SDLL10 - Beam of rectangular section variable (embed-embedded)

Summarized:

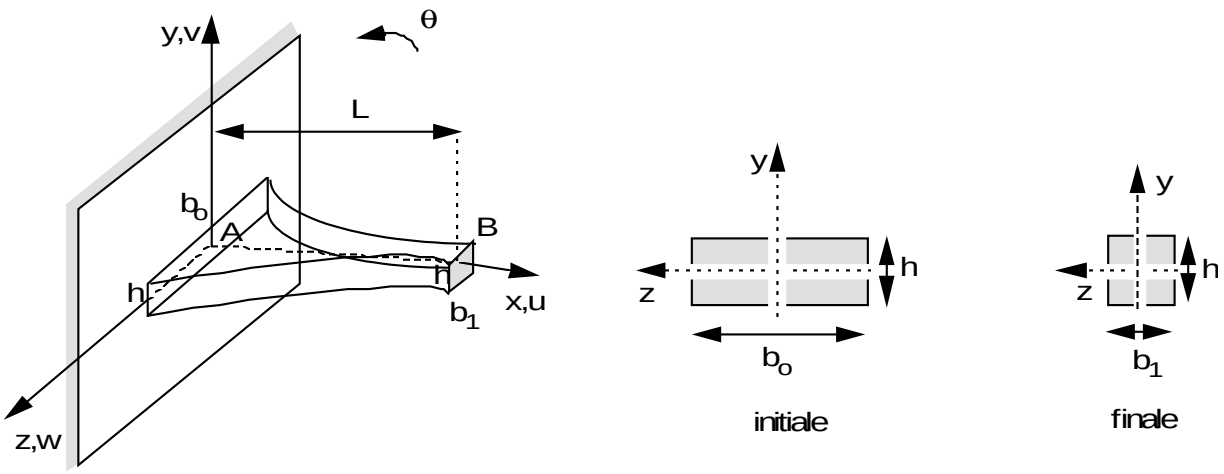
This plane problem consists in seeking the frequencies and the modes of vibration of a mechanical structure made up of an embed-embedded beam whose area of the cross-section varies exponentially. This test of Structural mechanics corresponds to a dynamic analysis of a linear model having a linear behavior. It understands only one modelization.

Via this problem, one tests the beam element in bending of variable Timoshenko of section as well as the computation of the frequencies and the modes of vibration by the method of Lanczos. One tests also the functionality “normalizes to 1. “at the point of maximum amplitude in translation” of the modes of vibration.

By means of a fine spatial discretization, the got results are in concord with the analytical results given in guide VPCS.

1 Problem of reference

1.1 Geometry



Length of beam: $L=0.6\,m$
Constant thickness: $h=0.01\,m$

Rectangular section:

Initial cross-section:
width: $b_0=0.03\,m$
area: $A_0=3.10^{-4}m^2$
main moment of inertia: $Iz_0=0.25\,10^{-8}m^4$

Variation of the section:
$b=b_0e^{-2\alpha x}$ with $\alpha=1.$
$A=A_0e^{-2\alpha x}$
$Iz=Iz_0e^{-2\alpha x}$

Coordinates of the points (m):

	A	B
x	0.	0.6
y	0.	0.

1.2 Material properties

$E=2.10^{11}\,Pa$
$\nu=0.3$
$\rho=7\,800.\,kg/m^3$

1.3 Boundary conditions and loadings

Points A and B : embedded $u=v=0.$ $\theta=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is that given in file SDLL10/89 of the guide VPCS which presents the method of calculating in the following way:

The pulsation ω_i is given by the roots of the equation:

$$1-\cos(rL)\,\mathrm{ch}(sL)+\frac{s^2-r^2}{2rs}\mathrm{sh}(sL)\,\sin(rL)=0$$

with:

$$\lambda_i^4=\frac{\rho\ A_0\ \omega_i^2}{E\ I_{z0}}\quad ;\quad r=\sqrt{\alpha^2+\lambda_i^2}\quad ;\quad s=\sqrt{\lambda_i^2-\alpha^2}\ \text{si}\left(\lambda_i^2-\alpha^2\right)>0$$

The components of translation ν of the mode $F_i(x)$ are then:

$$\phi_i(x)=e^{\alpha x}\left[\cos(rx)-ch(rx)+\frac{\cos(rL)-ch(sL)}{r\,sh(sL)-s\sin(rL)}(s\sin(rx)-r\,sh(sx))\right]$$

2.2 Results of reference

the first 4 eigenfrequencies and eigen modes normalized to 1 for the largest component in translation.

2.3 Uncertainty on the analytical

solution Solution.

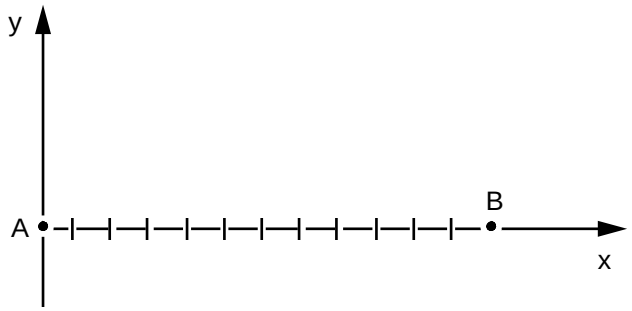
2.4 Bibliographical references

- Dynamic analysis Working group. Committee of Validation of the Software packages of Structural analysis. French company of the Mechanics (1988).

3 Modelization A

3.1 Characteristic of the modelization

Beam element POU_D_T (Straight beam of Timoshenko)



Cutting: beam: AB 120 meshes SEG2 of section variable.

Limiting conditions:

in all nodes DDL_IMPO (TOUT: "OUI', DZ: 0. , DRX: 0. , DRY: 0.)

with the nodes ends (THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: (AB) DX: 0. , DY: 0. , DRZ: 0.)

Names of the nodes:

Not A	$x=0.$	*N1*
	$x=0.1$	*N21*
	$x=0.2$	*N41*
	$x=0.3$	*N61*
	$x=0.4$	*N81*
	$x=0.5$	*N101*
Not B	$x=0.6$	*N121*

3.2 Characteristic of the mesh

Many nodes: 121
Number of meshes and types: 120 SEG2

3.3 Quantities tested and Order

results of the eigen mode	Frequency Reference	% tolerance
1	the 143.303	1.6
2	396.821	0.45
3	779.425	0.3
4	1289.577	0.9

eigen modes of *Aster* were normalized to 1. at the point of maximum amplitude in translation as in the reference.

Eigen mode $F_i(x)$ normalized with 1 at the point of maximum amplitude

	i	$x=0.1$	$x=0.2$	$x=0.3$	$x=0.4$	$x=0.5$
Reference		0.2349	0.6962	0.98960	0.8505	0.3507
Aster	1	0.2363	0.6970	0.9895	0.8516	0.3529
% difference		0.583	0.119	0.	0.132	0.631
% tolerance		0.6	0.15	0.1	0.15	0.7
Reference		– 0.4653	– 0.7558	0.	0.9232	0.6941
Aster	2	– 0.4670	– 0.7555	– 2.910-4	0.9226	0.6971
% difference		0.37	– 0.041	– 2.910-4	– 0.063	0.435
% tolerance		0.4	0.1	1.10-3	0.1	0.45
Reference		0.6278	0.1969	– 0.7783	0.2406	0.9366
Aster	3	0.6290	0.1952	– 0.7782	0.2377	0.9387
% difference		0.192	– 0.89	– 0.014	– 1.226	0.228
% tolerance		0.2	0.9	0.1	1.23	0.25
Reference		– 0.666	0.4832	0.	– 0.5901	0.9937
Aster	4	– 0.6656	0.4840	4.610-4	– 0.5919	0.9928
% difference		– 0.081	0.18	4.610-4	0.31	– 0.089
% tolerance		0.1	0.2	1.10-3	0.35	0.1

3.4 Remarks

Computations carried out by:

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MODE_ITER_SIMULTMETHODE: "TRI_DIAG"
                    OPTION: '    PLUS_PETITE'NMAX_FREQ: 4
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Contents of the file results:

the first 4 eigenfrequencies, eigenvectors.

4 Summary of suitable

the Modelization results (frequencies and eigen modes with less 2%) with a fine mesh.

A computation carried out on a coarse mesh (12 meshes) shows more important variations with the reference solution. This is especially due to the way in which the modes are normalized.