SDLD105 - Transient response of a system mass-springs with a seisme with static correction

Summarized

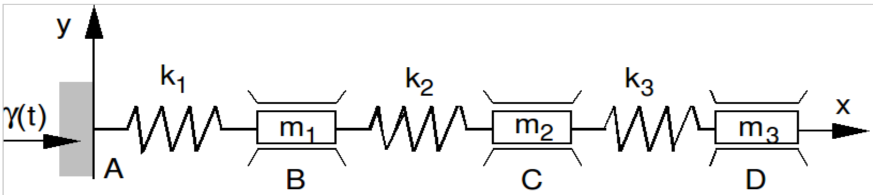
This case test, inspired by the case test VPCS SDLD04, consists in calculating the undamped transient response of a embed-free system mass-springs linear subjected to an imposed acceleration.

Its purpose is to show an example of static correction *a posteriori* and the interest of this correction.

1 Problem of reference

1.1 Geometry

One calculates the response of a linear system composed of three masses and three springs with an acceleration imposed on the level of its point of anchorage (A) :



1.2 Properties of the materials

- stiffness of connection: $k_1=1000\,N/m$ and $k_2=k_3=100\,N/m$;
- point masses: $m=m_1=m_2=m_3=1\,kg$.
- modal damping of $5\,\%$ for all Boundary conditions

1.3 modes and loadings

Boundary conditions

only authorized displacements are the translations according to the axis x .
The point A is clamped: $dx=dy=dz=drx=dry=drz=0$.

Loading

the point of anchorage A is subjected to a harmonic acceleration of frequency f_{ex}. The computation is made of 0 with $20\,s$.

1.4 Initial conditions

the system is initially at rest: at $t=0$, $dx(0)=0$ and $dx/dt(0)=0$ in any point.

2 Reference solution

the purpose of the case test is to show the truncation effect modal on a very simple example of seismic computation and to illustrate the interest of static correction *a posteriori*. The reference solution is thus the direct transient computation carried out with operator DYNA_VIBRA .

The system mass-springs has three degrees of freedom. It is thus associated with three modes of which the calculated frequencies are:
$f_1=0{,}946\,Hz$, $f_2=2{,}533\,Hz$ and $f_3=5{,}305\,Hz$.

The excitation frequency of the harmonic seismic signal was selected with $f_{ex}=2\,Hz$ to keep in modal base only the first two modes. One complies with the rule to twice retain the modes until the maximum frequency of excitation.

The table of the unit effective modal masses gives interesting information:

Effective mass unit			
NUME_MODE	FREQUENCY	MASS_EFFE_UN_DX	CUMUL_DX
1	9.48538E-01	6.82972E-01	6.82972E-01
2	2.53344E+00	5.03369E-02	7.33309E-01
3	5.30513E+00	2.66691E-01	1.00000E+00

One observes that the third mode will have a negligible dynamic response because its eigenfrequency is worth $f_3=5{,}305\,Hz$, beyond $2\times f_{ex}=4{,}0\,Hz$. On the other hand its unit effective modal mass in the direction x is worth $26{,}7\,\%$. It is thus not negligible and this mode can affect the response of the system mass-springs by its quasi-static contribution. It is the goal of the static correction which to take it into account.

If one looks at now the geometry of mode 3. It is noted that it is mainly on the node $N02$ that one will be able to observe the effect of the mode and thus the effect of static correction.

	Mode 1	Mode 2	Mode 3
Node	DX	DX	DX
$N01$ (not A)	0.00000E+00	0.00000E+00	0.00000E+00
$N02$ (not B)	5.08430E-02	9.84653E-02	9.93841E-01
$N03$ (not C)	5.41213E-01	8.33623E-01	-1.10279E-01
$N04$ (not D)	8.39347E-01	-5.43487E-01	1.09069E-02

Indeed the table, upon reading of the components modal, it appears that the component of the mode 3 on all the nodes is minority compared to the other modes, except for the node $N02$.

2.1 Results of reference

One takes for results of reference the results given by DYNA_VIBRA with complete modal base to the nodes $N02$ and $N04$ time $t=19{,}4\,s$.

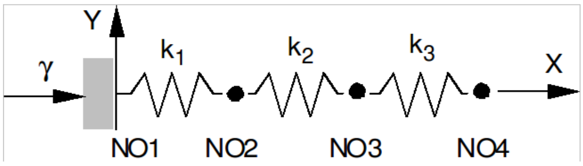
2.2 Uncertainty on the solution

Accuracy on integration in time in DYNA_VIBRA

3 Modelization A

3.1 Characteristic of the modelization

springs and point masses are modelled by discrete elements with 3 degrees of freedom DIS_T :



The node $NO1$ is embedded and subjected to an imposed acceleration $\gamma(t)$.

3.2 Characteristics of the mesh

Many nodes: 4
Number of meshes and types: 3 DIS_T

3.3 Quantities tested and Eigenfrequencies

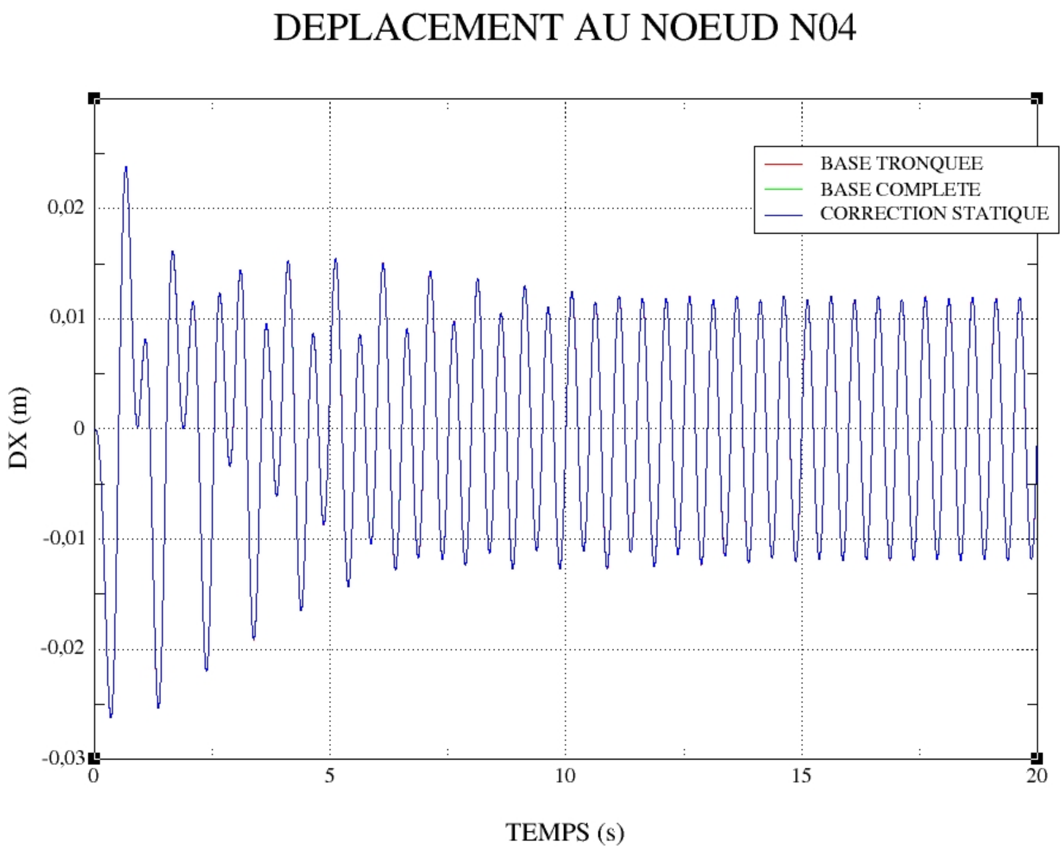
results (in Hz) of the system:

Number of the mode	*Code_Aster*
1	0,94853
2	2,53344
3	5,30513

Transient computation by modal synthesis
One tests the taking into account of a loading in the form of vector project on modal base, modal component, vector project and of modal component simultaneously as well as the taking into account of the modes neglected by static correction.

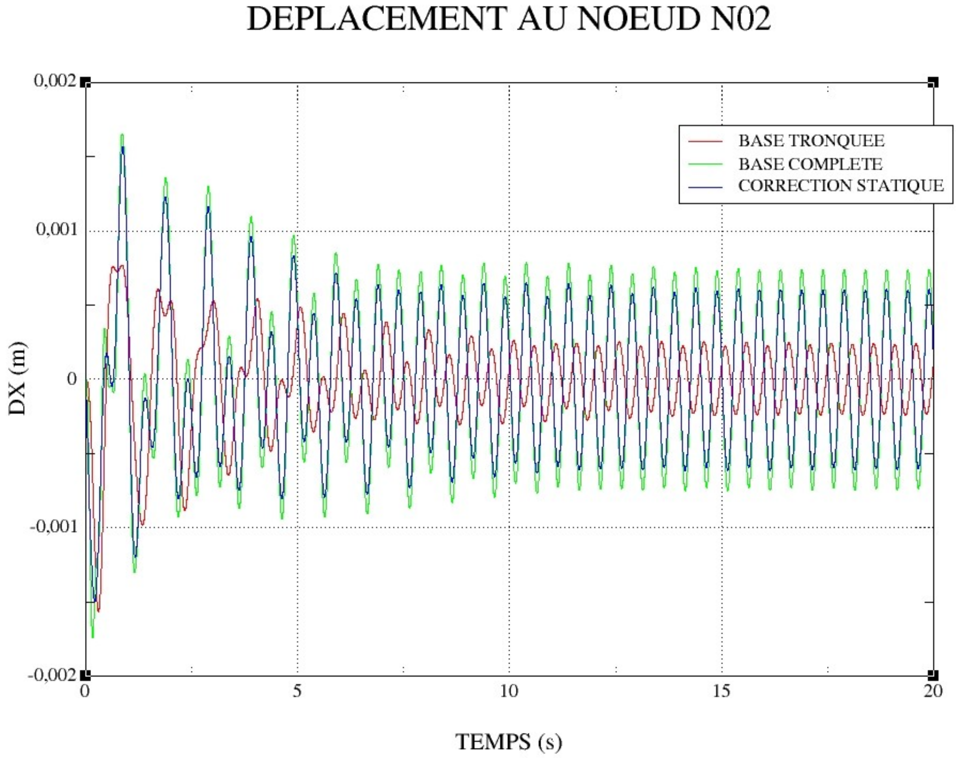
Values of the displacement relative of the node $NO4$ to time $t=19{,}4\,s$:

modal base	Reference
bases truncated	-0.011301
base supplements	-0.011301
static correction	-0.011301



Values of the displacement relative of the node $NO2$ to time $t=19,4\,s$:

modal base	Reference	Computation	Tolerance	Error
bases truncated	7.331625E-04	-1.647491E-04	122.5%	150%
bases supplements	7.331625E-04	7.331624852E-04	0.0%	1.0E-06%
static correction	7.331625E-04	5.961668E-04	18.7%	20.0%



One illustrated Ci - above the interest of static correction: as envisaged with the reading of the modal deformed shapes static correction is not visible for $NO4$ but plays an important role for $NO2$. Without static correction with $NO2$ displacement is out of phase and its amplitude undervalued of 50%. With static correction the error remains visible (lower than 20%) but computation remains realistic. On a less caricatural computation the error will be less sensitive. It is also noticed that the amplitude of displacement to the node $NO2$ is of two orders smaller than that with the node $NO4$.

4 Summary of the results

the case test is an example of implementation of static correction a posteriori. On a system especially calculated to show its effects, it shows that static correction can reduce in an important way the error due to the truncation effect modal on the “high frequencies”. On an industrial study one can expect a less visible effect.