SDLL113 - Substructuring transient dynamics : beam in simple tension

Summarized:

The structure considered is an annular beam of section in simple tension, embedded on a side, and subjected at its end with a force of the Heaviside type. Its response transient dynamics is calculated by under - structuring.

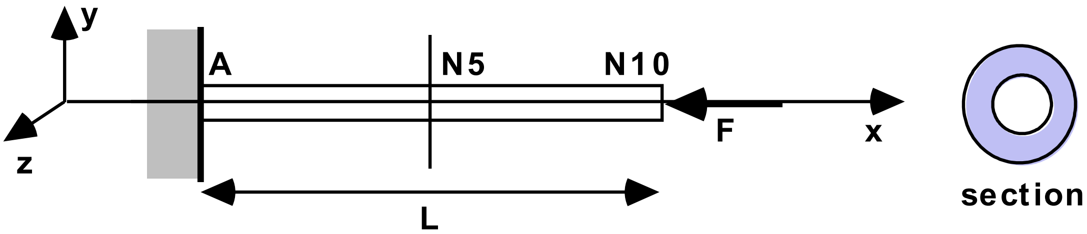
The beam is modelled by elements of the beams type of Timoshenko (models linear). Two modelizations are proposed according to whether the beam is damped or not. The damping tested is of type RAYLEIGH (proportional damping).

The results of reference result from a direct transient computation by modal recombination without under - structuring. This test thus makes it possible to validate the computational tools of transient response per substructuring, in the linear case.

A third modelization dealing with the same damped problem exactly that second was added in order to treat a case of use of a method of condensation dynamic where the resolution of the dynamic problem on physical base operates on a mixed model made up of the macro-element of the right part of structure and a physical model reduced to its left part. The results same as those of the second modelization like are exactly expected.

1 Problem of reference

1.1 Geometry



Length of beam: $L=1\,m$

Section: Interior radius $=0.09\,m$
Radius external $=0.10\,m$

1.2 Properties of the materials

$E=10^{10}\,Pa$ $\nu=0.3$ $\rho=1.10^{4}\,kg/m^{3}$

Modelization a: not of damping
Modelization b: Proportional damping (RAYLEIGH):

$$C=\alpha K+\beta M \quad \text{with} \quad \alpha=6.5\,10^{-6}\,s \quad \text{and} \quad \beta=16.0\,s^{-1}.$$

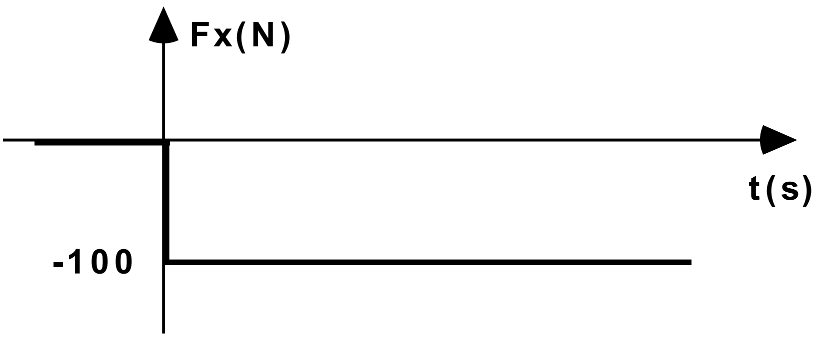
These values correspond to a reduced damping of 1% on the first mode of structure.

1.3 Boundary conditions and loadings

On all structure one imposes $DY=DZ=DRX=DRY=DRZ=0$.

On the point A one imposes the condition of fixed support $DX=0$.

In $N10$ one applies a constant force from time $t=0$: $F_x=-100\,N$.



1.4 Initial conditions

the structure is initially at rest.

2 Reference solution

2.1 Method of calculating used for the reference solution

There exists an analytical solution detailed in the reference [bib1].

The following notations are adopted:

E	:	Young's modulus
ρ	:	density
L	:	length of the bar
A	:	section of the bar
N	:	normal force directed according to the axis X
α, β	:	damping coefficients of Rayleigh

One also poses:

$\omega_n=(2\mathrm{n}-1)\frac{\pi}{2}$ where $n=1,2,3,...$

$$\varepsilon_n=\frac{1}{2}(\alpha\omega_n+\beta/\omega_n)$$

displacement in an unspecified $M(x)$ point is given by:

$$u(x,t)=\frac{Nx}{EA}+\frac{8\mathrm{NL}}{\pi^2 EA}\sum_{n=1}^{\infty}(-1)^n\frac{e^{-\omega_n\varepsilon_n t}}{(2\mathrm{n}-1)^2}\left\{\cos\left(\sqrt{1-\varepsilon_n^2}\omega_n t\right)+\frac{\varepsilon_n}{\sqrt{1-\varepsilon_n^2}}\sin\left(\sqrt{1-\varepsilon_n^2}\omega_n t\right)\right\}$$

2.2 Results of reference

the values of the fields of displacement, velocity and acceleration of the loose lead (node $N10$) are worth at time $t=0.0195\,s$:

	Displacement (m)	**Velocity** $(m.s^{-1})$	**Acceleration** $(m.s^{-2})$
Computation without damping	– 8.3766x10 – 7	1.6753 X 10 $^{-3}$	0
Computation with structural damping	– 1.00462x10 – 6	1.20384x10 $^{-3}$	– 1.21564

2.3 Uncertainty on the analytical

solution Solution.

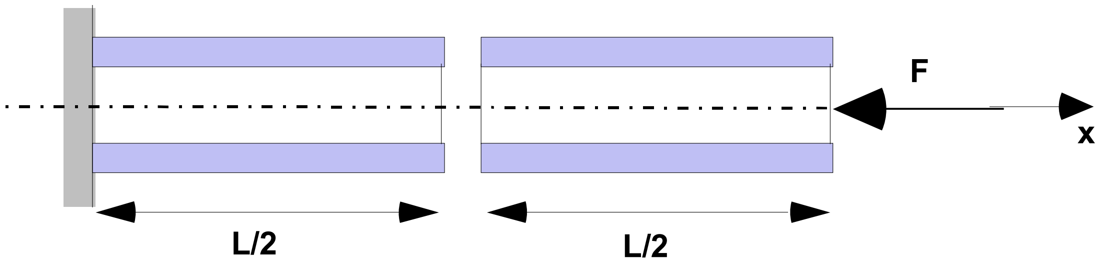
2.4 Bibliographical references

[1] G. ROBERT: Analytical solutions in dynamics of structures. Ratio Samtech n°121, March 1996.

3 Modelization A

3.1 Characteristic of the modelization

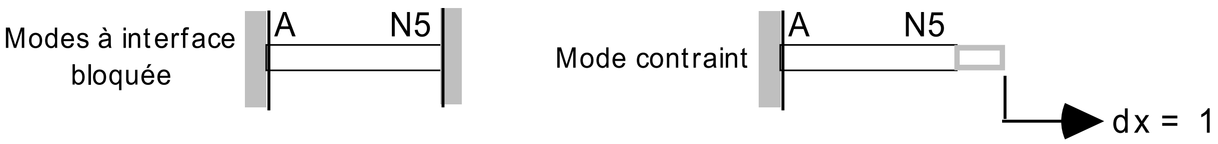
the beam is cut out in two parts of equal size. Each substructure considered is with a grid in segments to which are affected of the elements of the type `POU_D_T`.



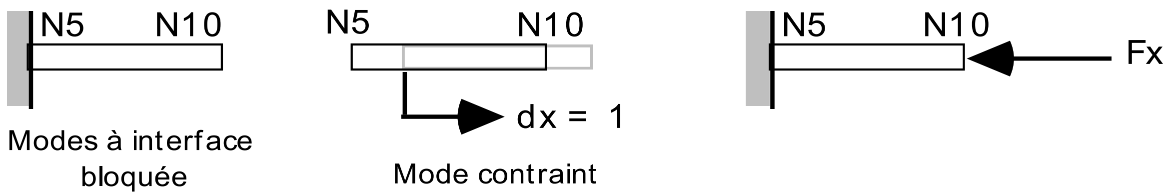
The structure is studied using the method of substructuring transitory with interfaces of the type "Craig-Bampton" (blocked interfaces).

Modal base used is made up of 4 eigen modes for substructure of left, of 5 eigen modes for substructure of right to which are added the constrained modes associated with the degrees of freedom with interface (to the number of 2).

Projection base of the substructure of left:



Projection base of the substructure of right:



3.2 Characteristics of the meshes

The mesh of the complete beam to carry out the computation of reference shows the following characteristics:

file of mesh type `Aster (.mail)`
Many nodes = 11
Number of meshes = 10 SEG2

The mesh of the half-beam to carry out computation by substructuring, shows the following characteristics:

file of the type `Ideas (.msup)`
Many nodes = 6
Number of meshes = 5 SEG2

3.3 Features tested

One tests the features of dynamic computation per substructuring as well as the restitution in physical space.

3.4 Quantities tested and results

the values are restored on a mesh squelette made up of two substructures. The mesh initial which contains 6 nodes is duplicated to create substructure line. The node of end thus corresponds to node 12.

Identification	Reference (beam supplements)	Substructuring	Difference (%)
Node 12: displacement (m)	– 6.2818E-7	– 6.2818E-7	< 0.1
Node 12: velocity $(m.s^{-1})$	2.0957E-3	2.0957E-3	< 0.1
Node 12: acceleration $(m.s^{-2})$	1.1139E+1	1.1139E+1	< 0.1

3.5 Remarks

One can be astonished that the adopted reference corresponds to the complete beam modelled by 10 elements and not to the analytical solution. It is that the development of the solution in series of the eigen modes converges very slowly: the modal solution is very far away from the theoretical solution here. The relevant comparison is thus well that selected.

The computation by modal recombination is realized on the basis of the complete structure modal base (11 modes) taking into account the adopted discretization. In the same way, the dimension of the projection base used for computation by dynamic substructuring is of 11 (substructure of left: 4 eigen modes + 1 constrained mode; substructure of right: 5 eigen modes + 1 constrained mode). It is thus normal to obtain an excellent agreement between the modelization of the complete beam and that of the beam divided into two substructures.

4 Modelization B

4.1 Characteristic of the modelization

the characteristics of this modelization are identical to the preceding one (modelization A). The only difference lies in the fact that the structure is damped. The damping used is of type proportional:

$$C=\alpha K+\beta M \quad \text{with} \quad \alpha=6.5\text{x}10^{-6} s \quad \text{and} \quad \beta=16.0\, s^{-1} .$$

These values correspond to a reduced damping of 1% on the first mode of structure.

4.2 Characteristics of the mesh

the characteristics of the mesh are also identical to those of the modelization A (cf [§ 3.2]).

4.3 Features tested

One tests the features of dynamic computation, with catch in depreciation account, by substructuring as well as the restitution in physical space.

4.4 Quantities tested and results

the results are restored on a mesh squelette made up of two substructures. The mesh initial which contains 6 nodes is thus duplicated to create substructure line. The node of end thus corresponds to node 12.

Identification	Reference (models complete)	Substructuring	Difference (%)
The node is outside the field of definition with a right profile of the EXCLU type node: 12 displacement (m)	– 9.54882E-7	– 9.54882E-7	< 0.1
The node is outside the field of definition with a right profile of the EXCLU type node: 12 velocity $(m.s^{-1})$	1.22190E-3	1.22190E-3	< 0.1
The node is outside the field of definition with a right profile of the EXCLU type node: 12 acceleration $(m.s^{-2})$	– 1.91712E+0	– 1.91712E+0	< 0.1

4.5 Remarks

One can be astonished that the adopted reference corresponds to the complete beam modelled by 10 elements and not to the analytical solution. The important differences between the numerical and theoretical solutions are ascribable with the reduced number of elements. The use of 50 elements instead of 10 would have made it possible to approach theoretical acceleration with a margin of 1%. That put except for, one can note that the use of a method of substructuring provides the same results as those of the complete beam.

The computation by modal superposition is realized on the basis of the complete structure modal base (11 modes). In the same way, the dimension of the projection base used for computation by under - dynamic structuring is of 11 (substructure of left: 4 eigen modes + 1 constrained mode; substructure of right: 5 eigen modes + 1 constrained mode). It is thus normal to obtain an excellent agreement between the modelization of the complete beam and its modelization in two substructures.

5 Modelization C

5.1 Characteristic of the modelization

As in the preceding modelization (modelization B), the structure is deadened with the same damping of the type proportional and the same coefficients of Rayleigh (cf [§ 5.1]).
The difference lies in the use of a method of condensation dynamic of the right part of the structure (including this time 6 elements) which is then represented by a dynamic macro-element of substructuring. The resolution of the dynamic problem takes place on a mixed model made up of the macro-element of the right part of structure where is exerted the applied force and of a physical model reduced to its left part including 4 elements where there is the fixed support.

5.2 Characteristics of the mesh

The mesh used is the initial mesh complete (cf §1.1) obtained by duplication then assembly of the mesh of the modelization A what gives 10 elements and 11 nodes.

5.3 Features tested

One tests the features of dynamic computation with condensation of part of structure as well as the restitution in physical space.

5.4 Quantities tested and results

the results are restored on the complete initial mesh assembling two substructures of the preceding modelizations. The node of end corresponds to the node $N10$ (cf appears of the §1.1). It same as those of the modelization B like is exactly hoped for.

Identification	**Reference (models complete)**	**dynamic Condensation**	**Difference (%)**
The node is outside the field of definition with a right profile of the EXCLU type node: $N10$ displacement (m)	– 9.54882E-7	– 9.54882E-7	< 0.1
The node is outside the field of definition with a right profile of the EXCLU type node: $N10$ velocity ($m.s^{-1}$)	1.22190E-3	1.22190E-3	< 0.1
The node is outside the field of definition with a right profile of the EXCLU type node: $N10$ acceleration ($m.s^{-2}$)	– 1.91712E+0	– 1.91712E+0	< 0.1

6 Summary of the results

As well in the undamped case as in the deadened case, the results got using the model complete and by substructuring do not present significant variations. The operators of linear transient computation by substructuring are thus validated.

In the deadened case, the agreement between the solutions numerical and analytical would have been better by taking more elements (50 instead of 10 for example).

Lastly, let us announce that the results got by *Code_Aster* were compared with results got by the SAMCEF software. They are included in the table below. It is noted that in the undamped case, the two software provides nearby results, quite as far away from the analytical solution.

Identification	**undamped Case**		**damped Case**	
	Code_Aster	the SAMCEF software	*Code_Aster*	the SAMCEF software
$N12$, displacement (m)	– 6.282E-7	– 6.290E-7	– 9.549E-7	– 9.557E-7
$N12$, velocity (m/s)	2.096E-3	2.080E-3	1.222E-3	1.222E-3
$N12$, acceleration (m/s^2)	1.114E+1	1.075E+1	– 1.917E+0	– 1.910E+0