SDLD106 – System masses spring with damping under harmonic vibration

Summarized

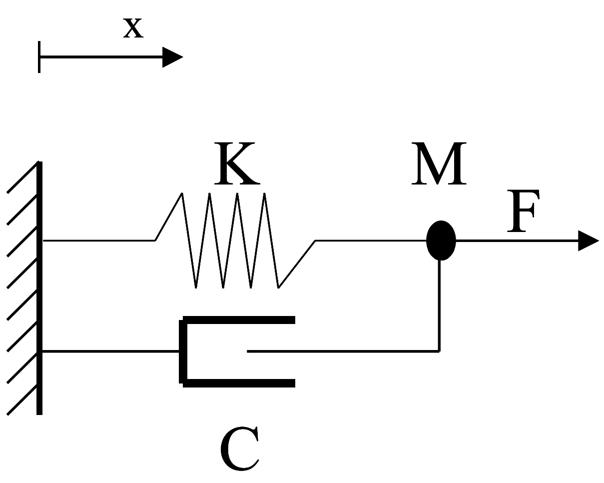
This case test makes it possible to validate the elementary matrix of a discrete element. One calculates the eigenfrequency of an oscillator to a degree of freedom (spring-mass system), the transient response due to a sinewave excitation and the forced response due to a harmonic excitation.

The results of reference are got analytically.

1 Problem of reference

1.1 Geometry

the diagram of the system is presented on the following figure:



1.2 Properties of the material

the properties of the material are the following ones:

stiffness $K:4\pi^2 N/m$
masses $M:100\,kg$
damping $C=0.4\pi\,Ns/m$

1.3 Boundary conditions and loadings

spring is embedded with the one of its ends and subjected to a force F at the loose lead. The point mass can move only according to the direction x .

1.4 Initial conditions

At initial time, the mass is motionless and it is with its equilibrium position.

$x(0)=0\,m$
$v(0)=0\,m/s$

2 Reference solution

2.1 Method of calculating

One proposes to calculate the eigenfrequency of the oscillator, the response due to a sinewave excitation and the response due to a harmonic excitation.

2.1.1 Computation of the eigenfrequency

For the validation of the computation of the eigenfrequency, we consider the system without damping. Thus, the displacement of the loose lead of spring is governed by the following relation:

$$M\ddot{x}+Kx=F \qquad (1)$$

the eigenfrequency of this oscillator is:

$$f_0=\frac{1}{2\pi}\sqrt{\frac{K}{M}}=\frac{\omega_0}{2\pi}$$ (f_0 : eigenfrequency ω_0 : own pulsation)

2.1.2 Computation of the transient response

For the validation of the computation of the transient response, we consider the system without damping.
With the initial condition $x(0)=0$ and $\dot{x}(0)=0$, if a sinusoidal force is applied $F(t)=F\sin(\Omega t)$, at the loose lead of spring, the solution of the differential equation (1) is:

$$x(t)=\frac{F\left(\sin\Omega t-\frac{\Omega}{\omega_0}\sin\omega_0 t\right)}{M(\omega_0^2-\Omega^2)}$$

For this computation of response to a sinewave excitation, we chose: $\Omega=1\,rd/s$ and $F=1\,N$.

2.1.3 Computation of the harmonic response

One then proposes to calculate the response of the damped oscillator due to a harmonic excitation.

The displacement of the loose lead of spring is governed by the following relation:

$$M\ddot{x}+C\dot{x}+Kx=F \qquad (2)$$

By applying a sinusoidal force $F(t)=F\sin(\Omega t)$, at the loose lead of spring, and by adopting the complex notation, one obtains the forced response: $\hat{x}(\Omega)=\frac{F}{K-\Omega^2 M+jC}$

For this computation of the harmonic forced response, we chose: $0.5\,\omega_0\leq\Omega\leq1.5\,\omega_0$

3 Modelization A

3.1 Characteristic of the modelization

the oscillator is modelled using elements `2D_DIS_TR`.

3.2 Characteristics of the mesh

Number and type of meshes: 1 element of type `SEG2` and element of type `a POI1.`

3.3 Quantities tested and Eigenfrequency

results:

Quantity tested	Reference	Tolerance
$f_0=\frac{\omega_0}{2\pi}$	$0.1\,Hz$	1.E-4

sinusoidal Response at time $t_1=1\,s$:

Quantity tested	Reference	harmonic
$x(t_1=1\,s)$	$1.55346\,10^{-3}\,m$	Tolerance

5.E-4 Response with the frequency $\Omega_1=1.5\,\omega_0$:

Quantity tested	Reference	Tolerance
$\hat{x}(\Omega)=1.5\omega_0$	$-2.022511\,10^{-2}\ \ -5.15690\,10^{-4}\,i\,m$	5.E-3

the three tests are doubled tests of NON-regression with a tolerance of 1.E-6.

4 Summary of the results

the got results are in concord with the theoretical solution.