SDLD104 - Extrapolation of local measurements on a complete model (discrete)

Summarized:

It is about a test of linear dynamics discrete.

The goal is to test command PROJ_MESU_MODAL in the case of a discrete system. This command makes it possible to project experimental dynamic transient responses in a certain number of points on a modal base of a numerical modelization.

This test contains 2 modelizations where projection is done on a concept of the type [mode_meca], the difference being given by the way in which this concept is manufactured.

For the 2 modelizations, provided experimental measurements are identical and make it possible to test the search of the nodes in opposite, the taking into account of a local directional sense and the processing of a sampling in constant time or not, for measurements in displacement.

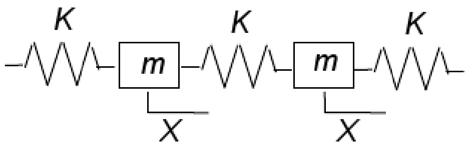
In both cases, the reference solution is analytically given (by Maple); projection is carried out in the favorable configuration where the number of modes is equal to the number of measurements.
The responses in displacement obtained after projection are identical to the displacements of reference provided in data.

The values velocities and the accelerations deduced from the displacements obtained after projection are close to those obtained analytically. The weak noted variations are due to the errors of approximation generated by the determination via a linear diagram in time of the velocities and accelerations.

1 Problem of reference

1.1 Description of the system

We consider the system represented by the diagram below:



1.2 Masses and stiffness

three springs are of identical stiffness: $k=1000\,N/m$.
The two masses are equal to $m=10\,kg$.

1.3 Boundary conditions and loading

the two ends are clamped.
The loading is a thrust load in tension applied to the mass $m1$, sinusoidal according to time, of pulsation ω .

2 Reference solutions

2.1 Method of calculating used for the reference solution

the analytical solution of this problem is presented below.

Modes and frequencies of vibration:
The following system characterizes the dynamics of the masses:

$$\begin{cases} m\,\ddot{x}_1 + 2\mathrm{k}\,x_1 - k\,x_2 = 0 \\ m\,\ddot{x}_2 + 2\mathrm{k}\,x_2 - k\,x_1 = 0 \end{cases}$$ éq 2.1-1

What is equivalent to the following system:

$$\begin{cases} m(\ddot{x}_1 + \ddot{x}_2) + k(x_1 + x_2) = 0 \\ m(\ddot{x}_1 - \ddot{x}_2) + 3\mathrm{k}(x_1 - x_2) = 0 \end{cases}$$ éq the 2.1-2

2 eigenfrequencies of the system are thus given by:

$$\omega_1 = \sqrt{\frac{k}{m}} \quad \text{and} \quad \omega_2 = \sqrt{\frac{3\mathrm{k}}{m}}$$ éq 2.1-3

and the associated modal strains are:

$$\Phi_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \Phi_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ éq the 2.1-4

generalized matrixes are:

$$\bar{M} = \Phi^T M \Phi = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2\mathrm{m} & 0 \\ 0 & 2\mathrm{m} \end{pmatrix}$$
$$\bar{K} = \Phi^T K \Phi = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2\mathrm{k} & -k \\ -k & 2\mathrm{k} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2\mathrm{k} & 0 \\ 0 & 6\mathrm{k} \end{pmatrix}$$ éq 2.1-5

Transient response:

The sinusoidal force is applied to the first mass: $F = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(\omega t)$

The checked dynamic system is the following:

$$M\ddot{X}+KX=F \qquad \text{éq 2.1-6}$$

While projecting on the basis of eigen mode, we obtain:

$$\Phi^T M\Phi\ddot{\eta}+\Phi^T K\Phi\eta=\Phi^T F \qquad \text{éq 2.1-7}$$

Is:

$$\begin{pmatrix}2\mathrm{m} & 0\\ 0 & 2\mathrm{m}\end{pmatrix}\begin{pmatrix}\ddot{\eta}_1\\ \ddot{\eta}_2\end{pmatrix}+\begin{pmatrix}2\mathrm{k} & 0\\ 0 & 6\mathrm{k}\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}\sin(\omega t) \qquad \text{éq 2.1-8}$$

We thus end to the following decoupled system:

$$\begin{cases} m\ddot{\eta}_1+k\eta_1=\dfrac{1}{2}\sin(\omega t)\\ m\ddot{\eta}_2+3k\eta_2=\dfrac{1}{2}\sin(\omega t)\end{cases} \qquad \text{éq 2.1-9}$$

the solution of this system is given by:

$$\begin{cases} \eta_1(t)=A_1\cos(\omega_1 t)+B_1\sin(\omega_1 t)+\dfrac{\sin(\omega t)}{2\mathrm{m}(\omega_1^2-\omega^2)}\\ \eta_2(t)=A_2\cos(\omega_2 t)+B_2\sin(\omega_2 t)+\dfrac{\sin(\omega t)}{2\mathrm{m}(\omega_2^2-\omega^2)}\end{cases} \qquad \text{éq 2.1-10}$$

displacements in physical space are obtained by the formula of Ritz:

$$X=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\Phi\eta=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}=\begin{pmatrix}\eta_1+\eta_2\\ \eta_1-\eta_2\end{pmatrix} \qquad \text{éq 2.1-11}$$

One from of deduced the statements from $x_1(t)$ and $x_2(t)$:

$$\begin{cases} x_1(t)=A_1\cos(\omega_1 t)+B_1\sin(\omega_1 t)+A_2\cos(\omega_2 t)+B_2\sin(\omega_2 t)+\dfrac{\sin(\omega t)}{2\mathrm{m}}\left(\dfrac{1}{\omega_1^2-\omega^2}+\dfrac{1}{\omega_2^2-\omega^2}\right)\\ x_2(t)=A_1\cos(\omega_1 t)+B_1\sin(\omega_1 t)-A_2\cos(\omega_2 t)+B_2\sin(\omega_2 t)+\dfrac{\sin(\omega t)}{2\mathrm{m}}\left(\dfrac{1}{\omega_1^2-\omega^2}-\dfrac{1}{\omega_2^2-\omega^2}\right)\end{cases}$$

éq 2.1-12

At initial time, the system is at rest, from where final statements of $x_1(t)$ and $x_2(t)$:

$$\begin{cases} x_1(t)=\dfrac{1}{2\text{m}}\left[\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_1}\sin(\omega_1 t)}{\omega_1^2-\omega^2}+\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_2}\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \\ x_2(t)=\dfrac{1}{2\text{m}}\left[\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_1}\sin(\omega_1 t)}{\omega_1^2-\omega^2}-\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_2}\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \end{cases}$$ éq the 2.1-13

velocities of the two masses are calculated by deriving displacements compared to time:

$$\begin{cases} \dot{x}_1(t)=\dfrac{\omega}{2\text{m}}\left[\dfrac{\cos(\omega t)-\cos(\omega_1 t)}{\omega_1^2-\omega^2}+\dfrac{\cos(\omega t)-\cos(\omega_2 t)}{\omega_2^2-\omega^2}\right] \\ \dot{x}_2(t)=\dfrac{\omega}{2\text{m}}\left[\dfrac{\cos(\omega t)-\cos(\omega_1 t)}{\omega_1^2-\omega^2}-\dfrac{\cos(\omega t)-\cos(\omega_2 t)}{\omega_2^2-\omega^2}\right] \end{cases}$$ éq 2.1-14

accelerations of the two masses are calculated by deriving the velocities compared to time:

$$\begin{cases} \ddot{x}_1(t)=\dfrac{\omega}{2\text{m}}\left[\dfrac{\omega\sin(\omega t)-\omega_1\sin(\omega_1 t)}{\omega_1^2-\omega^2}+\dfrac{\omega\sin(\omega t)-\omega_2\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \\ \ddot{x}_2(t)=\dfrac{\omega}{2\text{m}}\left[\dfrac{\omega\sin(\omega t)-\omega_1\sin(\omega_1 t)}{\omega_1^2-\omega^2}-\dfrac{\omega\sin(\omega t)-\omega_2\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \end{cases}$$ éq 2.1-15

2.2 Results of reference

the comparison of the results relates to displacements, velocities and accelerations along the axis of the two masses, at five different times.

2.3 Uncertainty on the solution

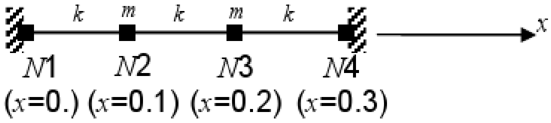
the reference solution is exact.

The model discrete represents perfectly the problem arising (modal base is complete; there is thus no approximation related to a possible modal truncation). Number of modes of the base of modal projection is equal to the number of measurements, therefore the solution of the inversion is exact (in opposition to an approximate solution of generalized inverse problems). If the search of the nodes in opposite is good, the displacements obtained after projection must be in perfect adequacy with the experimental values. The velocities and accelerations are determined by derivative of the modal contributions identified via a diagram of linear approximation in time, thus being able to generate some errors.

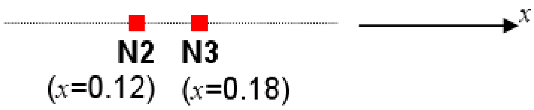
3 Modelization A

3.1 Characteristic of the modelization and the meshes

numerical Mesh:
The mesh numerical is carried out directly with the Aster format . It comprises 4 nodes and 3 meshes discrete.



Experimental mesh:
The mesh of measurement understands only 2 point elements and 2 nodes:



3.2 Characteristics of measurements

provided experimental measurements are:

- With the node is outside the field of definition with a right profile of the EXCLU type node: $N3$
  The data are the displacements axial, multiplied by $-1/\sqrt{2}$, and applied in the direction $-X$. The local directional sense indicated in the command file is $(45.0.0.)$
  the sampling of time is constant: initial time is $0\,s$, time step is $10^{-3}\,s$ and the number of times is 1001 (either until a final time of $1\,s$).

- With the node is outside the field of definition with a right profile of the EXCLU type node: $N2$
  The data are the axial displacements, applied in the direction x.
  The sampling of time is variable: all times are indicated of $0\,s$ to $1\,s$, by step of $10^{-3}\,s$ (1001 times on the whole).

  The values result from the analytical computation carried out with Maple.

3.3 Characteristics of modal base

the two only modes are stored in a concept of the mode_meca `type` created by the command `MODE_ITER_SIMULT`. Their eigenfrequencies are identical to the analytical eigenfrequencies.

3.4 Values tested

Identification		Reference	*Code_Aster*	difference
	to T = 0.1 S	1.745 10-4	1.745 10-4	0.01%
	with T = 0.3 S	6.797 10-4	6.797 10-4	0.01%

DEPL_X (m)	with the node $N2$ (mass 1)	with T = 0.5 S	– 1.217 10-3	– 1.217 10-3	0.01%
		with T = 0.7 S	5.214 10-4	5.214 10-4	– 0.01%
		with T = 0.9 S	9.031 10-4	9.031 10-4	0.00%
DEPL_X (m)	with the node $N3$ (mass 2)	with T = 0.1 S	9.154 10-6	9.154 10-6	0.00%
		with T = 0.3 S	6.414 10-4	6.414 10-4	0.00%
		with T = 0.5 S	– 8.636 10-4	– 8.636 10-4	0.00%
		with T = 0.7 S	– 1.107 10-4	– 1.107 10-4	0.03%
		with T = 0.9 S	1.633 10-3	1.633 10-3	0.02%
VITE_X (m/s)	with the node $N2$ (mass 1)	with T = 0.1 S	4.586 10-3	4.616 10-3	0.65%
		with T = 0.3 S	– 7.598 10-3	– 7.663 10-3	0.85%
		with T = 0.5 S	– 1.581 10-4	– 8.000 10-5	7.81 10-5 m/s
		with T = 0.7 S	9.382 10-3	9.354 10-3	– 0.30%
		with T = 0.9 S	– 7.481 10-3	– 7.537 10-3	0.75%
VITE_X (m/s)	with the node $N3$ (mass 2)	with T = 0.1 S	4.328 10-4	4.405 10-4	1.79%
		with T = 0.3 S	3.671 10-3	3.640 10-3	– 0.84%
		with T = 0.5 S	– 1.539 10-2	– 1.536 10-2	– 0.20%
		with T = 0.7 S	2.453 10-2	2.457 10-2	0.15%
		with T = 0.9 S	– 1.899 10-2	– 1.912 10-2	0.68%
ACCE_X (m/s^2)	with the node $N2$ (mass 1)	with T = 0.1 S	6.112 10-2	6.100 10-2	– 0.20%
		with T = 0.3 S	– 1.306 10-1	– 1.300 10-1	– 0.46%
		with T = 0.5 S	1.571 10-1	1.600 10-1	1.85%
		with T = 0.7 S	– 5.657 10-2	– 5.800 10-2	2.53%
		with T = 0.9 S	– 1.124 10-1	– 1.130 10-1	0.53%
ACCE_X (m/s^2)	with the node $N3$ (mass 2)	with T = 0.1 S	1.562 10-2	1.618 10-2	3.58%
		with T = 0.3 S	– 6.031 10-2	– 6.223 10-2	3.18%
		with T = 0.5 S	5.102 10-2	5.374 10-2	5.33%
		with T = 0.7 S	7.428 10-2	7.043 10-2	– 5.19%

with T = 0.9 S	– 2.364 10-1	– 2.263 10-1	– 4.28%

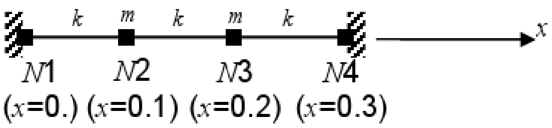
Note::

The velocity with the node $N2$ at time $t=0.5s$ being relatively close to zero, the comparison is produced for this case in absolute value.

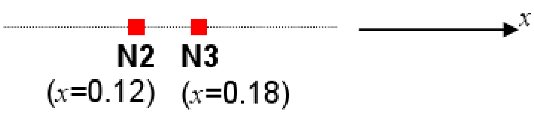
4 Modelization B

4.1 Characteristic of the modelization and the meshes

numerical Mesh:
The mesh numerical is carried out directly with the Aster format . It comprises 4 nodes and 3 meshes discrete.



Experimental mesh:
The mesh of measurement understands only 2 point elements and 2 nodes:



4.2 Characteristic of measurements

provided experimental measurements are:

- With the node is outside the field of definition with a right profile of the EXCLU type node: $N3$

  The data are the displacements axial, multiplied by $-1/\sqrt{2}$, and applied in the direction $-x$. The local directional sense indicated in the command file is $(45.0.0.)$
  the sampling of time is constant: initial time is $0\,s$, time step is $10^{-3}\,s$ and the number of times is 1001 (i.e until a final time of). $1\,s$ With

- the node is outside the field of definition with a right profile of the EXCLU type node: $N2$

  The data are the axial displacements, applied in the direction. X
  The sampling of time is variable: all times are indicated of to $0\,s$, $1\,s$ by step of (1001 $10^{-3}\,s$ times on the whole).

  The values result from the analytical computation carried out with Maple. Characteristics

4.3 of modal base

the two only modes are stored in a concept of type _meca `mode, created` by the command DEFI `_BASE_MODALE.` L'interface, of Craig-Bampton type, is placed on the degree of freedom in displacement following of x the node (corresponding $N2$ to the mass). $m1$ Modal base thus contains a dynamic mode (with blocked $N2$) and a static mode. Values

4.4 tested Identification

Reference		Code_Aster	difference	to
	T = 0.1 S	10-4 1.745	10-4 0.01%	with

		1.745			
		T = 0.3 S 6.797	10-4 6.797	10-4 0.01%	DEPL
_X with (m mass	the node with $N2$ 1) with	T = 0.5 S –	1.217 10-3 –	1.217 10-3 0.01%	()
		T = 0.7 S 5.214	10-4 5.214	10-4 –	0.01% with
		T = 0.9 S 9.031	10-4 9.031	10-4 0.00%	with
		T = 0.1 S 9.154	10-6 9.154	10-6 0.00%	with
		T = 0.3 S 6.414	10-4 6.414	10-4 0.00%	DEPL
_X with (m mass	the node with $N3$ 2) with	T = 0.5 S –	8.636 10-4 –	8.636 10-4 0.00%	()
		T = 0.7 S –	1.107 10-4 –	1.107 10-4 0.03%	with
		T = 0.9 S 1.633	10-3 1.633	10-3 0.02%	with
		T = 0.1 S 4.586	10-3 4.616	10-3 0.65%	with
		T = 0.3 S –	7.598 10-3 –	7.663 10-3 0.85%	VITE
_X with (m/s mas s	the node with $N2$ 1) with	T = 0.5 S –	1.581 10-4 –	8.000 10-5 7.81	10-5 m/s ()
		T = 0.7 S 9.382	10-3 9.354	10-3 –	0.30% with
		T = 0.9 S –	7.481 10-3 –	7.537 10-3 0.75%	with
		T = 0.1 S 4.328	10-4 4.405	10-4 1.79%	with
		T = 0.3 S 3.671	10-3 3.640	10-3 –	0.84% VITE
_X with (m/s mas s	the node with $N3$ 2) with	T = 0.5 S –	1.539 10-2 –	1.536 10-2 –	0.20% ()
		T = 0.7 S 2.453	10-2 2.457	10-2 0.15%	with
		T = 0.9 S –	1.899 10-2 –	1.912 10-2 0.68%	with
		T = 0.1 S 6.112	10-2 6.100	10-2 –	0.20% with
		T = 0.3 S –	1.306 10-1 –	1.300 10-1 –	0.46% ACCE_X
with (m/s^2 ma ss	the node with $N2$ 1) with	T = 0.5 S 1.571	10-1 1.600	10-1 1.85%	()
		T = 0.7 S –	5.657 10-2 –	5.800 10-2 2.53%	with
		T = 0.9 S –	1.124 10-1 –	1.130 10-1 0.53%	with
		T = 0.1 S 1.562	10-2 1.618	10-2 3.58%	with
		T = 0.3 S –	6.031 10-2 –	6.223 10-2 3.18%	ACCE
_X with (m/s^2 ma ss	the node with $N3$ 2) with	T = 0.5 S 5.102	10-2 5.374	10-2 5.33%	()
		T = 0.7 S 7.428	10-2 7.043	10-2 –	5.19% with
		T = 0.9 S –	2.364 10-1 –	2.263 10-1 –	4.28% Note:

:

The velocity with the node at $N2$ time being $t=0.5\,s$ relatively close to zero, the comparison is produced for this case in absolute value. Summary

5 of the results For

the two modelizations, the responses in displacement obtained after projection are identical to the displacements of reference calculated analytically with Maple and provided in data.

The values velocities and the accelerations deduced from the displacements obtained after projection are close to those obtained analytically. The weak noted variations are due to the errors of approximation generated by the determination by a linear diagram in time of the velocities and accelerations.