SDLD320 - Transient response of a system librede 3 masses and 2 springs under harmonic excitation

Summarized:

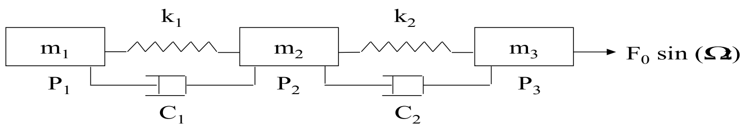
One considers the transient analysis of a discrete system masses/linear spring with three degrees of freedom completely free. This system has a non-proportional damping. A sinewave excitation is applied at an end of the system.

In this problem, one tests, through a discrete model, the computation of the transient response of a system whose rigid modes are not built-in. One is interested only in the transient regime. For that, one will seek the solution by an integration on complete modal base.

The got results (displacement, velocity and acceleration) are compared with an average of results coming from industrial codes and a numerical integration method of type β - Newmark improved.

1 Problem of reference

1.1 Geometry



1.2 Properties of the materials

Stiffness of connection: $k_1=4.10^9\,N.m^{-1}$, $k_2=5.33\,10^8\,N.m^{-1}$

Point masses: $m_1=10^6\,kg$, $m_2=m_3=12.10^6\,kg$

One-way Viscous damping: $C_1=1.2566\,10^6\,kg.s^{-1}$, $C_2=9.0478\,10^6\,kg.s^{-1}$

1.3 Boundary conditions and loadings

completely free System.

Loading at the point P_3 following the axis x : $F(t)0=F_0\sin(\Omega t)$ for $t\geq 0$ with $F_0=5.10^4\,N$ and $\Omega=19\pi\,rad.s^{-1}$.

1.4 Initial conditions

the system is at rest with $t=0$: $u(0)=0$ and $\frac{du}{dt}(0)=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

the search of the transient response of this problem to damping nonproportional, and where the rigid modes are not fixed, can be carried out by numerical integration in real space:

$$[M]\{\ddot{u}_n\}+[C]\{\dot{u}_n\}+[K]\{u_n\}=\{F\}\ .$$

For that, the response was calculated with two industrial codes:

- PERMAS: Diagram of integration of Newmark ($\alpha=0{,}25$, $\delta=0{,}5$) $\Delta t=10^{-4s}$,
  Diagram of integration with cubic interpolation of Hermit [bib1] $\Delta t=10^{-4s}$,
- ABAQUS: Diagram of integration of Hilber-Hughes-Taylor [bib2] ($\alpha=-0{,}05$) $\Delta t=10^{-4s}$,

and integration method of β - Newmark improved [bib3]:

$$\left[\frac{[M]}{\Delta t^2}+\frac{[C]}{2\Delta t}+\frac{[K]}{3}\right]\{u_{n+2}\} = \left\{\frac{\{F_{n+2}\}+\{F_{n+1}\}+\{F_n\}}{3}\right\}+\left[\frac{2[M]}{\Delta t^2}-\frac{[K]}{3}\right]\{u_{n+1}\}$$
$$+\left[-\frac{[M]}{\Delta t^2}+\frac{[C]}{2\Delta t}-\frac{[K]}{3}\right]\{u_n\}$$

where n $n+1$, $n+2$ the computations carried out at times indicate respectively t_n , $t_{n+1}=t_n+\Delta t$ and $t_{n+2}=t_n+2\Delta t$ where Δt is the increment of appointed time.

To start, one takes:

- u_0 et $u_{-1}=u_0-\Delta t\ \dot{u}_0$
- $F_{-1}=2F_0-F_1$

Time step adopted is $\Delta t=10^{-5s}$.

2.2 Results of reference

Displacement, velocity and acceleration of the point P_3 .

Differential of displacement enters the points P_3 and P_1 .

2.3 Uncertainty on the Average

solution of numerical solutions.

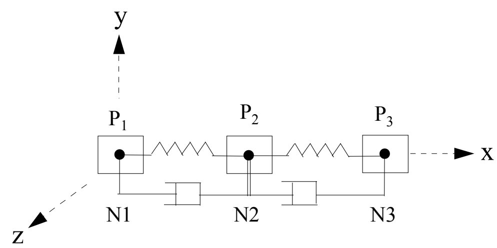
2.4 Bibliographical references

1) J.H. ARGYRIS, PC DUNE and T. ANGELOPOULOS "Non-linear oscillations using the finite technical element" comp. Meth. Appl. Mech. Engng., Vol.2, 1972, pp. 203-254
2) H.M. HILBER, T.J.R. HUGHES and R.L. TAYLOR "Improved numerical dissipation for time integration algorithms in structural dynamics" Earthquake Engineering and Structural Dynamics, Vol.5, 1977, pp. 283-292 structural
3) N.M. NEWMARK "A method of computation for dynamics" Proceeding ASCE J.Eng.Mech. Div E-3, July 1959, pp. 67-94 Modelization

3 A Characteristic

3.1 of the modelization Discrete elements

of stiffness, damping and mass. Characteristics



of the elements: DISCRET: nodal

masses	M_TR_D_N linear	stiffness
	K_TR_D_L straight-line	
	depreciations	
	A_TR_D_L No	boundary conditions

, in all the nodes: free. Names of $DX, DY, DZ, DRX, DRY, DRZ$ the nodes

: . Method of calculating $P_1 = N1$ $P_2 = N2$ $P_3 = N3$

: Integration on

modal base supplements with Newmark (,), Time step $\alpha = 0{,}25$ $\delta = 0{,}5$:
then modal $\Delta t = 10^{-4s}$ recombination. Period of observation

: . Characteristics $5s$

3.2 of the mesh Many nodes

: 3 Number of meshes
and type: 2 meshes SEG2 Quantities tested

3.3 and Displacement results of

- the point Time Displacement P_3

Difference	Displacement	()	Reference
s (	**) Aster () (%**) m 0,09	***6,7395*** m	E
6 6,73326	E-6 -0,093	0,32 1,1019	E
5 1,10002	E-6 -0,171	1,18 3,6683	E
5 3,66122	E-5 -0,193	4,92 1,6615	E
4 1,65849	E-4 -0,181	Velocity	of the point

- Time Velocity P_3

Velocity	Difference	()	Reference
s (	**) Aster () (** %) $m.s^{-1}$	***0,05 1,3425*** $m.s^{-1}$	E
4 1,34131	E-4 -0,088	0,32 -6,4111	E
- 5 -6,41097	E-4 -0,002	1,18 1,6104	E
5 1,60598	E-5 -0,274	3,55 4,4262	E
5 4,41720	E-5 -0,203	Acceleration	

- of the point Time Acceleration P_3

	Acceleration	Difference ()	Reference
(s)	**Aster () (%**) $m.s^{-2}$ 0,09	***-3,5694*** $m.s^{-2}$	E
- 3 -3,56634	E-3 -0,086	0,18 -4,3924	E
- 3 -4,38933	E-3 -0,070	0,55 4,3766	E
3 4,37283	E-3 -0,086	1,18 4,2459	E
3 4,24264	E-3 -0,077	4,92 -4,2233	E
- 3 -4,21962	E-3 -0,087	relative	Displacement

- of the point compared to P_3 the point Time Difference P_1

()	u_3-u_1	u_3-u_1	Reference
s (	**) Aster () (** %) m	***0,18 8,0987*** m	E
6 8,04800	E-6 -0,626	0,55 -6,2246	E
- 6 -6,21194	E-6 -0,203	0,82 5,3064	E
6 5,34121	E-6 0,656	1,18 -4,5552	E
- 6 -4,52071	E-6 -0,757	1,92 -3,0416	E
- 6 -3,04417	E-6 0,085	3,55 1,8448	E
6 1,82742	E-6 -0,942	4,92 1,4832	E
6 1,47526	E-6 -0,535	Remarks	Besides

3.4

the comparison for the values tested, one checks that them kinematical variables others that those related to the translation according to remain null x . Summary of

4 the results to obtain

- a good accuracy of the results, it is initially necessary to obtain a precise and perfectly orthogonal modal base (MODE_ITER_SIMULT): by avoiding

  the multiple modes (different stiffness on the nonexcited degrees of freedom), by calculating

  the modes of rigid bodies correctly (to prefer the option "Centers" in MODE_ITER_SIMULT with the other options), by specifying

  method "JACOBI" for a modal complete extraction. The accuracy

- of the results is good as well for displacements as for the velocities and the accelerations. For the elastic

response of the system (relative displacements), the numerical $u_3 - u_1$ accuracy is a little less good because of the numerical office plurality of the errors on the absolute values.