SDLD301 - Spectral seismic response of a system 2 masses and 3 springs multimedia (correlated or uncorrelated excitations)

Summarized:

The problem consists in calculating the spectral response of a system 2 masses - 3 springs subjected to a multiple seismic excitation. The excitations are considered either uncorrelated and independent, or correlated between them.

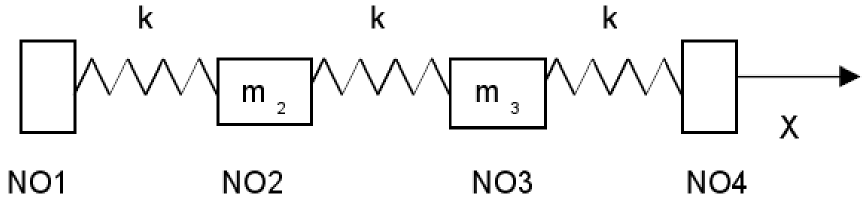
One tests the discrete element in tension, the computation of the eigen modes, the static modes and the spectral response by modal superposition via operator COMB_SISM_MODAL. Various office pluralities are tested during the computation of the responses of bearings. It is checked that, in the case of excitations equal to the bearings, computation out of mono-bearing and computation out of correlated multi-bearing provide the same one result.

The got results are in very good agreement with the analytical results of reference.

1 Problem of reference

1.1 Geometry

the structure is modelled by a set of 3 springs and of 2 point masses.



1.2 Properties of the material

- Stiffness of connection: $k_1 = k_3 = k = 100000\ N/m$; $k_2 = 2k = 200000\ N/m$
- Point mass: $m_2 = m_3 = m = 2533\,kg$.

1.3 Boundary conditions and loadings

- **boundary conditions**

only authorized displacements are the translations according to the axis x .
The points $NO1$ and $NO4$ are clamped:

$DX = DY = DZ = DRX = DRY = DRZ = 0$.

The other points are free in translation according to the direction x :

$DY = DZ = DRX = DRY = DRZ = 0$.

- **loading**

Modelization a: the structure is subjected to an uncorrelated multiple spectral excitation seismic.
The response sprectrums of oscillator in pseudo-acceleration are defined by:

- with the node is outside the field of definition with a right profile of the EXCLU type node:

$$NO1 \qquad SRO_{NO1} = \frac{a_1 \omega^2}{|\omega_1^2 - \omega^2|}$$

- with the node is outside the field of definition with a right profile of the EXCLU type node:

$$NO4 \qquad SRO_{NO4} = \frac{a_2 \omega^2}{|\omega_2^2 - \omega^2|}$$

with $\omega_1 = 2\pi f_1$ $\omega_2 = 2\pi f_2$

$f_1 = 1.5\,Hz$ $f_2 = 2.\,Hz$, $a_1 = a_2 = 0.5\,ms^{-2}$

They do not depend on damping.

Modelization b: the structure is subjected to a seismic excitation identical to the two bearings.
The response sprectrum of oscillator in pseudo-acceleration is defined by:

- with the node $NO1$ and the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$ $SRO = \dfrac{a_1 \omega^2}{|\omega_1^2 - \omega^2|}$

with $\omega_1 = 2\pi f_1$

$f_1 = 1.5\,Hz$, $a_1 = 0.5\,ms^{-2}$

It does not depend on damping.

1.4 Initial conditions

the system is initially at rest

2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the spectral response by modal superposition of a system mass-springs subjected to two distinct excitations. One determines the displacement of the masses and the reactions of bearing to the nodes $NO1$ and $NO4$ along the axis x .

One calculates analytically:

- eigenfrequencies f_i ,
- associated eigenvectors ϕ_{Ni} standardized compared to the modal mass,
- static modes of bearings ψ_j of the system,
- participation factors modal P_{ij} relating to the bearings,
- Rm_{ij} the maximum of response of each mode starting from the excitation spectrums,
- Rc_j the term of static correction.

These analytical computations are described in the file Matlab sdld301.55.

2.2 Reference variable

•**stiffness matrix** K

$$K=\begin{bmatrix} k & -k & 0 & 0 \\ -k & 3\mathrm{k} & -2\mathrm{k} & 0 \\ 0 & -2\mathrm{k} & 3\mathrm{k} & -k \\ 0 & 0 & -k & k \end{bmatrix}$$

$$K^p=\begin{bmatrix} 3\mathrm{k} & -2\mathrm{k} & -k & 0 \\ -2\mathrm{k} & 3\mathrm{k} & 0 & -k \\ -k & 0 & k & 0 \\ 0 & -k & 0 & k \end{bmatrix}$$

stamps partitionnée degrees of freedom of structure $2,3$, degrees of freedom of support $1,4$

$$K^p=\begin{bmatrix} k_{xx} & k_{xs} \\ k_{sx} & k_{ss} \end{bmatrix} \quad K_{xx}=\begin{bmatrix} 3\mathrm{k} & -2\mathrm{k} \\ -2\mathrm{k} & 3\mathrm{k} \end{bmatrix} \quad K_{xs}=\begin{bmatrix} -k & 0 \\ 0 & -k \end{bmatrix}$$

•**mass matrix** M

$$M=\begin{bmatrix}0&0&0&0\\0&m&0&0\\0&0&m&0\\0&0&0&0\end{bmatrix}$$

$$M^p=\begin{bmatrix}m&0&0&0\\0&m&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

stamps partitionnée degrees of freedom of structure $2,3$, degrees of freedom of support $1,4$

•**modal computation in clamped base**

$$K_{xx}=\begin{bmatrix}3\mathrm{k}&-2\mathrm{k}\\-2\mathrm{k}&3\mathrm{k}\end{bmatrix} \qquad m_{xx}=\begin{bmatrix}m&0\\0&m\end{bmatrix}$$

$$(k_{xx}-\lambda_i m_{xx})\phi_i=0 \quad \lambda_i=\omega_{pi}^2$$

$$\lambda_1=\frac{k}{m} \quad \lambda_2=\frac{5\mathrm{k}}{m}$$

- eigenfrequencies:

$$\Rightarrow \quad freq_1=\frac{\omega_{p1}}{2\pi}\,;\, freq_2=\frac{\omega_{p2}}{2\pi}$$

- not normalized eigen modes:

$$\bullet \quad \phi_1=\begin{pmatrix}0\\1\\1\\0\end{pmatrix} \quad \phi_2=\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

- generalized modal masses : $\mu_i={}^T\phi_i\,M\,\phi_i$

$$\bullet \quad \mu_1=2\mathrm{m} \quad \mu_2=2\mathrm{m}$$

- eigen modes normalized with the unit generalized modal mass ϕ_{Ni} :

$$\Rightarrow \quad \phi_{N1}=\frac{\phi_1}{\sqrt{\mu_1}} \quad \phi_{N2}=\frac{\phi_2}{\sqrt{\mu_2}}$$

- modal reactions Fm_i :

$$\Rightarrow \quad Fm_1 = K\,\phi_{N1} = \frac{k}{\sqrt{2\mathrm{m}}}\begin{pmatrix}1\\5\\-5\\1\end{pmatrix} \quad Fm_2 = K\,\phi_{N2} = \frac{k}{\sqrt{2\mathrm{m}}}\begin{pmatrix}1\\1\\1\\-1\end{pmatrix}$$

•**static modes of bearings** Ψ_j

Stamps static modes reduced with the degrees of freedom of static $\varphi_s = -k_{xx}^{-1}\,k_{xs}$

$$\varphi_s = -\frac{1}{5\mathrm{k}}\begin{bmatrix}3 & 2\\2 & 3\end{bmatrix}\begin{bmatrix}-k & 0\\0 & -k\end{bmatrix} = \frac{1}{5}\begin{bmatrix}3 & 2\\2 & 3\end{bmatrix}$$

•structure solution to a unit displacement of the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

displacements: $\psi_1 = \frac{1}{5}\begin{pmatrix}5\\3\\2\\0\end{pmatrix}$ nodal reactions: $Fs_1 = K\,\psi_1 = \frac{k}{5}\begin{pmatrix}-8\\0\\0\\-2\end{pmatrix}$

•static solution with a unit displacement of the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

displacements: $\psi_2 = \frac{1}{5}\begin{pmatrix}0\\2\\3\\5\end{pmatrix}$ nodal reactions: $Fs_2 = K\,\psi_2 = \frac{k}{5}\begin{pmatrix}-2\\0\\0\\-3\end{pmatrix}$

•participation factors modal out of multi-bearing : $P_{ij} = {}^T\phi_i M\,\psi_j$

•contribution of the dynamic mode 1 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

$$P_{11} = {}^T\phi_{N1} M\,\psi_1 = \frac{1}{5}\sqrt{\frac{m}{2}}$$

•contribution of the dynamic mode 1 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

$$P_{12} = {}^T\phi_{N1} M\,\psi_2 = \frac{-1}{5}\sqrt{\frac{m}{2}}$$

•contribution of the dynamic mode 2 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

$$P_{21} = {}^T\phi_{N2} M\,\psi_1 = \sqrt{\frac{m}{2}}$$

•contribution of the dynamic mode 2 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

$$P_{22} = {}^T\phi_{N2} M\,\psi_2 = \sqrt{\frac{m}{2}}$$

•participation factor of the dynamic mode 1 in the direction X:

$$P_{1\mathrm{X}} = P_{11} + P_{12}$$

•participation factor of the dynamic mode 2 in the direction X:

$$P_{2X} = P_{21} + P_{22}$$

•participation factors modal out of mono-bearing $P_i = \dfrac{\phi_{\mathrm{Ni}} M \psi_{R1}}{\mu_i}$

•contribution of the dynamic mode 1 :

$$P_1 = {}^T\phi_{N1} M \psi_{R1} = \phi_{N1} M (\psi_{s1} + \psi_{s2}) = P_{11} + P_{12}$$

•contribution of the dynamic mode 2 :

$$P_2 = {}^T\phi_{N2} M \psi_{R1} = \phi_{N2} M (\psi_{s1} + \psi_{s2}) = P_{21} + P_{22}$$

•participation factor of the dynamic mode 1 in the direction X :

$$P_{1\mathrm{X}} = P_1 + P_2$$

•response of the mode i with the motion of the bearing j out of multi-bearing

$$Rm_{ij} = r_i P_{ij} \frac{A_{ij}}{\omega_i^2} \text{ with } r_i = \phi_{\mathrm{Ni}}\, ou\, Fm_i$$

Modelization A :

$A_{11} = \dfrac{a_1 \, freq_1^2}{|f_1^2 - freq_1^2|}$: mode 1, the node is outside the field of definition with a right profile of the EXCLU type node: 1

$A_{12} = \dfrac{a_2 \, freq_1^2}{|f_2^2 - freq_1^2|}$ mode 1, the node is outside the field of definition with a right profile of the EXCLU type node: 2

$A_{21} = \dfrac{a_1 \, freq_2^2}{|f_1^2 - freq_2^2|}$ mode 2, the node is outside the field of definition with a right profile of the EXCLU type node: 1

$A_{22} = \dfrac{a_2 \, freq_2^2}{|f_2^2 - freq_2^2|}$ mode 2, node 2

Modelization B :

$A_{11} = A_{12} = \dfrac{a_1 \, freq_1^2}{|f_1^2 - freq_1^2|}$: mode 1 $\quad$ $A_{21} = A_{22} = \dfrac{a_1 \, freq_2^2}{|f_1^2 - freq_2^2|}$: response 2

•mode of the mode i out of mono-bearing

$$Rm_i = r_i P_i \frac{A_i}{\omega_i^2} \text{ with } r_i = \phi_{\mathrm{Ni}}\, ou\, Fm_i$$

combined Responses of the modal oscillators

Response of the mode 1 : $Rm_1 = \phi_{N1} P_1 \dfrac{A_1}{\omega_1^2} = Rm_{11} + Rm_{12}$

Response of the mode 2 : $Rm_2 = \phi_{N2} P_2 \dfrac{A_2}{\omega_2^2} = Rm_{21} + Rm_{22}$

•static correction

•static modes u_j solution of $k_{xs} u_{sj} = m_{xs} \phi_{sj}$:

modes ψ reduced to the degrees of freedom of structure: $\psi_{S1} = \frac{1}{5}\begin{pmatrix}3\\2\end{pmatrix}$ $\psi_{S2} = \frac{1}{5}\begin{pmatrix}2\\3\end{pmatrix}$

displacements: $u_1 = \frac{m}{25\mathrm{k}}\begin{pmatrix}0\\13\\12\\0\end{pmatrix}$ nodal reactions: $Fu_1 = \frac{m}{25}\begin{pmatrix}-13\\3\\10\\-1\end{pmatrix}$

displacements: $u_2 = \frac{m}{25\mathrm{k}}\begin{pmatrix}0\\12\\13\\0\end{pmatrix}$ nodal reactions: $Fu_2 = \frac{m}{25}\begin{pmatrix}-13\\3\\10\\-12\end{pmatrix}$

2.3 Uncertainty on the solution

No (exact analytical solution).

3 Modelization A

3.1 Characteristic of the modelization

the system is modelled by:

- 3 discrete elements `K_T_D_L`,
- 2 discrete elements `M_T_D_N`.

3.2 Characteristics of the mesh

The mesh consists of 3 meshes `SEG2`.

3.3 Quantities tested and results

3.3.1 Eigenfrequencies

MODE	Reference	Tolerance $(\%)$
1	1.000E+00	0.1
2	2.236E+00	0.1

3.3.2 static Modes for the training

Mode 1 : absolute displacements $DEPL$

NOEUD	Reference	Tolerance $(\%)$
$NO2$	0.4E+00	0.1
$NO3$	0.6E+00	0.1

Mode 2 : absolute displacements $DEPL$

NOEUD	Reference	Tolerance $(\%)$
$NO2$	0.4E+00	0.1
$NO3$	0.6E+00	0.1

3.3.3 static Modes for static correction

Mode 1 : absolute displacements $DEPL$

NOEUD	Reference	Tolerance $(\%)$
$NO2$	1.317E−02	0.1
$NO3$	1.216E−02	0.1

Mode 2 : absolute displacements $DEPL$

NOEUD	Reference	Tolerance $(\%)$
$NO2$	1.216E−02	0.1
$NO3$	1.317E−02	0.1

3.3.4 total Response on modal base supplements (computation uncorrelated multi-bearing)

the modes 1 and 2 are taken into account.

•**computation** $n°1$

COMB_MODE=' SRSS'

For each active degree of freedom 2 and 3 :

•response of the bearing $j=1$ (node $NO1$): $R_1=\sqrt{Rm_{11}^2+Rm_{21}^2}$ (office plurality on the modes 1 and 2)

•response of the bearing $j=2$ (node $NO4$): $R_2=\sqrt{Rm_{12}^2+Rm_{22}^2}$ (office plurality on the modes 1 and 2)

•total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the bearings)

absolute displacements: $DEPL$

NOEUD	Reference	Tolerance $(\%)$
$NO2$	$5.65E-03$	0.1
$NO3$	$5.65E-03$	0.1

• **computation** $n°2$

COMB_MODE=' ABS'

•response of the bearing $j=1$ (node $NO1$): $R_1=|Rm_{11}|+|Rm_{21}|$ (office plurality on the modes 1 and 2)

•response of the bearing $j=2$ (node $NO4$): $R_2=|Rm_{12}|+|Rm_{22}|$ (office plurality on the modes 1 and 2)

•total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the bearings)

absolute displacements: $DEPL$

NOEUD	Reference	Tolerance $(\%)$
$NO2$	$6.476E-03$	0.1
$NO3$	$6.476E-03$	0.1

• **computation** $n°3$

COMB_MODE=' DPC'

•response of the bearing $j=1$ (node $NO1$): $R_1=\sqrt{Rm_{11}^2+Rm_{21}^2}$ (office plurality on the modes 1 and 2)

•response of the bearing $j=2$ (node $NO4$): $R_2=\sqrt{Rm_{12}^2+Rm_{22}^2}$ (office plurality on the modes 1 and 2)

•total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the bearings)

absolute displacements: $DEPL$

NOEUD	Reference	*Tolerance* $(\%)$
$NO2$	$5.65E-03$	0.1
$NO3$	$5.65E-03$	0.1

- **computation** $n°4$

  COMB_MODE=' CQC'

  modal dampings = 0.05

•response of the bearing $j=1$ (node $NO1$): $R_1=\sqrt{\rho_{12} Rm_{11} Rm_{21}}$ (office plurality on the modes 1 and 2)

•response of the bearing $j=2$ (node $NO4$): $R_2=\sqrt{\rho_{12} Rm_{12} Rm_{22}}$ (office plurality on the modes 1 and 2)

•total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the bearings)

absolute displacements: $DEPL$

NOEUD	Reference	*Tolerance* $(\%)$
$NO2$	$5.65E-03$	0.1
$NO3$	$5.65157E-03$	0.1

- **computation** $n°5$

  COMB_MODE=' DSC'

  modal dampings = 0.05
  lasted: 15 S

•response of the bearing $j=1$ (node $NO1$): $R_1=\sqrt{\rho_{12} Rm_{11} Rm_{21}}$ (office plurality on the modes 1 and 2)

•response of the bearing $j=2$ (node $NO4$): $R_2=\sqrt{\rho_{12} Rm_{12} Rm_{22}}$ (office plurality on the modes 1 and 2)

•total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the bearings)

absolute displacements: $DEPL$

NOEUD	Reference	*Tolerance* $(\%)$
$NO2$	$5.649E-03$	0.1
$NO3$	$5.6521E-03$	0.1

4 Modelization B

4.1 Characteristic of the modelization B

the system is modelled by:

- 3 discrete elements K_T_D_L,
- 2 discrete elements M_T_D_N.

4.2 Characteristics of the mesh

The mesh consists of 3 meshes SEG2.

4.3 Quantities tested and results

4.3.1 Eigenfrequencies

MODE	Reference	*Tolerance* (%)
1	1.000E+00	0.1
2	2.236E+00	0.1

4.3.2 static Modes for the training

Mode 1 : absolute displacements $DEPL$

NOEUD	Reference	*Tolerance* (%)
NO2	0.6E+00	0.1
NO3	0.4E+00	0.1

Mode 2 : absolute displacements $DEPL$

NOEUD	Reference	*Tolerance* (%)
NO2	0.6E+00	0.1
NO3	0.4E+00	0.1

4.3.3 static Modes for static correction

Mode 1 : absolute displacements $DEPL$

NOEUD	Reference	*Tolerance* (%)
NO2	1.317E−02	0.1
NO3	1.216E−02	0.1

Mode 2 : absolute displacements $DEPL$

NOEUD	Reference	*Tolerance* (%)
NO2	1.216E−02	0.1
NO3	1.317E−02	0.1

4.3.4 total Response on modal base supplements

4.3.4.1 Computation mono-bearing

the modes 1 and 2 are taken into account.

•**computation** $n°1$

```
COMB_MODE=' SRSS'
```

For each ddl credit 2 and 3 :

•response of the mode 1 : $R_1=Rm_{11}+Rm_{12}$
•response of the mode 2 : $R_2=Rm_{21}+Rm_{22}$
•total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the modes)

absolute displacements: $DEPL$

NOEUD	**Reference**	***Tolerance*** $(\%)$
NO2	1.01321E−02	0.1
NO3	1.01321E−02	0.1

- **computation** $n°2$

```
COMB_MODE=' ABS'
```

•response of the mode 1 : $R_1=Rm_{11}+Rm_{12}$
•response of the mode 2 : $R_2=Rm_{21}+Rm_{22}$ R
•total response: $R=|R_1|+|R_2|$ (office plurality on the modes)

absolute displacements: $DEPL$

NOEUD	**Reference**	***Tolerance*** $(\%)$
NO2	0.01013	0.1
NO3	0.01013	0.1

- **computation** $n°3$

```
COMB_MODE=' DPC'
```

•response of the mode 1 : $R_1=Rm_{11}+Rm_{12}$
•response of the mode 2 : $R_2=Rm_{21}+Rm_{22}$
•total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the modes)

absolute displacements: $DEPL$

NOEUD	**Reference**	***Tolerance*** $(\%)$
NO2	0.01013	0.1
NO3	0.01013	0.1

- **computation** $n°4$

  COMB_MODE=' CQC'

  modal dampings = 0.05

•response of the mode 1 : $R_1 = Rm_{11} + Rm_{12}$
•response of the mode 2 : $R_2 = Rm_{21} + Rm_{22}$

•total response: $R = \sqrt{\rho_{12} R_1 R_2}$ (office plurality on the modes)

absolute displacements: $DEPL$

NOEUD	**Reference**	***Tolerance*** $(\%)$
NO2	0.01013	0.1
NO3	0.01013	0.1

- **computation** $n°5$

  COMB_MODE=' DSC'

  modal dampings = 0.05
  lasted: 15 second

•response of the mode 1 : $R_1 = Rm_{11} + Rm_{12}$
•response of the mode 2 : $R_2 = Rm_{21} + Rm_{22}$
•total response: $R = \sqrt{\rho_{12} R_1 R_2}$ (office plurality on the modes)

absolute displacements: $DEPL$

NOEUD	**Reference**	***Tolerance*** $(\%)$
NO2	0.01013	0.1
NO3	0.01013	0.1

4.3.4.2 Computation multi-bearing correlated

•**computation** $n°6$

COMB_MODE=' SRSS'

For each active degree of freedom 2 and 3 :

•response of the mode 1 : $R_1 = Rm_{11} + Rm_{12}$ (office plurality on the bearings)
•response of the mode 2 : $R_2 = Rm_{21} + Rm_{22}$ (office plurality on the bearings)
•total response: $R = \sqrt{R_1^2 + R_2^2}$ (office plurality on the modes)

absolute displacements: $DEPL$

NOEUD	Reference	*Tolerance* $(\%)$
$NO2$	$1.01321E-02$	0.1
$NO3$	$1.01321E-02$	0.1

4.3.5 total Response on incomplete modal base (computation mono-bearing with static correction)

Modal base made up of mode 2 only.

•**computation** $n°7$

```
COMB_MODE=' ABS'
```

For each active degree of freedom 2 and 3 :

•response of the mode $i=2$ (node $NO4$): $R_2=Rm_{21}+Rm_{22}$ (office plurality on the bearings 1 and 2)
•total response: $R=\sqrt{R_2^2+U^2}$ (office plurality modal response and static correction)

absolute displacements: $DEPL$

NOEUD	Reference	*Tolerance*
$NO2$	0.02302302705	0.001
$NO3$	0.02302302705	0.001

- **computation** $n°8$

```
COMB_MODE=' SRSS'
```

For each active degree of freedom 2 and 3 :

•response of the mode $i=2$ (node $NO4$): $R_2=Rm_{21}+Rm_{22}$ (office plurality on the bearings 1 and 2)
•total response: $R=\sqrt{R_2^2+U^2}$ (office plurality modal response and static correction)

absolute displacements: $DEPL$

NOEUD	Reference	*Tolerance*
$NO2$	0.02302302705	0.001
$NO3$	0.02302302705	0.001

- **computation** $n°9$

```
COMB_MODE=' DPC'
```

For each active degree of freedom 2 and 3 :

•response of the mode $i=2$ (node $NO4$): $R_2=Rm_{21}+Rm_{22}$ (office plurality on the bearings 1 and 2)
•total response: $R=\sqrt{R_2^2+U^2}$ (office plurality modal response and static correction)

absolute displacements: $DEPL$

NOEUD	Reference	Tolerance
$NO2$	0.02302302705	0.001
$NO3$	0.02302302705	0.001

- **computation** $n°10$

```
COMB_MODE=' CQC'
```

modal dampings = 0.05

For each active degree of freedom 2 and 3 :

•response of the mode $i=2$ (node $NO4$): $R_2=Rm_{21}+Rm_{22}$ (office plurality on the bearings 1 and 2)
•total response: $R=\sqrt{R_2^2+U^2}$ (office plurality modal response and static correction)

absolute displacements: $DEPL$

NOEUD	Reference	Tolerance
$NO2$	0.02302302705	0.001
$NO3$	0.02302302705	0.001

- **computation** $n°11$

```
COMB_MODE=' DSC'
```

For each active degree of freedom 2 and 3 :

•response of the mode $i=2$ (node $NO4$): $R_2=Rm_{21}+Rm_{22}$ (office plurality on the bearings 1 and 2)
•total response: $R=\sqrt{R_2^2+U^2}$ (office plurality modal response and static correction)

absolute displacements: $DEPL$

NOEUD	Reference	Tolerance
$NO2$	0.02302302705	0.001
$NO3$	0.02302302705	0.001

5 Summary of the results

the results got with *Code_Aster* are in conformity with the analytical results of reference.