SDLL06 - Transient response of a embed-free column

Summarized

In this case test, one analyzes the transient response of an undamped embed-free beam, modelled by a spring-mass system and subjected to an unspecified dynamic loading.

One tests the discrete element in bending, the computation of the eigen modes by the method of Lanczos and the computation of the transient response by modal recombination of structure subjected either to an accelerogram (modelization A) or with an equivalent imposed force (modelization B).

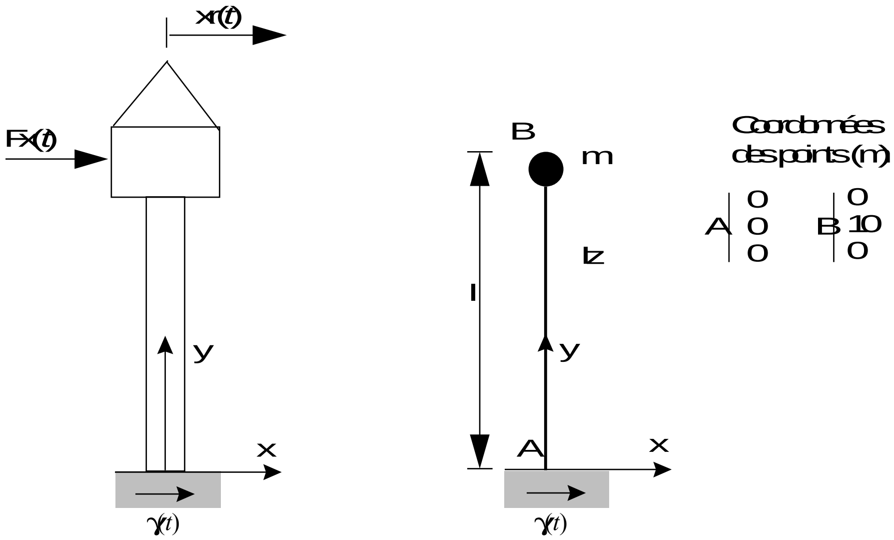
The diagram of Eulerian is used.

The got results are in concord with the results of reference (analytical results).

1 Problem of reference

1.1 Geometry

It is a problem suggested initially in the reference [bib1] and contained in [bib2].



- beam: AB massless slender beam length AB , $l=10\ m$ and main moment of inertia $I_Z=0{,}3285\ m^4$.
- point mass in B : $m=43{,}8\ 10^3\ kg$

1.2 Properties of the materials

Modulus Young: $E=4.\ 10^{10}\ Pa$
Density: $\rho=0\ kg/m^3$

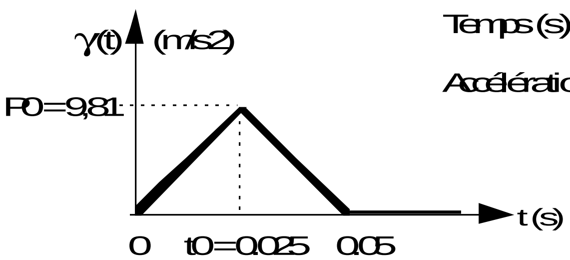
1.3 Boundary conditions and loadings

Boundary conditions:

Only authorized displacements are the translations according to the axis x .
The point A is clamped: $dx=dy=dz=drx=dry=drz=0$.

Loadings:

- modelization a: transverse acceleration at point a: $\gamma(t)$



Temps (s)	0	0.025	0.05
Accélération suivant x (m s^{-2})	0	9.81	0

- modelization b: forces transverse with point: B $Fx(t)$ with $Fx(t)=-m.\gamma(t)$

1.4 Initial conditions

the system is at rest: at $t=0$ $dx(0)=0$, $dx/dt(0)=0$ in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

the problem is treated by a model to a degree of freedom. The column is regarded as an undamped and nonheavy beam hurled stiffness $k=3EI_Z/l^3=3{,}942.10^7\ N/m$. The superstructure located at the top of the column is modelled by a point mass $m=43{,}8\,10^3\ kg$.

The two loading cases lead to the computation of the response of a system with a degree of freedom subjected to an acceleration $\gamma(t)$ of an unspecified form:

$\ddot{x}_r+\omega^2 x_r=-\gamma(t)$ with $\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{3EI_z}{ml^3}}$ the eigenfrequency of the system and x_r the relative displacement of the point B compared to the point A . The solution is obtained by integration of the integral of Duhamel [bib3]:

$$x_r(t)=-\frac{m}{\omega}\int_0^t \gamma(t)\sin\omega(t-\tau)\,d\tau$$

2.2 Results of reference

Displacement relating to the point B .

For a triangular imposed acceleration, one can calculate the integral of Duhamel analytically [bib3]:

$$t<t_0 \quad : \quad x_r=-\frac{P_0}{\omega^2 t_0}\left(t-\frac{\sin\omega t}{\omega}\right)$$

$$t_0<t<2t_0 \quad : \quad x_r=-\frac{P_0}{\omega^2 t_0}\left(2t_0-t-\frac{2\sin\omega(t-t_0)}{\omega}-\frac{\sin\omega t}{\omega}\right)$$

$$t_0<t<2t_0 \quad : \quad x_r=-\frac{P_0}{\omega^3 t_0}\left(2\sin\omega(t-t_0)-\sin\omega(t-2t_0)-\sin\omega t\right)$$

2.3 Uncertainty on the solution

No if one calculates the integral of Duhamel analytically [bib3]. About the accuracy of the numerical integration method employed to compute: the integral of Duhamel ([bib1], [bib2]): method of Simpson with 40 points per period.

2.4 Bibliographical references

1) R.W. Clough and J. Penzien: Dynamics of New York structures, Mac Graw-Hill, 1975, p. 102 - 105

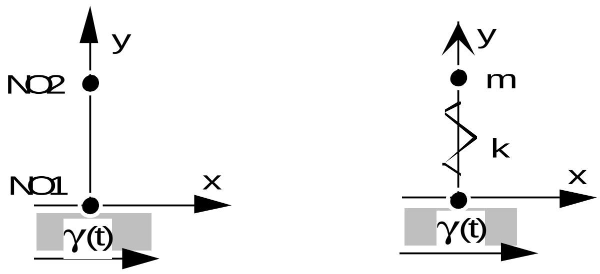
2) Technical Guide VPCS AFNOR - 1990

3) J.S. Przemieniecki: Theory of matrix structural analysis New York, Mac Graw-Hill, 1968, p. 351-357

3 Modelization A

3.1 Characteristic of the modelization

the elements is modelled by discrete elements with 6 degrees of freedom DIS_TR.



The node $NO1$ is subjected to an imposed acceleration $\gamma(t)$. One calculates the relative displacement of the node $NO2$ compared to the displacement of the node $NO1$ and one it compared to analytically calculated displacement.
Temporal integration is carried out with the algorithm of Eulerian (time step: $5.10^{-4} s$).

3.2 Characteristics of the mesh

The mesh consists of 2 nodes and a discrete element (DIS_TR).

3.3 Quantities tested and relative

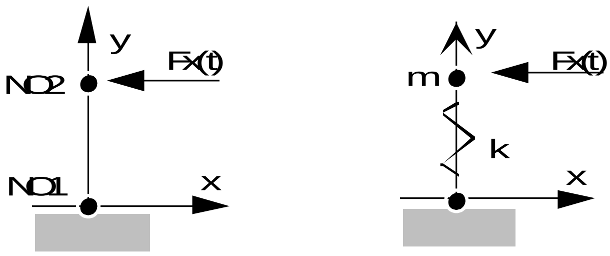
Displacement results of node NO1 (in meters).

Time (S)	analytical Computation	*Code_Aster*	Error (%)
0,010	– 6,511E-05	– 6,495E-05	0
0,015	– 2,185E-04	– 2,183E-04	0
0,020	– 5,139E-04	– 5,136E-04	– 0,058
0,024	– 8,809E-04	– 8,806E-04	– 0,039
0,026	– 1,115E-03	– 1,115E-03	– 0,041
0,030	– 1,679E-03	– 1,679E-03	– 0,014
0,035	– 2,523E-03	– 2,523E-03	– 0,004
0,040	– 3,457E-03	– 3,457E-03	0
0,045	– 4,412E-03	– 4,412E-03	0,004
0,049	– 5,143E-03	– 5,143E-03	0,005
0,051	– 5,485E-03	– 5,485E-03	0,005
0,055	– 6,109E-03	– 6,109E-03	0,005
0,060	– 6,765E-03	– 6,765E-03	0,005
0,065	– 7,269E-03	– 7,269E-03	0,005
0,070	– 7,610E-03	– 7,610E-03	0,005
0,075	– 7,779E-03	– 7,780E-03	0,005
0,080	– 7,774E-03	– 7,775E-03	0,004
0,085	–7,595E-03	– 7,595E-03	0,004

4 Modelization B

4.1 Characteristic of the modelization

the elements are modelled by discrete elements with 6 degrees of freedom `DIS_TR`.



The node $NO2$ is subjected to an imposed force $Fx(t)$. One calculates the relative displacement of the node $NO2$ compared to the displacement of the node $NO1$ and one it compared to displacement calculated in the references [bib1] and [bib2].
Temporal integration is carried out with the algorithm of Eulerian (time step: $10^{-3} s$).

4.2 Characteristics of the mesh

It is the same mesh as for modelization A.

4.3 Grandeurs tested and relative

Displacement results of the node $NO1$ (in meters).

Time (s)	References [bib1], [bib2]	*Code_Aster*	Error (%)
0,01	– 6,500E-05	– 6,447E-05	– 0,82
0,02	– 5,130E-04	– 5,127E-04	– 0,064
0,03	– 1,679E-03	– 1,678E-03	– 0,037
0,04	– 3,457E-03	– 3,457E-03	0,013
0,05	– 5,316E-03	– 5,317E-03	0,022
0,06	– 6,764E-03	– 6,766E-03	0,035
0,07	– 7,609E-03	– 7,611E-03	0,027
0,08	– 7,774E-03	– 7,776E-03	0,024
0,09	– 7,244E-03	– 7,246E-03	0,028
0,1	– 6,068E-03	– 6,069E-03	0,014
0,12	– 2,242E-03	– 2,242E-03	– 0,017
0,14	2,367E-03	2,369E-03	0,071
0,16	6,149E-03	6,152E-03	0,041
0,18	7,783E-03	7,785E-03	0,029
0,2	6,698E-03	6,699E-03	0,018

5 Summary of the results and general remarks

The model simplified presented in this case test makes it possible to validate method of resolution numerical. To deal with the real physical problem, it would be necessary to take into account the effects of inertia (mass of the column, effect of inertia of rotation around B of the superstructure) and of compression of the column (inertia loading).

For the modelization A, the mistake made with time step of $5.10^{-4}s$ is about $0,01\%$; for the modelization B (time step of $10^{-3}s$) it is about $0,6\%$.

One will be able to supplement this case test by checking the convergence of the results for other values of time step and by comparing the results got with other diagrams of integration.