr/min}$		
Bearin P1 g	$K_{yy} = 110.10^6 \text{ N/m}$	$K_{zz} = 35.10^7 \text{ N/m}$
	$K_{yz} = 20.10^4 \text{ N/m}$	$K_{zy} = -20.10^4 \text{ N/m}$
	$c_{yy} = 11.10^4 \text{ Ns/m}$	$c_{zz} = 26.10^4 \text{ Ns/m}$
	$c_{yz} = -2.10^2 \text{ Ns/m}$	$c_{zy} = 2.10^2 \text{ Ns/m}$
Bearin P2 g	$K_{yy} = 80.10^6 \text{ N/m}$	$K_{zz} = 14.10^7 \text{ N/m}$
	$K_{yz} = 20.10^4 \text{ N/m}$	$K_{zy} = -20.10^4 \text{ N/m}$
	$c_{yy} = 9.10^4 \text{ Ns/m}$	$c_{zz} = 13.10^4 \text{ Ns/m}$
	$c_{yz} = -2.10^2 \text{ Ns/m}$	$c_{zy} = 2.10^2 \text{ Ns/m}$
$\Omega = 10000 \text{ tr/min}$		
Bearin P1 g	$K_{yy} = 115.10^6 \text{ N/m}$	$K_{zz} = 33.10^7 \text{ N/m}$
	$K_{yz} = 35.10^4 \text{ N/m}$	$K_{zy} = -35.10^4 \text{ N/m}$
	$c_{yy} = 10.10^4 \text{ Ns/m}$	$c_{zz} = 20.10^4 \text{ Ns/m}$
	$c_{yz} = -2.10^2 \text{ Ns/m}$	$c_{zy} = 3.10^2 \text{ Ns/m}$
Bearin P2 g	$K_{yy} = 90.10^6 \text{ N/m}$	$K_{zz} = 14.10^7 \text{ N/m}$
	$K_{yz} = 30.10^4 \text{ N/m}$	$K_{zy} = -30.10^4 \text{ N/m}$
	$c_{yy} = 8.10^4 \text{ Ns/m}$	$c_{zz} = 10.10^4 \text{ Ns/m}$
	$c_{yz} = -2.10^2 \text{ Ns/m}$	$c_{zy} = 2.10^2 \text{ Ns/m}$

$$\Omega = 14000 \text{ tr/min}$$

$$\begin{array}{l} \text{Bearin } P1 \\ \text{g} \end{array} \quad \begin{array}{ll} K_{yy} = 120.10^6 \text{ N/m} & K_{zz} = 30.10^7 \text{ N/m} \\ K_{yz} = 70.10^4 \text{ N/m} & K_{zy} = -70.10^4 \text{ N/m} \\ c_{yy} = 7.10^4 \text{ Ns/m} & c_{zz} = 15.10^4 \text{ Ns/m} \\ c_{yz} = -3.10^2 \text{ Ns/m} & c_{zy} = 4.10^2 \text{ Ns/m} \end{array}$$

$$\begin{array}{l} \text{Bearin } P2 \\ \text{g} \end{array} \quad \begin{array}{ll} K_{yy} = 100.10^6 \text{ N/m} & K_{zz} = 14.10^7 \text{ N/m} \\ K_{yz} = 60.10^4 \text{ N/m} & K_{zy} = -60.10^4 \text{ N/m} \\ c_{yy} = 6.10^4 \text{ Ns/m} & c_{zz} = 8.10^4 \text{ Ns/m} \\ c_{yz} = -3.10^2 \text{ Ns/m} & c_{zy} = 3.10^2 \text{ Ns/m} \end{array}$$

1.3 Boundary conditions

to block motions of type rigid body in the direction x , one blocks the degrees of freedom DX and DRX with the node bearing $B1$.

2 Reference solution

2.1 Method of calculating

the results of reference are given by ROTORINSA, code with the finite elements intended to envisage the dynamic behavior of rotors in bending. The following parameters were used for the results of reference:

- The computation relates to a number of modes in rotation $NVES = 8 + 4$, in ROTORINSA.
- The beach rotational speeds is defined of 0 with 20000 *tr/mn* a step 500 *tr/mn*.

2.2 Quantities and results of reference

the results of ROTORINSA give the frequencies of the modes in bending.

The computation modes in rotation is carried out with Code_Aster by means of the same modelization as ROTORINSA. The results of Code_Aster give at the same time the frequencies of the modes of bending, torsion and tension/compression. The number of calculated modes is 12.

2.3 References

- [1] ROTORINSA, software finite elements intended to envisage the dynamic behavior of rotors in bending, LaMCoS UMR5259, INSA-Lyon.

3 Modelization A

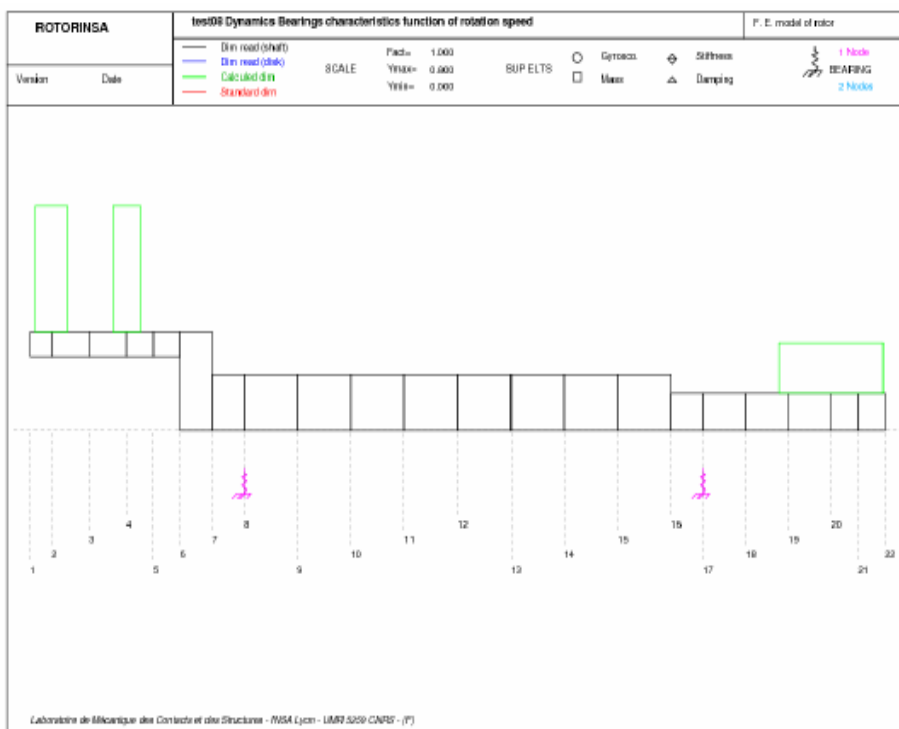
3.1 Characteristic of the modelization

It acts of a system of rotating shafts with positive rotational speeds.

3.2 Characteristics of the mesh

the rotor is with a grid in 21 finite elements of shaft of the type POU_D_T and comprises 5 discrete elements of the type DIS_TR for the modelization of the discs and the bearings.

Many nodes: 22
Number and type of elements: 21SEG2
5POI1



Appears 1-b: Characteristic of the model finite elements under ROTORINSA

3.3 Quantities tested and Eigenfrequencies

results according to rotational speed (direct computation):

The values of the first 8 frequencies of bending for the velocities 0 tr/mn and 20000 tr/mn , for the two software, are presented in the table below.

N° Fréq in flexion Vitesse	of rotation (tr/min)	ROTORINSA		Code_AsterFacteur	
		$F(\text{Hz})$	of amortissementA mortissementré uit10130.695	$F(\text{Hz})$	
			6.37866E-01	131.547	6.37613E-01
	20000		1.24817E-01	123.706	1.24678E-01
20227.9 25			9.58757E-02	227.716	9.49991E-02
	20000		2.17961E-01	200.7282. 17632E-01	
30313.3 11			2.27364E-01	313.935	2.27701E-01
	20000		1.81652E-01	286.912	1.81388E-01
40381.5 29			6.52812E-01	375.305	6.61825E-01
	20000		4.32727E-01	309.825	4.31714E-01
50390.8 02			1.77361E-01	390.241	1.74485E-01
	20000		2.54912E-01	369.917	2.54178E-01
601448. 01			1.42312E-01	1439.66	1.54371E-01
	20000		5.58764E-01	412.422	5.63110E-01
701554. 84			1.07354E-01	1554.35	1.11688E-01
	20000		4.73230E-01	824.952	4.70031E-01
802050. 12			3.08294E-01	1928.78	3.04586E-01
	20000		4.93828E-01	1006.17	5.08301E-01

Table 2-a: Eigenfrequencies of type bending for ROTORINSA and Code_Aster (direct computation)

the criteria of tolerance into relative are of 10% on the results with the stop and of 5% on the results with 20000 tr/min. Except for the tolerance, the frequencies obtained are in adequacy with those of ROTORINSA.

Eigenfrequencies according to rotational speed (computation in 2 stages):

The values of the first 8 frequencies of bending for the velocities 0 tr/min and 20000 tr/min , for the two software, are presented in the table below.

N° Fréq in flexion Vitesse	of rotation (tr/min)	ROTORINSA		Code_AsterFacteur	
		F (Hz)	of amortissementA mortissementréd uit10130.695	F (Hz)	
			6.37866E-01	130.909	6.37982E-01
	20000		1.24817E-01	123.701	1.24816E-01
20227.9 25			9.58757E-02	227.890	9.56146E-02
	20000		2.17961E-01	200.634	2.17876E-01
30313.3 11			2.27364E-01	313.433	2.27817E-01
	20000		1.81652E-01	286.805	1.81739E-01
40381.5 29			6.52812E-01	380.699	6.55069E-01
	20000		4.32727E-01	308.825	4.32616E-01
50390.8 02			1.77361E-01	390.883	1.76940E-01
	20000		2.54912E-01	370.399	2.54825E-01
601448. 01			1.42312E-01	1448.54	1.45247E-01
	20000		5.58764E-01	410.199	5.60285E-01
701554. 84			1.07354E-01	1556.64	1.07779E-01
	20000		4.73230E-01	814.475	4.72976E-01
802050. 12			3.08294E-01	2009.06	3.09277E-01
	20000		4.93828E-01	998.343	4.95564E-01

Table 2-b: Eigenfrequencies of type bending for ROTORINSA and Code_Aster (computation in 2 stages)

the criteria of tolerance into relative are of 5% on the results with the stop and of 1% on the results with 20000 tr/min. Except for the tolerance, the frequencies obtained are in better adequacy with those of ROTORINSA (by comparison with the method of calcul modal direct). Indeed, it is this method of calculating modal in 2 stages which is used in ROTORINSA.

In Code_Aster, one observes also frequencies and modes of torsion and modes of tension/compression. These modes are not calculated by ROTORINSA, because it models only the behavior in bending.

The values of the first frequency in torsion for the velocities 0 tr/min and 20000 tr/min , for Code_Aster, are presented in table below

N° Fréq in Torsion	Rotational speed (tr/min)	F (Hz)
1	0	5.81803E+02
	20000	5.81803E+02

Table 2-c: Frequencies of torsion given by Code_Aster

the value of the first frequency in tension for the velocities 0 tr/mn and 20000 tr/mn , for Code_Aster, are presented in table below

N° Fréq in Torsion	Rotational speed (tr/min)	F (Hz)
1	0	1.67224E+03
	20000	1.67224E+03

Table 2-c: Frequencies of tension given by Code_Aster

In short in the table below, are presented, the numbers the frequencies calculated and used in the layout of the diagram of Campbell in Code_Aster.

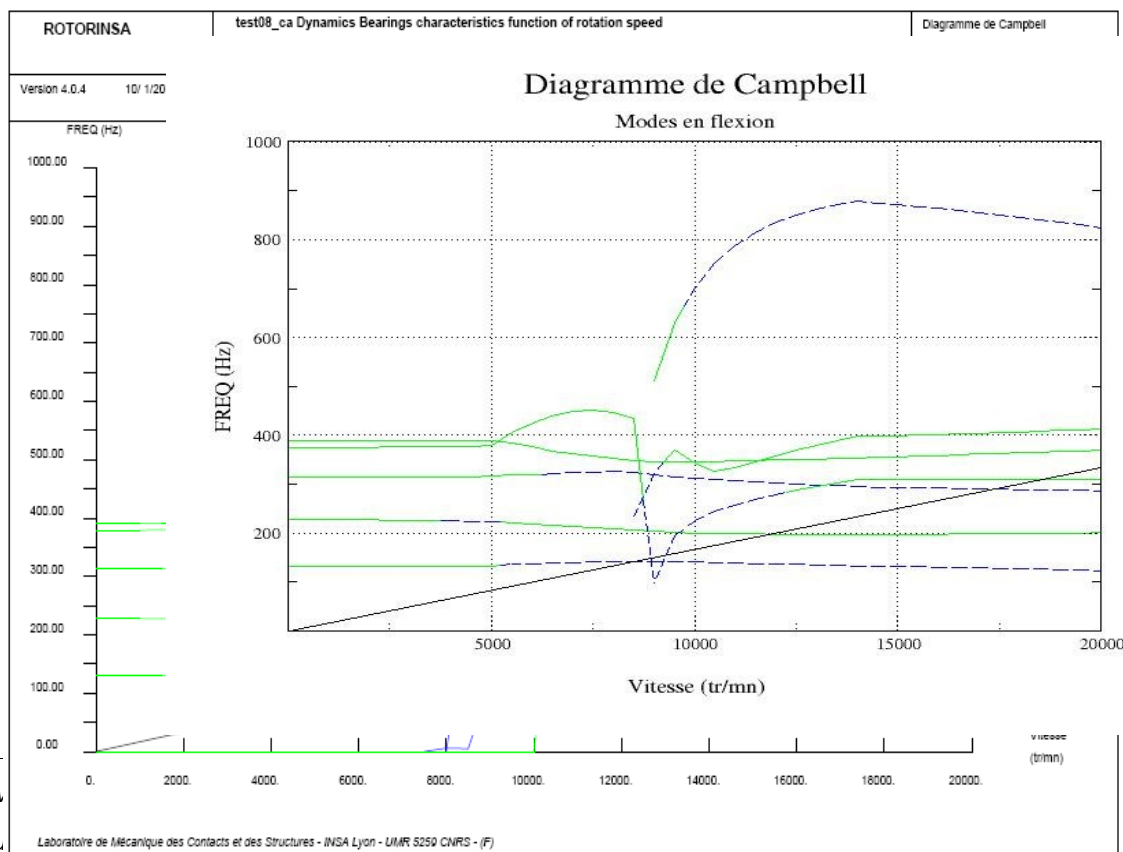
Number of values clean detected: 12
Many frequencies requested for the layout: 8

	calculated	traced
Many total frequencies	12	8
Many frequencies in bending	8	7
Many frequencies torsion	1	1
Number of frequencies tension/compression	1	0

Table 2-d: Calculated and traced frequencies (Code_Aster)

Diagram of Campbell:

The diagram of Campbell obtained in Code-Aster while following the modes of bending per comparison of form (MAC matrixes) corresponds perfectly to that obtained by ROTORINSA.

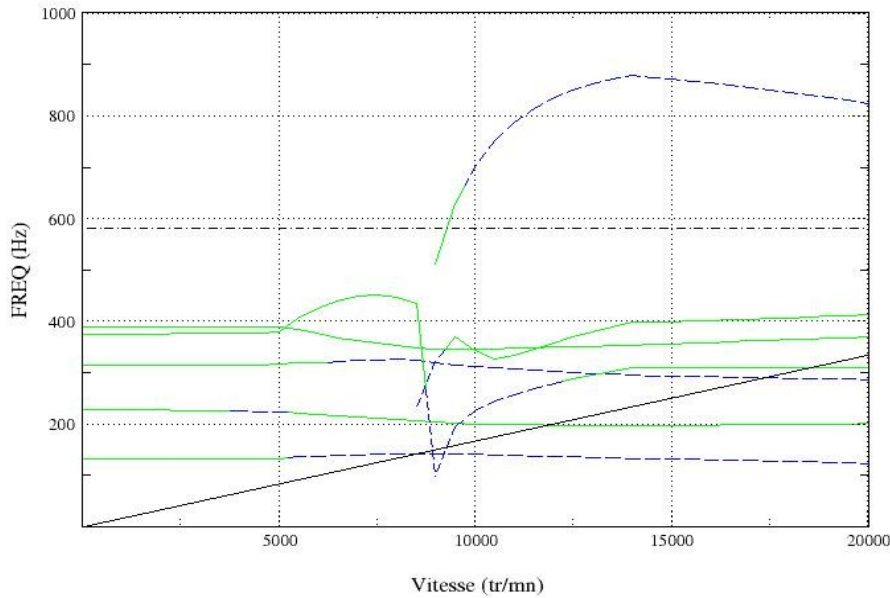


or in part

Appears 3-b: Diagram of Campbell in bending given by Code_Aster

One observes in more the follow-up of the modes of torsion and tension/compression. For this application, the modes of tension/compression are located beyond the interval of frequencies considered. L be modes of torsion are invariants compared to the vistsess of rotation and thus the curves of evolution are horizontal lines.

Diagramme de Campbell



Appear 3-c: Diagram of Campbell in bending and torsion

For the modes of torsion: line color black, style an indent, a dotted line.

Critical velocities (Points of intersection with the lines $Y = SX$):

The possible critical velocities due to the unbalances or synchronous revolving forces at the speed of the rotor, are obtained by the intersections of the right of slope $S=1$ with the curves of evolution of the frequencies.

Table below watch that the points of intersection for the modes in bending obtained are in perfect adequacy with those of ROTORINSA.

ROTORINSA $S=1$		Code-Aster $S=1$	
Velocity (tr / mn)	Frequency (Hz)	Velocity (tr / mn)	Frequency (Hz)
8490.71	141.51	8494.57	141.58
9232.07	153.87	9287.67	154.79
11874.83	197.91	11880.25	198.00
17407.04	290.12	17413.65	290.23
18526.74	308.78	18579.12	309.65

Table 4-a: Critical velocities by Code_Aster and ROTORINSA

4 Summary of the results

This benchmark makes it possible to validate the functionality Digraph of Campbell for lines of trees whose characteristics in stiffness and damping depend on rotational speed since one finds the same results by Code_Aster and ROTORINSA.