SDLD321 - Response transient dynamics of a harmonic oscillator with variable damping

Abstract:

The system considered is a harmonic oscillator with 1 degree of freedom under harmonic excitation with resonance. Various depreciation will be considered:

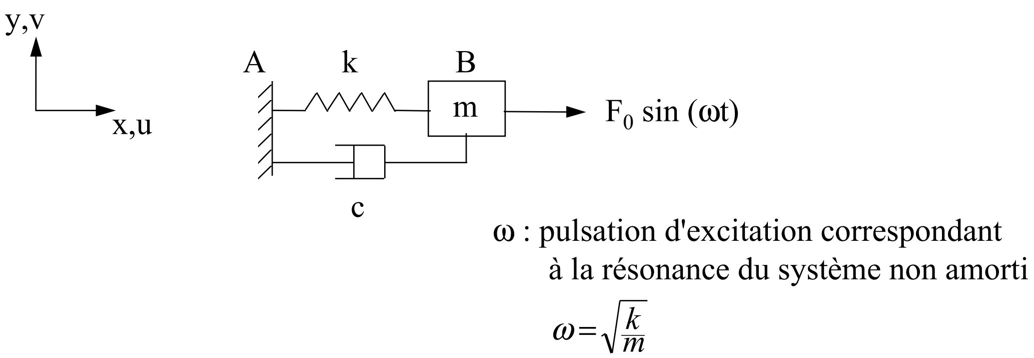
- critical damping,
- average damping,
- very weak damping.

Via this problem, one tests various algorithms DYNA_TRAN_MODAL `of the command` [U4.54.03] and their capacities with dealing with problems with extreme damping. The results are compared with the exact analytical solutions.

1 Problem of reference

1.1 Geometry

the system is composed of a mass, a spring and a damper. He admits a single degree of freedom in translation.



1.2 Material properties

Stiffness of connection: $k=25.10^3\,N.m^{-1}$

Point mass: $m=10\,kg$

Viscous damping:

$c=c_{critique}$; $c=0{,}01\,c_{critique}$; $c=10^{-5}c_{critique}$
with $c_{critique}=1\,000\,kg.s^{-1}$

1.3 Boundary conditions and loadings

End A clamped.

Force harmonic according to X with the resonance frequency with point: B

$F(t)=F_0\sin(\omega t)$ for $t\geq 0$ with $F_0=5\,N$ and $\omega=\sqrt{\frac{k}{m}}=50\,rad.s^{-1}$.

1.4 Initial conditions

the system is at rest with $t=0$: $u(0)=0$ and $\frac{du}{dt}(0)=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

the simple oscillator checks the following equation:

$$m\ddot{u}+c\dot{u}+k\,u=F_0\sin(\omega t)$$

with $u(0)=0$ and $\dot{u}(0)=0$

ω : own pulsation of the oscillator $\omega=\sqrt{\frac{k}{m}}$

The damping criticizes is $c_{critique}=2\mathrm{m}\,\omega$.

The solution for $c=c_{critique}$ is:

$$u(t)=\frac{F_0}{2\mathrm{k}}\left[e^{-\omega t}(1+\omega t)-\cos(\omega t)\right]$$

The solution for a subcritical damping such as $\frac{c}{c_{critique}}=\xi$ is:

$$u(t)=e^{-\xi\omega t}\left(\frac{F_0}{2\xi k}\cos(\omega_D t)+\frac{F_0\omega}{2k\,\omega_D}\sin(\omega_D t)\right)-\frac{F_0}{2\xi k}\cos(\omega t)$$

with $\omega_D=\omega\sqrt{1-\xi^2}$

2.2 Results reference

Displacement and velocity of the point B .

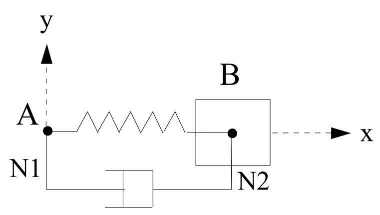
2.3 Uncertainty on the solution

exact analytical Solution.

3 Modelization A

3.1 Characteristic of the modelization

Discrete elements of stiffness, damping and mass.



Characteristics of the elements:

`DISCRET` :	nodal mass	`M_T_D_N`
	linear stiffness	`K_T_D_L`
	straight-line depreciation	`A_T_D_L` ($c=c_{critique}$)

Boundary conditions: with node $N1$ `DDL_IMPO` `DX = DY = DZ = 0.`

Names of the nodes: $P_1=N1$ $P_2=N2$.

Methods of calculating:

- Integration on modal base with Newmark ($\alpha=0{,}25$, $\delta=0{,}5$)
  Time step $\Delta t=10^{-3}s$
- Integration on modal base with Eulerian
  Time step $\Delta t=10^{-3}s$

Lasted of observation: $0{,}5\,s$.

3.2 Characteristics of the mesh

Many nodes: 2

Number of meshes and type: 1 mesh `SEG2`

3.3 Quantities tested and Displacement

- results of the point B

Time (s)	Displacement Reference (m)	Displacement NEWMARK **Aster** (m)	Tolerance (%)	Displacement EULER **Aster** (m)	Tolerance (%)
0,06	1,18914 E-4	1,18886 E-4	0.5%	1,18886 E-4	0.5%
0,12	– 9,42819 E-5	– 9,42574 E-5	0.5%	– 9,47822 E-5	0.6%

0,19	9,97958 E-5	9,97765 E-5	0.5%	9,96206 E-5	0.5%
0,25	– 9,97748 E-5	– 9,97526 E-5	0.5%	– 9,99152 E-5	0.5%
0,31	9,78457 E-5	9,78210 E-5	0.5%	9,83436 E-5	0.6%
0,38	– 9,88705 E-5	– 9,88530 E-5	0.5%	– 9,84730 E-5	0.5%
0,44	9,99961 E-5	9,99754 E-5	0.5%	9,99525 E-5	0.5%

- Velocity of the point B

Time (s)	**Velocity Reference** ($m.s^{-1}$)	**Velocity NEWMARK** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)	**Velocity EULER** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)
0,03	3,31400 E-3	3,31363 E-3	0.5%	3,32568 E-3	0.5%
0,09	– 5,13760 E-3	– 5,13729 E-3	0.5%	– 5,13627 E-3	0.65%
0,16	4,93337 E-3	4,93354 E-3	0.5%	4,93088 E-3	0.5%
0,22	– 5,00087 E-3	– 5,00087 E-3	0.5%	– 5,00133 E-3	0.5%
0,28	4,95298 E-3	4,95284 E-3	0.5%	4,95297 E-3	0.5%
0,35	– 4,87813 E-3	– 4,87836 E-3	0.5%	– 4,87801 E-3	0.5%
0,41	4,98415 E-3	4,98423 E-3	0.5%	4,98409 E-3	0.5%
0,47	– 4,99041 E-3	– 4,99035 E-3	0.5%	– 4,99043 E-3	0.5%

3.4 Remarks

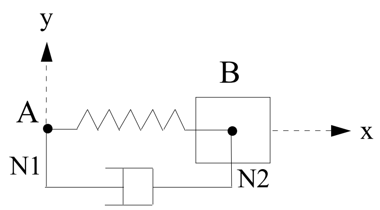
the results are tested on the level of the peaks for the grain of observation selected (10-2s) where the values are most significant.

The mode becomes quasi-permanent after the first period, it is what one must observe by carrying out a transient analysis.

4 Modelization B

4.1 Characteristic of the modelization

Discrete elements of stiffness, damping and mass.



Characteristics of the elements:

`DISCRET` : nodal mass `M_T_D_N`
linear stiffness `K_T_D_L`
straight-line depreciation `A_T_D_L` ($c=0{,}01\,c_{critique}$)

Boundary conditions: with node $N1$ `DDL_IMPO` `DX = DY = DZ = 0.`

Names of the nodes: $P_1=N1$ $P_2=N2$.

Methods of calculating:

- Integration on modal base with Fu-Devogelaere
  Time step $\Delta t=10^{-3}s$
- Integration on the basis of modal base with Δt adaptive order 2
  Time step initial $\Delta t=10^{-5}s$
  Steps maximum $\Delta t=10^{-3}s$

Period of observation: $5s$.

4.2 Characteristics of the mesh

Many nodes: 2

Number of meshes and type: 1 mesh `SEG2`

4.3 Quantities tested and Displacement

- results of the point B

Time (s)	Displacement Reference (m)	Displacement DEVOG Aster (m)	Tolerance (%)	Displacement ADAPT_ORDRE2 Aster (m)	Tolerance (%)
0,06	3,06503 E-4	3,06503 E-4	0.5%	3,06521 E-4	0.5%
0,13	– 5,93807 E-4	– 5,93807 E-4	0.5%	– 5,93729 E-4	0.5%
0,25	– 1,17872 E-3	– 1,17872 E-3	0.5%	– 1,17890 E-3	0.5%
0,69	2,91788 E-3	2,91788 E-3	0.5%	2,91744 E-3	0.5%
1,01	– 3,83901 E-3	– 3,83901 E-3	0.5%	– 3,83567 E-3	0.5%
2,32	6,68206 E-3	6,68206 E-3	0.5%	6,68656 E-3	0.5%
3,64	– 8,19821 E-3	– 8,19821 E-3	0.5%	– 8,204 E-3	0.5%
4,96	9,00847 E-3	9,00847 E-3	0.5%	9,0143 E-3	0.5%

Time (s)	Displacement Reference (m)	Displacement RUNGE_KUTTA_54 Aster (m)	Tolerance (%)	Displacement RUNGE_KUTTA_32 Aster (m)	Tolerance (%)
0,06	3,06503 E-4	3.06420E-04	0.5%	3.06443E-04	0.5%
0,13	– 5,93807 E-4	-5.93619E-04	0.5%	-5.93713E-04	0.5%
0,25	– 1,17872 E-3	-1.178373E-3	0.5%	-1.17845E-3	0.5%
0,69	2,91788 E-3	2.91701E-3	0.5%	2.91706E-3	0.5%
1,01	– 3,83901 E-3	-3.83786E-3	0.5%	-3.83772E-3	0.5%
2,32	6,68206 E-3	6.68009E-3	0.5%	6.67939E-3	0.5%
3,64	– 8,19821 E-3	-8.19578E-3	0.5%	-8.19318E-3	0.5%
4,96	9,00847 E-3	9.00579E-3	0.5%	9.00479E-3	0.5%

- Velocity of the point B

Time (s)	Velocity Reference ($m.s^{-1}$)	Velocity DEVOG Aster ($m.s^{-1}$)	Tolerance (%)	Velocity *ADAPT_ORDRE2* Aster ($m.s^{-1}$)	Tolerance (%)
0,04	8,95997 E-3	8,95997 E-3	0.5%	8,9722 E-3	0.5%
0,10	– 2,33271 E-2	– 2,33271 E-2	0.5%	– 2,33499 E-2	0.5%
0,22	– 5,20590 E-2	– 5,20590 E-2	0.5%	– 5,2113 E-2	0.5%
0,66	1,40500 E-1	1,40500 E-1	0.5%	1,40591 E-1	0.5%
1,04	1,99889 E-1	1,99889 E-1	0.5%	1,99933 E-1	0.5%
2,36	– 3,39933 E-1	– 3,39933 E-1	0.5%	– 3,39725 E-1	0.5%
3,68	4,10585 E-1	4,10585 E-1	0.5%	4,10008 E-1	0.5%
5,00	– 4,4531 E-1	– 4,45308 E-1	0.5%	– 4,44429 E-1	0.5%

Time (s)	Velocity Reference ($m.s^{-1}$)	Velocity RUNGE_KUTTA_54 Aster ($m.s^{-1}$)	Tolerance (%)	Velocity *RUNGE_KUTTA_32* Aster ($m.s^{-1}$)	Tolerance (%)
0,04	8,95997 E-3	8.89561E-3	0.5%	8,95719E-3	0.5%
0,10	– 2,33271 E-2	-2,33194E-2	0.5%	-2,33211E-2	0.5%
0,22	– 5,20590 E-2	-5,20435E-2	0.5%	-5,20573E-2	0.5%
0,66	1,40500 E-1	1,40458E-1	0.5%	1,40475E-1	0.5%
1,04	1,99889 E-1	1,99829E-1	0.5%	1,99809E-1	0.5%
2,36	– 3,39933 E-1	-3,39832E-1	0.5%	-3,39767E-1	0.5%
3,68	4,10585 E-1	4,10463E-1	0.5%	4,10403E-1	0.5%
5,00	– 4,4531 E-1	-4,45308E-1	0.5%	-4,45145E-1	0.5%

4.4 Remarks

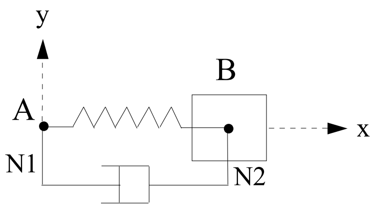
the results is tested on the level of the peaks where the values are most significant.

The period of selected observation makes it possible to see the effect of damping. However, in this interval, the response of the point B remains always transitory but one is close to the permanent mode whose scale of displacement is $10^{-2} m$.

5 Modelization C

5.1 Characteristic of the modelization

Discrete elements of stiffness, damping and mass.



Characteristics of the elements:

`DISCRET` :	nodal mass	`M_T_D_N`
	linear stiffness	`K_T_D_L`
	straight-line depreciation	`A_T_D_L` ($c=10^{-5}c_{critique}$)

Boundary conditions: with node $N1$ `DDL_IMPO` `DX = DY = DZ = 0.`

Names of the nodes: $P_1=N1$ $P_2=N2$.

Methods of calculating:

- Integration on modal base with Newmark ($\alpha=0{,}25$, $\delta=0{,}5$)
  Time step $\Delta t=10^{-3s}$
- Integration on modal base with Eulerian
  Time step $\Delta t=10^{-3s}$

Lasted of observation: $5s$.

5.2 Characteristics of the mesh

Many nodes: 2

Number of meshes and type: 1 mesh `SEG2`

5.3 Quantities tested and Displacement

- results of the point B

Time (s)	Displacement Reference (m)	Displacement NEWMARK *Aster* (m)	Tolerance (%)	Displacemen t EULER *Aster* (m)	Tolerance (%)
0,06	3,11105 E-4	3,10936 E-4	0.5%	3,11181 E-4	0.5%
0,13	– 6,13250 E-4	– 6,13016 E-4	0.5%	– 6,13380 E-4	0.5%

0,25	– 1,25380 E-3	– 1,25304 E-3	0.5%	– 1,25418 E-3	0.5%
0,69	3,44945 E-3	3,44691 E-3	0.5%	3,45069 E-3	0.5%
1,01	– 4,88729 E-3	– 4,89081 E-3	0.5%	– 4,88547 E-3	0.5%
2,32	1,12876 E-2	1,12475 E-2	0.5%	1,13069 E-2	0.5%
3,64	– 1,77960 E-2	– 1,77100 E-2	0.5%	– 1,78360 E-2	0.5%
4,96	2,43613 E-2	2,42198 E-2	0.5%	2,44242 E-2	0.5%

- Velocity of the point B

Time (s)	**Velocity Reference** ($m.s^{-1}$)	**Velocity NEWMARK** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)	**Velocity EULER** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)
0,04	9,09284 E-3	9,08897 E-3	0.5%	9,08230 E-3	0.5%
0,10	– 2,39724 E-2	– 2,39637 E-2	0.5%	– 2,40269 E-2	0.5%
0,22	– 5,49964 E-2	– 5,49680 E-2	0.5%	– 5,48752 E-2	0.5%
0,66	1,64958 E-1	1,64879 E-1	0.5%	1,64882 E-1	0.5%
1,04	2,56456 E-1	2,56547 E-1	0.5%	2,57280 E-1	0.5%
2,36	– 5,79010 E-1	– 5,80019 E-1	0.5%	– 5,81033 E-1	0.5%
3,68	8,97631 E-1	9,00729 E-1	0.5%	9,00668 E-1	0.5%
5,00	– 1,21164	– 1,21829	0.5%	– 1,21531	0.5%

5.4 Remarks

the results are tested on the level of the peaks where the values are most significant.

In the interval of observation, one remains very below permanent mode in resonance whose scale of displacement is $10\,m$.

6 Summary of the results

For the modelization A, the results got as well in displacement as of velocity have an absolute error largely lower than 1% compared to the analytical solution.

The diagram of integration of Newmark is shown more precise than the diagram of Eulerian.

A 1% of critical damping (modelization B), the diagram of Fu-Devogelaere integration is of a frightening accuracy (not error compared to the reference solution).

The diagram with time step adaptive of order 2 also gives results to very small percentage of error.

For very weak depreciation (modelization C), one will note a better accuracy for the diagram of integration of the Eulerian type than for a diagram of the Newmark type. For this last, the error increases according to time but remains lower all the same than 1% .