SDLL101 - Vibration of a beam with prestressing

Summarized:

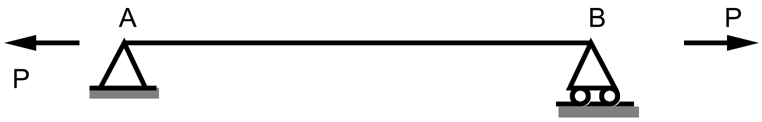
This plane problem consists in seeking the frequencies of vibration of a mechanical structure made up of a hurled beam, of circular section, under tension embed-slide. This test of Structural mechanics corresponds to a dynamic analysis of a linear model having a linear behavior. This test comprises two modelizations.

In the first modelization, one tests the beam element of Timoshenko subjected to a prestressing, the computation of the geometrical stiffness and the computation of the eigenfrequencies by the method of Lanczos. In the second modelization, one tests the beam element of Eulerian - Bernouilli subjected to a prestressing, the computation of the geometrical stiffness and the computation of the eigenfrequencies by the method of Bathe and Wilson.

The got results are in concord with the results of guide VPCS. One notices a shift to the top of the frequencies of vibration when prestressing in the beam increases.

1 Problem of reference

1.1 Geometry



circular Section full
diameter $d=0.01\,m$

Length with the beam
$L=2\,m$

1.2 Material properties

$E=2\,10^{11}\,N/m^2$
$\nu=0.3$
$\rho=7\,800.\,kg/m^3$

1.3 Boundary conditions and loadings

- Beam pose-posed,
- 4 loadings are studied $P=0.$ $P=10.$ $P=100.$, $P=1\,000.\,N$

2 Reference solution

2.1 Method of calculating used for the reference solution

the equation of vibration of a prestressed beam is:

$$EI_z \frac{\partial^4 y}{\partial x^4} + P \frac{\partial^2 y}{\partial x^2} = -\rho S \frac{\partial^2 y}{\partial x^2}$$

prestressed tension if $P > 0$, of compression if $P < 0$, and led to the eigenfrequencies of bending (assumption of Eulerian-Bernoulli)

$$f_i = \frac{i^2 \pi}{2L^2} \left(1 + \frac{PL^2}{EI_z i^2 \pi^2} \right)^{1/2} \left(\frac{EI_z}{\rho S} \right)^{1/2}, \ i = 1,2,3, \ldots$$

2.2 Results of reference

the first 5 eigenfrequencies.

2.3 Uncertainty on the analytical

solution Solution (assumption of the beams of Eulerian-Bernouilli).

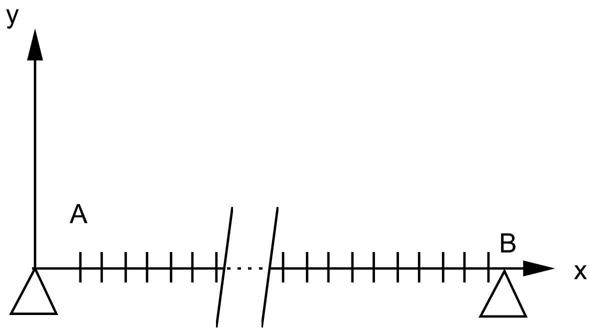
2.4 Bibliographical references

1) Robert D. BLEVINS Formulated for natural frequency and shape mode - 1979 p.144 (rectified formula 8.20).

3 Modelization A

3.1 Characteristic of the modelization

Beam elements POU_D_T (Straight beam of Timoshenko)



Cutting: 10 beam elements

the node is outside the field of definition with a right profile of the EXCLU type node: A translations in x and y blocked
the node is outside the field of definition with a right profile of the EXCLU type node: B translation in y blocked.

Note:

The applied force P generates B a reaction of it $-P$ in A .

3.2 Characteristics of the mesh

Many nodes:	21
Number of meshes and types:	20 SEG2

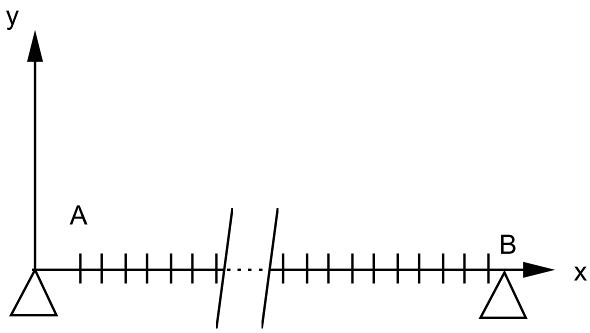
3.3 Quantities tested and results

Pre stress/order of the eigen mode		Reference
$P=0$	1	4.97137
	2	19.8851
	3	44.7414
	4	79.5403
	5	124.2818
$P=10$	1	5.0728
	2	19.9874
	3	44.8439
	4	79.6429
	5	124.3844
$P=100$	1	5.9090
	2	20.8860
	3	45.7561
	4	80.5600
	5	125.3037
$P=1\,000$	1	11.2577
	2	28.3462
	3	54.0370
	4	89.2134
	5	134.1511

4 Modelization B

4.1 Characteristic of the modelization

Beam elements POU_D_E (Beam of Eulerian-Bernouilli)



Cutting: 19 beam elements

the node is outside the field of definition with a right profile of the EXCLU type node: A translations in x and y blocked

the node is outside the field of definition with a right profile of the EXCLU type node: B translation in y blocked.

Note:

The applied force P generates B a reaction of it $-P$ in A .

4.2 Characteristics of the mesh

Many nodes:	21
Number of meshes and types:	20 SEG2

4.3 Quantities tested and results

Pre stress/order of the eigen mode		Reference
$P=0$	1	4.97137
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$P=1\,000$	1	11.2577
	2	28.3462
	3	54.0370
	4	89.2134
	5	134.1511

5 Summary of the results

the got results are in concord with the results of reference. It is noticed well that the frequencies of vibration increase when prestressing increases.