SDLL02 - Beam hurled, embed-free, folded up on it even

Summarized:

This two-dimensional problem consists in searching the frequencies and the modes of vibration of a mechanical structure, made up of a hurled, embedded beam free and folded up on itself.

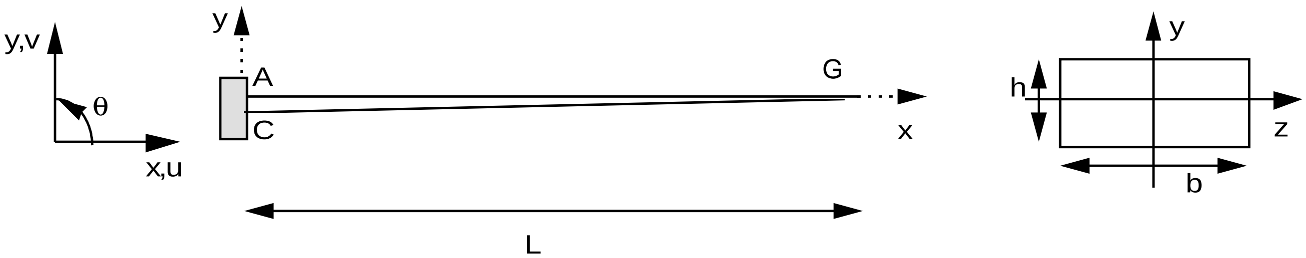
The posed problem does not have physical meaning. It on the other hand makes it possible to validate the search of the multiple eigenfrequencies of bending and the search of the double modes in a subspace of order 2.

In this test, three different modelizations are carried out:

- in the first modelization, the boundary conditions are imposed using parameters of Lagrange (command `AFFE_CHAR_MECA`) and the values and eigenvectors are calculated by the method of Lanczos (command `MODE_ITER_SIMULT`, method: "`TRI_DIAG`"),
- in the second modelization, the boundary conditions are imposed by removing degrees of freedom in the mass matrixes and of stiffness (command `AFFE_CHAR_CINE`) and the values and eigenvectors are calculated by the method of Bathe and Wilson (command `MODE_ITER_SIMULT`, method: "`JACOBI`"),
- in the third modelization, one checks the behavior of modelization `COQUE_C_PLAN` in dynamics. The eigenvalues and the eigen modes are calculated with command `MODE_ITER_SIMULT` and the method of SORENSEN.

1 Constituting problem of

1.1 reference



B the geometrical characteristics of the beam the model mechanical are the following ones:

Length: $L=0.5\,m$

Rectangular cross-section:

Height:	$h=0.005\,m$
Width:	$b=0.050\,m$
Area:	$A=2.5\,10^{-4}\,m^2$
Main moment of inertia:	$I_z=5.208\,10^{-10}\,m^4$

The coordinates (in meters) of the points characteristic of all the beams are:

	A	B	C
x	0.	0.5	0.
y	0.	0.	0.

1.2 Material properties

the properties of the material constituting the beam are:

$E=2.1\,10^{11}\,Pa$

$\nu=0.3$

$\rho=7\,800.\,kg/m^3$

1.3 Boundary conditions and loadings

the boundary condition which characterizes this problem is the fixed support of the point A and is written:

$u=v=0.\quad \theta=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is that given in file SDLL02/89 of the guide VPCS which presents the method of calculating in the following way:

By the method of stiffness dynamic, one shows that the folded up beam admits double frequencies, solution of:

$$\cos(\lambda)=0 \qquad \Rightarrow \quad \lambda_i=(2i-1)\frac{\pi}{2}$$

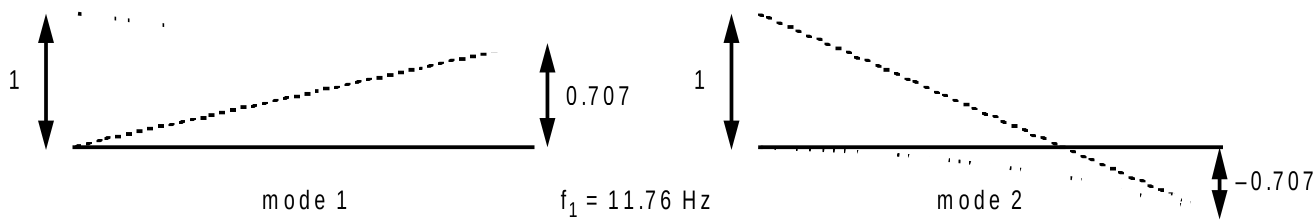
$$f_i=\frac{1}{2\pi}\frac{\lambda_i^2}{L^2}\sqrt{\frac{EI_z}{\rho A}} \qquad i=1,2,...$$

For a rectangular section, one obtains:

$$f_i=(2i-1)^2\pi\frac{R}{8L^2}\sqrt{\frac{E}{12\rho}} \quad i=1,2,...$$

This formulation neglects the shear deformations and of inertia of rotation (beam of Eulerian-Bernoulli).

For the eigen modes, the forms are given in guide VPCS. They are normalized to 1 or – 1 at the point of greater amplitude. There are results only for modes 1,2,3,4,7 and 8. For example, the forms of the first two eigen modes are the following ones:



2.2 Results of reference

the results of reference are the first eight eigenfrequencies and the displacements of the points B, C for eigen modes 1,2,3,4,7 and 8. In *Code_Aster*, the modes are normalized to 1 at the point of greater amplitude (command `NORM_MODE`). To be able to make comparisons with the results of reference, the latter were corrected (multiplication by – 1 if necessary).

2.3 Uncertainty on the solution

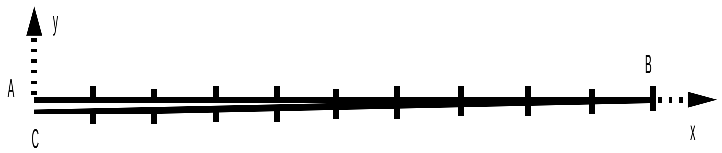
It does not have there uncertainty on the solution because it is analytical.

2.4 Bibliographical references

1) PIRANDA J.: Course and Directed Works of Vibrations of Structures - Mechanical Option - École Nationale Supérieure of Mechanics and Micromechanics - Laboratory of Mechanics Applied - Besancon (France (1983).)

3 Modelization A

3.1 Characteristic of the modelization



One cut out the beam in 20 meshes SEG2 (10 for part AB and 10 for the part BC).

The modelization used for the beams is that of Eulerian Bernoulli (POU_D_E).

Two-dimensional solutions are sought. One can thus block for all the nodes displacement DZ and the rotations DRX and DRY .

The end of the beam (not A) is clamped from where in this point:

$DX=DY=0. \quad DRZ=0.$

3.2 Characteristics of the mesh

The mesh contains 21 nodes and 20 meshes of type SEG2.

The points characteristic of the mesh are the following:

Not $A=A$ Not $B=B$ Not $C=C$

3.3 Quantities tested and results

For the frequencies of vibration of structure, there are the following results:

Identification	Reference
Frequency 1	11.76
Frequency 2	11.76
Frequency 3	105.88
Frequency 4	105.88
Frequency 5	294.10
Frequency 6	294.10
Frequency 7	576.44
Frequency 8	576.44

For the modes of vibration of structure, one has the following results:

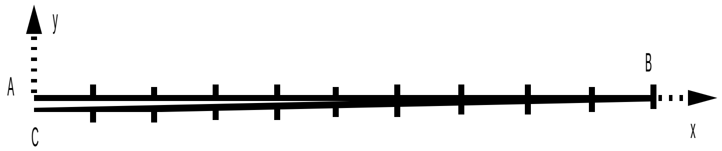
Identification	Points	Quantity	Reference
Frequency 1	B	DY	– 0.707
	C	DY	1.
Frequency 2	B	DY	0.707
	C	DY	1.
Frequency 3	B	DY	0.707
	C	DY	1.
Frequency 4	B	DY	– 0.370
	C	DY	0.523
Frequency 7	B	DY	0.707
	C	DY	1.
Frequency 8	B	DY	– 0.388
	C	DY	0.549

3.4 Remarks

For the eigenfrequencies, the got results are correct. It is the same for the results got concerning the eigen modes. The tolerance is lower than 2% for all the modes except for the mode 2 where the tolerance lies between 2 and 3% .

4 Modelization B

4.1 Characteristic of the modelization



One cut out the beam in 20 meshes SEG2 (10 for the part AB and 10 for the part BC).

The modelization used for the beams is that of Eulerian Bernouilli (POU_D_E).

Two-dimensional solutions are sought. One can thus block for all the nodes displacement DZ and the rotations DRX and DRY .

The end of the beam (not A) is clamped from where in this point:

$$DX=DY=0. \quad DRZ=0.$$

4.2 Characteristics of the mesh

The mesh contains 21 nodes and 20 meshes of type SEG2.

The points characteristic of the mesh are the following:

Not $A=A$ Not $B=B$ Not $C=C$

4.3 Quantities tested and results

For the frequencies of vibration of structure, there are the following results:

Identification	Reference
Frequency 1	11.76
Frequency 2	11.76
Frequency 3	105.88
Frequency 4	105.88
Frequency 5	294.10
Frequency 6	294.10
Frequency 7	576.44
Frequency 8	576.44

For the modes of vibration of structure, one has the following results:

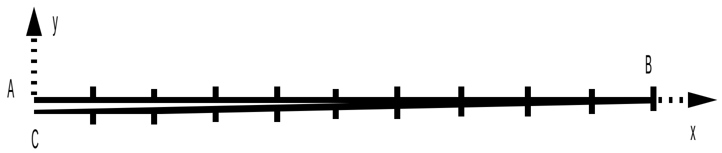
Identification	Points	Quantity	Reference
Frequency 1	B	DY	– 0.707
	C	DY	1.
Frequency 2	B	DY	0.707
	C	DY	1.
Frequency 3	B	DY	0.707
	C	DY	1.
Frequency 4	B	DY	– 0.370
	C	DY	0.523
Frequency 7	B	DY	0.707
	C	DY	1.
Frequency 8	B	DY	– 0.388
	C	DY	0.549

4.4 Remarks

For the eigenfrequencies, the got results are correct. It is the same for the results got concerning the eigen modes. The tolerance is lower than 2% for all the modes except for the mode 2 where the tolerance lies between 3 and 4% .

5 Modelization C

5.1 Characteristic of the modelization



One cut out the beam in 20 meshes SEG3 (10 for part AB and 10 for the part BC).

The modelization used is COQUE_C_PLAN.

The end of the beam (not A) is clamped from where in this point:

$DX = DY = DRZ = 0$.

5.2 Characteristics of the mesh

The mesh contains 41 nodes and 20 meshes of type SEG3.

The points characteristic of the mesh are the following:

Not $A = A$ Not $B = B$ Not $C = C$

5.3 Quantities tested and results

For the frequencies of vibration of structure, there are the following results:

Identification	Reference
Frequency 1	11.76
Frequency 2	11.76
Frequency 3	105.88
Frequency 4	105.88
Frequency 5	294.10
Frequency 6	294.10
Frequency 7	576.44
Frequency 8	576.44

For the modes of vibration of structure, one has the following results:

Identification	Points	Quantity	Reference
Frequency 1	B	DY	– 0.707
	C	DY	1.
Frequency 2	B	DY	0.707
	C	DY	1.
Frequency 3	B	DY	0.707
	C	DY	1.
Frequency 4	B	DY	– 0.370
	C	DY	0.523
Frequency 7	B	DY	0.707
	C	DY	1.
Frequency 8	B	DY	– 0.388
	C	DY	0.549

5.4 Remarks

In this case, where the results are independent of the Young modulus, it is not necessary to modify the Young modulus retained for the modelization, as in the case of the linear static analysis, to take account of the real width of the beam.

6 Summary of the Modelizations

results A and B of type Beam:
With the problem is dealt with a very good accuracy on the first eight frequencies (tolerance $< 0.1\%$) for the two modelizations tested. The components of modes 3,4,7 and 8 are also obtained with a good accuracy about 0.1%. The accuracy on mode 1 is about 1% for the method of Lanczos and of 0.5% the method of Bathe and Wilson. With regard to mode 2, the accuracy is degraded: it is about 2.7% for the method of Lanczos and about 3.3% for the method of Bathe and Wilson. Complementary tests (use of the method of Lanczos by imposing boundary conditions by the command AFFE_CHAR_CINE) make it possible to think that these differences come from the research method of the eigenvalues used.

Modelization COQUE_C_PLAN
the accuracy on the results is good for the first three frequencies, the error is about 0.6%. It is degraded as the frequency increases, the error passes from the 4th frequency to 8th of 1.5% to 6.4%. More the frequency is high plus the difference between the double frequencies is important. The error on the modes is satisfactory for the first 7 modes ($<0.7\%$), it is higher for 8th ($<4\%$). A finer mesh should make it possible to better represent the modal deformed shapes associated with the high frequencies.