SDLL15 - Beam hurled, embed-free, with mass or offset inertia

Summarized:

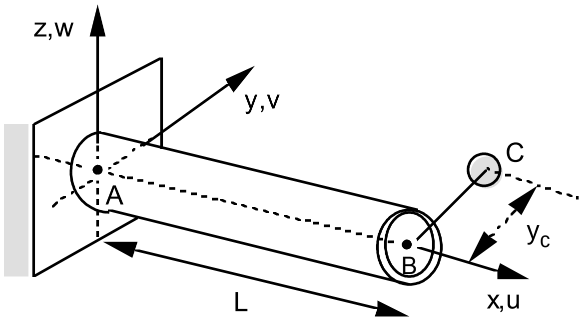
This three-dimensional problem consists in calculating the frequencies and the modes of vibration of a mechanical structure made up of a straight beam hurled, embed-free, with tubular section and of an unbalance attached at the loose lead of the beam. This test of Structural mechanics corresponds to a dynamic analysis of a linear model having a linear behavior. It comprises only one modelization.

This problem makes it possible to test the beam element of Eulerian Bernouilli, the model of point mass and modal computation by the method of Lanczos.

The got results are in concord with those of guide VPCS. Two computations carried out (eccentricity of the point mass null or different from zero) make it possible to highlight the coupling of the various modes when the point mass is offset.

1 Problem of reference

1.1 Geometry



Coordinated of the points (in m):

	A	B	C
x	0.	10.	10.
y	0.	0.	yc
z	0.	0.	0.

length of beam: $AB=L=10\,m$

point mass in C : $m_c=1000\,kg$

Tubular section:

external diameter	$de=0.350\,m$
internal diameter	$di=0.320\,m$
area	$A=1.57865\,10^{-2}\,m^2$
polar	$Iy=Iz=2.21899\,10^{-4}\,m^4$
inertia inertia	$Ip=4.43798\,10^{-4}\,m^4$

2 studied cases:
1) $yc=0.$
2) $yc=1.\,m$

1.2 Material properties

$E=2.1\,10^{11}\,Pa$

$\rho=7800\,kg/m^3$

1.3 Boundary conditions and loadings

Not A clamped: ($u=v=w=0$, $\theta_x=\theta_y=\theta_z=0$).

1.4 Initial conditions

Without object for the modal analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is that given in file SDLL15/89 of the guide VPCS which presents the method of calculating in the following way:

The problem with not offset mass leads to decoupled modes:

- traction and compression (effect of the mass alone),
- torsion (effect of inertia around neutral fiber),
- bending in the planes x, y and x, z (effect of the mass).

The various eigenfrequencies are given with a model by finite elements of beam of Eulerian (slender beam).

For the first mode with an unbalance, a method of Rayleigh gives the approximate formula:

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{3 E I_z}{L^3 (m_c + 0.24 M)}}$$

with M = total mass of the beam.

When the mass is offset, the modes of bending (x, z) and torsion are coupled, as well as the modes of bending (x, y) and traction and compression.

For the eigen mode, the components at the point B make it possible to calculate the components at the center of gravity of the mass (not C) by:

$$\begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix} = \begin{bmatrix} u_B \\ v_B \\ w_B \end{bmatrix} + \begin{bmatrix} 0 & z_c & -y_c \\ -z_c & 0 & +x_c \\ +y_c & -x_c & 0 \end{bmatrix} \begin{bmatrix} \theta_{xB} \\ \theta_{yB} \\ \theta_{zB} \end{bmatrix}$$

For this test:

$$u_c = u_B = -\theta_{zB}$$

$$v_c = v_B$$

$$w_c = w_B + \theta_{xB}$$

2.2 Results of reference

Case 1: the first 10 eigen modes.

Case 2: the first 8 eigen modes.

2.3 Uncertainty on the solution

Problem 1:	$f1$ analytical solution other frequencies $\pm 1\%$
Problem 2:	$\pm 1\%$

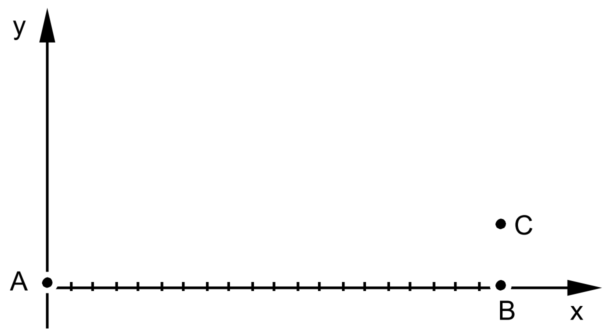
2.4 Bibliographical references

1) Dynamic analysis Working group. Commission of Validation of the Software packages of Structural analysis. French company of Mécaniens. (1988)

3 Modelization A

3.1 Characteristic of the modelization

Beam element POU_D_E and discrete element DIS_TR



Cutting: beam: AB 20 meshes SEG2.

Limiting conditions:

with ending node A
the DDL_IMPO: (THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: A DX: 0. , DY: 0. , DZ: 0. , DRX: 0. , DRY: 0. , DRZ: 0.)

Nodal mass in B with an eccentricity $ey=0.$ Case 1
$ey=1.$ Case 2

Names of the nodes: Points $A=N100$ $B=N200$

3.2 Characteristic of the mesh

Many nodes: 21
Number of meshes and types: 20 SEG2

3.3 Quantities tested and Natural

Case	results of the eigen mode	Frequency Hz Reference	Aster	% difference
	bending 1,2	1.65	1.6554	0.33
	bending 3,4	16.07	16.0712	0.
CAS 1	bending 5,6	50.02	50.0240	0.
	tension 1	76.47	76.4727	0.
$yc=0.$	torsion 1	80.47	80.4688	0.
	bending 7,8	103.20	103.20444	0.
	f_z+t_o 1	1.636	1.6363	0.
	f_y+t_r 2	1.642	1.6416	0.
CAS 2	f_y+t_r 3	13.46	13.4551	0.
	f_z+t_o 4	13.59	13.5919	0.5
$yc=1.$	f_z+t_o	28.90	28.8972	0.
	f_y+t_r 6	31.96	31.9594	0.
	f_z+t_o 7	61.61	61.6091	0.
	f_y+t_r 8	63.93	63.9289	0.
Mode	θ_{xB}	0.03	3.039 10-2	1.321
1	w_C/w_B	1.030	1.030	0.
2	u_C/v_B	– 0.148	– 0.148	0.
3	u_C/v_B	– 2.882	– 2.880	0.07
4	w_C/w_B	– 0.922	– 0.923	0.108
5	θ_{xB}	– 1.922	– 1.92268	0.036

with: $f_z+t_o=flexion\ x, z+torsion$ $f_y+t_r=flexion\ x, y+traction$

3.4 Remarks

Computations carried out by:

```
MODE_ITER_SIMULTMETHODE    : "TRI_DIAG"
                        OPTION: ' PLUS_PETITE'NMAX_FREQ: 10 Cases 1
                                                          8 Cases
2
```

In the test, one cannot check the values of the ratios $\frac{u_C}{v_B}$ for modes 2 and 3 (except manually). With regard to the values of $\frac{w_C}{w_B}$, the technique is the following one: if one imposes $w_B=1$ (command NORM_MODE), one then has $\frac{w_C}{w_B}=1+\theta_{xB}$ and one can make checks on the values of θ_{xB} .

Contents of the file results:

Case 1: the first 11 eigenfrequencies, modal eigenvectors and parameters.
Case 2: the first 9 eigenfrequencies, modal eigenvectors and parameters.

4 Summary of the results

The modelization of unbalance gives exact results for the 8 frequencies of reference.

The accuracy of the eigen modes is about 0.1% until mode 4.