SDLD325 - Response transient dynamics of a deadened spring-mass system with 2 degrees of freedom

Summarized:

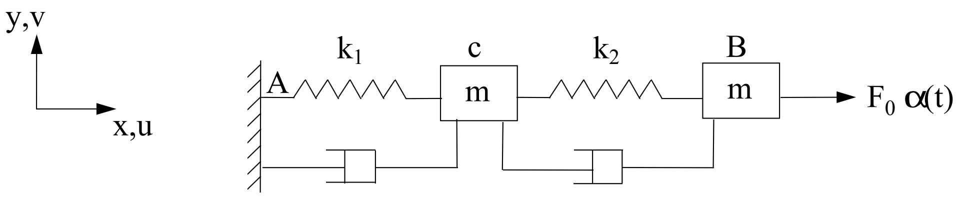
This problem consists in analyzing the dynamic response of a system made up of a set of masses - spring-dampers with 2 degrees of freedom from which the stiffness of springs is very different under excitation of type crenel into 1 degree of freedom.

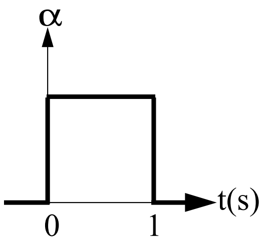
Via this problem, one tests the sensitivity of diagrams of integration on physical space or modal space screw - to - screw of the ratio of the stiffness.

The results in displacement and velocity are compared with an average of results coming from industrial codes and a numerical integration method of improved Newmark type.

1 Problem of reference

1.1 Geometry





1.2 Material properties

Stiffness of connection: $k=28.10^3\ N.m^{-1}$

2 cases:

- $k_1=k/10 \quad k_2=10\mathrm{k}$
- $k_1=10\mathrm{k}$, $k_2=k/10$

Point mass: $m=10\,kg$

One-way viscous damping: $c=50\,kg.s^{-1}$

1.3 Boundary conditions and loadings

clamped A End.

Applied force at the end B : $F(t)=F_0\alpha(t)$ with $\begin{cases}\alpha(t)=1\,\mathrm{si}\ 0\leq t\leq 1\,s\\ \alpha(t)=0\,\mathrm{sinon}\end{cases}$

and $F_0=5\mathrm{N}$.

1.4 Initial conditions

the system is at rest with $t=0$: $u(0)=0$ and $\frac{du}{dt}(0)=0$

2 Reference solution

2.1 Method of calculating used for the reference solution

the search of the transient response of this problem to damping nonproportional can be carried out by numerical integration in real space:

$$[M]\{\ddot{u}_n\}+[C]\{\dot{u}_n\}+[K]\{u_n\}=\{F\}$$

For that, the response was calculated with two industrial codes:

- PERMAS: Diagram of integration of Newmark ($\alpha=0{,}25$ and $\delta=0{,}5$) $\Delta t=10^{-4s}$;
- ABAQUS: Diagram of integration of Hilbert-Hugues-Taylor [bib1] ($\alpha=-0{,}05$) $\Delta t=10^{-4s}$;

and integration method of β - Newmark improved [bib2]:

$$\left[\frac{[M]}{\Delta t^2}+\frac{[C]}{2\Delta t}+\frac{[K]}{3}\right]\{u_{n+2}\}=\left[\frac{\{F_{n+2}\}+\{F_{n+1}\}+\{F_n\}}{3}\right]+\left[\frac{2[M]}{\Delta t^2}-\frac{[K]}{3}\right]\{u_{n+1}\}+\left[\frac{[M]}{\Delta t^2}+\frac{[C]}{2\Delta t}-\frac{[K]}{3}\right]\{u_n\}$$

where n $n+1$, $n+2$ the computations carried out at times indicate respectively t_n , $t_{n+1}=t_n+\Delta t$ and $t_{n+2}=t_n+2\Delta t$ where Δt is the increment of appointed time.

To start, one takes:

- u_0 and $u_{-1}=u_0-\Delta t\,\dot{u}_0$
- $F_{-1}=2F_0-F_1$

time step adopted is $\Delta t=10^{-5s}$.

2.2 Results reference

Displacement and velocity of the point end B .

2.3 Uncertainty on the Average

solution of numerical solutions.

2.4 Bibliographical references

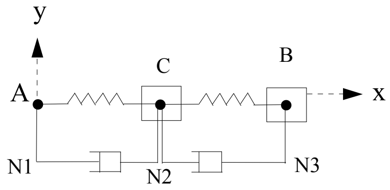
1) H.M. HILBERT, T.J.R HUGUES and R.L. TAYLOR "Improved numerical dissipation for time integration algorithms in structural dynamics" Earthquake Engineering and Structural Dynamics, Vol.5, 1977, pp. 283-292 structural

2) N.M. NEWMARK "A method of computation for dynamics" Proceeding ASCE J.Eng.Mech. DIV E-3, July 1959, pp. 67-94 Modelization

3 A Characteristic

3.1 of the modelization Discrete elements

of stiffness, damping and mass. Characteristics



of the elements: DISCRET: nodal

mass M_T_D_N	linear	stiffness		
	K_T_D_L (,)	straight-line $k_{N2N3}=10\text{k}$	depreciation	$k_{N1N2}=k/10$
	A_T_D_L Boundary conditions			

: with node DDL_IMPO. Names $N1$ of the nodes $DX=DY=DZ=0$

: . Methods of calculating $A=N1$ $C=N2$ $B=N3$

: Integration on

- physical space with Newmark (,) Time step $\alpha=0{,}25$ $\delta=0{,}5$ Integration on $\Delta t=10^{-3}s$
- modal base supplements with Eulerian then modal recombination $\Delta t=10^{-3}s$ Time step Integration on the basis of
- complete modal base with adaptive order Δt 2 Time step initial then modal $\Delta t=10^{-3}s$ recombination Integration on
- complete modal base with formula adaptive Δt the method of the Runge-Kutta type of order (32). The tolerance of relative error is of. Integration 10^{-5} on
- modal base supplements with formula adaptive Δt the method of the Runge-Kutta type of order (54). The tolerance of relative error is of formula. Period 10^{-6}

: 3 S. Caractéristiques

3.2 of the mesh Many nodes

: 3 Number of meshes

and type: 2 meshes SEG2 Quantities tested

3.3 and Displacement results (

-) of the point Time m Reference B

()	**0,27 3,0927**
E s	
3 0,53	8,7953 E
4 0,80	2,4669 E
3 1,25	-1,0980 E
- 3	7,8754 E
4 1,78	-5,6508 E
- 4	4,0502 E
4 2,31	-2,9012 E
- 4	2,0831 E
4 2,85	-1,4943 E
- 4	() of

* the point Time $m.s^{-1}$ Reference B

()	**0,11 1,8347**
E s	
2 0,39	-1,3140 E
- 2	9,3509 E
3 0,93	-6,7080 E
- 3	-1,5863 E
- 2	1,1157 E
2 1,64	-7,9838 E
- 3	5,7108 E
3 2,17	-4,0998 E
- 3	2,9405 E
3 2,71	-2,1073 E
- 3	1,5105 E
3	

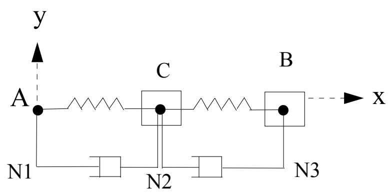
3.4 the results

are tested on the level of the respective peaks of displacement and velocity where the values are most significant. Modelization

4 B Characteristic

4.1 of the modelization Discrete elements

of stiffness, damping and mass. Characteristics



of the elements: DISCRET: nodal

mass linear stiffness
M_T_D_N
K_T_D_L (,) straight-line depreciation $k_{N1N2}=10\text{k}$ $k_{N2N3}=k/10$
A_T_D_L Boundary conditions

: with node DDL_IMPO. Names $N1$ of the nodes $DX=DY=DZ=0$

: . Methods of calculating $A=N1$ $C=N2$ $B=N3$

: Integration on

- physical space with Newmark (,) Time step $\alpha=0{,}25$ $\delta=0{,}5$
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: . Characteristics $2{,}5\,s$

4.2 of the mesh Many nodes

: 3 Number of meshes
and type: 2 meshes SEG2 Quantities tested

4.3 and Displacement results (

-) of the point Time m Reference B

()	**0,19 2,9334**
E s	
3 0,38	1,0959 E
3 0,57	2,2468 E
3 0,76	1,5260 E
3 0,95	1,9773 E
3 1,19	-1,2107 E
- 3	7,5880 E
4 1,57	-4,7553 E
- 4	2,9796 E
4 1,95	-1,8668 E
- 4	1,1694 E
4 2,33	-7,3246 E
- 5	() of

- the point Time $m.s^{-1}$ Reference B

()	**0,09 2,4261**
E s	
2 0,28	-1,5210 E
- 2	9,5332 E
3 0,66	-5,9745 E
- 3	3,7438 E
3 1,08	-2,6037 E
- 2	1,6302 E
2 1,46	-1,0204 E
- 2	6,3887 E
3 1,85	-4,0059 E
- 3	2,5114 E
3 2,23	-1,5743 E
- 3	9,8676 E
4	

4.4 the results

are tested on the level of the respective peaks of displacement and velocity where the values are most significant. Summary of

5 the results For the two

modelizations, the results are precise with an error lower than 1%. Integration

on modal base with a diagram with adaptive step of order 2 gives the best results for a restricted computing time.