# SDLL09 - Vibration of a slender beam of variable rectangular section (embed-free)

**Summarized:**

This plane problem consists in seeking the frequencies of vibration of a free clamped beam with rectangular variable section. This test comprises only one modelization.

The variation of section of the beam is either homothetic, or nonhomothetic. The characteristics of the section of the beam are given according to meshes in two different ways:

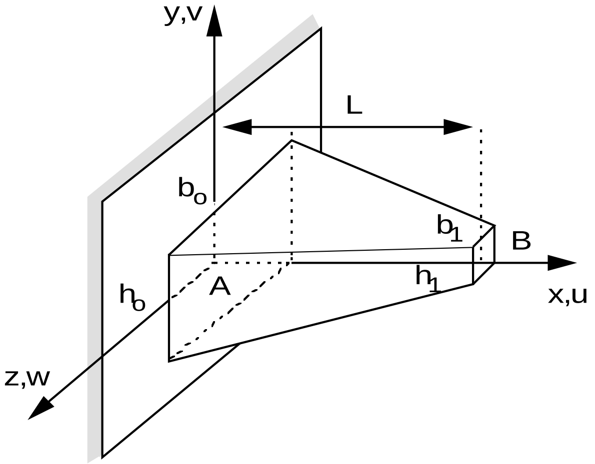
section and inertias,
height and width.

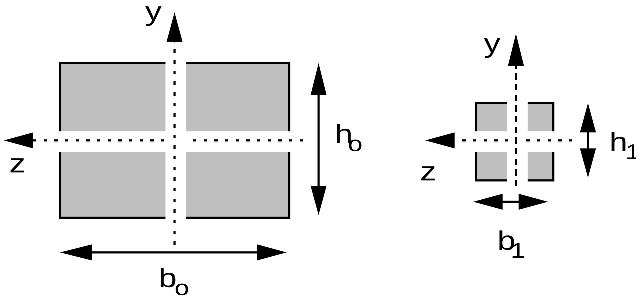
This problem thus makes it possible to test the beam element with variable section for a prismatic structure as well as computation of the frequencies of vibration by inverse iterations. In addition, in operator `AFFE_CARA_ELEM`, one tests the remanence of certain key words.

The got results are in concord with those given in guide VPCS.

# 1 Problem of reference

## 1.1 Geometry




Sections rectangulaires

**Length of beam:**

$L=1\,m$

**Rectangular section:**

| | **Cross-section initial** | | **final Cross-section** |
|---|---|---|---|
| | **Case 1** | **Cases 2** | |
| height: | $h_o=0.04\,m$ | $=0.04\,m$ | $h_1=0.01\,m$ |
| width: | $b_o=0.04\,m$ | $=0.05\,m$ | $b_1=0.01\,m$ |
| area: | $A_o=1.6\,10^{-3}\,m^2$ | $=2.10^{-3}\,m^2$ | $A_1=1.10^{-4}\,m^2$ |
| inertia: | $Iz_o=2.1333\,10^{-7}\,m^4$ | $=2.6667\,10^{-7}\,m^4$ | $Iz_1=8.3333\,10^{-10}\,m^4$ |

**Coordinates of the points ( $m$ ):**

| | $A$ | $B$ |
|---|---|---|
| $x$ | 0. | 1. |
| $y$ | 0. | 0. |
| $z$ | 0. | 0. |

## 1.2 Material properties

$E=2.10^{11}\,Pa$

$\rho=7\,800\,kg/m^3$

## 1.3 Boundary conditions and loadings

Point: $A$ embedded $u=v=0$ $\theta=0.$

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

the reference solution is that given in file SDLL09/89 of the guide VPCS which presents the method of calculating in the following way:

Exact computation by numerical integration of the differential equation of the bending of the beams (Theory of Eulerian-Bernouilli).

$$\frac{\partial^2\left(EI_z\frac{\partial^2 v}{\partial x^2}\right)}{\partial x^2}=-\rho A\frac{\partial^2 v}{\partial t^2}$$

where $I_z$ and $A$ vary with the X-coordinate.

One obtains:

$$f_i=\frac{1}{2\pi}\lambda_i(\alpha,\beta)\frac{h_1}{L^2}\sqrt{\frac{E}{12\rho}}$$

with:

$$\alpha=\frac{h_0}{h_1}=4$$
$$\beta=\frac{b_0}{b_1}=4\ ou\ 5$$

| | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ |
|---|---|---|---|---|---|
| $\beta=4$ | 23.289 | 73.9 | 165.23 | 299.7 | 478.1 |
| $\beta=5$ | 24.308 | 75.56 | 167.21 | 301.9 | 480.4 |

## 2.2 Results of reference

the first 5 eigen modes of bending.

## 2.3 Uncertainty on the semi-analytical

solution Solution.

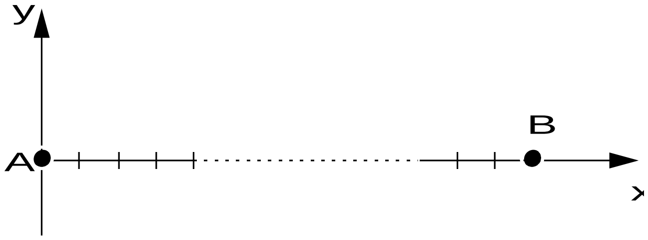
## 2.4 Bibliographical references

H.H. MABIE, C.B. ROGERS, Transverse vibrations of double-tapered cantilever beams - Newspaper of the Acoustical Society of America, n° 51, p. 1771-1774 (1972).

# 3 Modelization A

## 3.1 Characteristic of the modelization

**Modelization** : Beam elements POU_D_E




| | |
|---|---|
| Cutting: | beam: $AB$ 30 meshes SEG2 of section variable<br>15 meshes in "General section"<br>15 meshes in "Rectangular section" |
| limiting Conditions:<br>in all the nodes<br>at end $A$ | DDL_IMPO: (TOUT: "OUI' DZ: 0. , DRX: 0. , DRY: 0. )<br>(The node is outside the field of definition with a right profile of the EXCLU type node: A DX: 0. , DY: 0. , DRZ: 0. ) |
| Names of the nodes: | Not $A=N100$<br>Not $B=N200$ |

## 3.2 Characteristic of the mesh

Mesh: Many nodes: 31
Number of meshes and types: 30 SEG2

## 3.3 Quantities tested and results

| Identification | Reference |
|---|---|
| | **Frequency in** $HZ$ |
| Case 1 $h_0/h_1=4$ $b_0/b_1=4$ | |
| homothetic | |
| bending 1 | 54.18 |
| bending 2 | 171.94 |
| bending 3 | 384.40 |
| bending 4 | 697.24 |
| bending 5 | 1112.28 |
| Cases 2 $h_0/h_1=4$ $b_0/b_1=5$ | |
| nonhomothetic | |
| bending 1 | 56.55 |
| bending 2 | 175.19 |
| bending 3 | 389.01 |
| bending 4 | 702.36 |
| bending 5 | 1117.63 |

# 4 Summary of the results

Good establishment of the variable beam element of section with a fine mesh.

A coarser modelization would be sufficient.