SDLD103 - Seismic response of a system 3 masses and 4 springs multimedia

Summarized

the problem consists in analyzing the response of a mechanical structure of standard embed-embedded and undamped beam, modelled by a system 3 masses and 4 springs and subjected to an unspecified seismic loading.

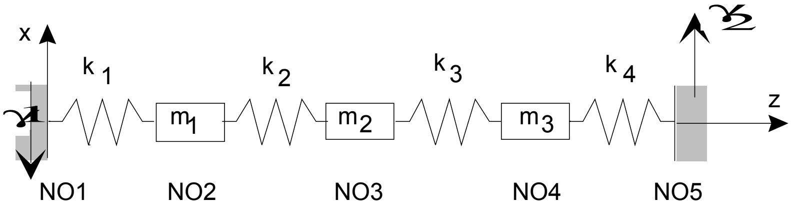
One tests the discrete element in tension and rotation, the computation of the eigen modes and the static modes and the computation of the transient response by modal superposition of a structure subjected to an accelerogram of translation (modelization A) or rotation (modelization B).

The got results are in very good agreement with the results of reference (analytical results).

1 Problem of reference

1.1 Geometry

the beam is modelled by a set of 4 springs and of 3 point masses.



1.2 Material properties

Stiffness of connection: $k=k_1=k_2=k_3=k_4=104\ N/m$;
point mass: $m=m_1=m_2=m_3=10\ kg$.

1.3 Boundary conditions and loadings

Boundary conditions :
Only authorized displacements are the translations according to the axis x .
The points $NO1$ and $NO5$ are clamped: $dx=dy=dz=drx=dry=drz=0$.
The other points are free in translation according to the direction x : $dy=dz=drx=dry=drz=0$.

Loading :
Anchorage the points of $NO1$ and $NO5$ each one to a transverse acceleration $\gamma_1(t)=a t^2$ with $a=2.10^5 m/s^4$ in $NO1$ and $\gamma_2(t)=0\, m/s^2$ in $NO5$.

1.4 Initial conditions

the system is at rest: at $t=0$ $dx(0)=0$, $dx/dt(0)=0$ in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

the problem consists in calculating the response of a system with five degrees of freedom subjected to two accelerations $\gamma_1(t)$ and $\gamma_2(t)$ distinct of an unspecified form. It is exposed in detail in the reference [bib2].

One initially calculates the eigenfrequencies f_i, the eigenvectors associated standardized compared to the modal mass Φ_{Ni} and the static modes Ψ with the system (analytical values). One calculates then the generalized response of the system multimedia by solving analytically the integral of Duhamel [bib1]. Lastly, one restores on physical base the vector of relative displacements (on the active degrees of freedom) X_r, which allows us, after having calculated the vector of displacements of training X_e, to calculate the vector of absolute displacements $X_a = X_r + X_e$.

2.2 Results of reference

- Computation of the three eigenfrequencies f_i, the eigenvectors associated standardized compared to the modal mass Φ_{Ni} and of the static modes Ψ with the system

$$\begin{cases} f_1 = \dfrac{1}{2\pi\sqrt{(2+\sqrt{2})m/2\mathrm{k}}} = 3.85\,Hz \\ f_2 = \dfrac{1}{2\pi\sqrt{m/2\mathrm{k}}} = 7.12\,Hz \\ f_3 = \dfrac{1}{2\pi\sqrt{(2-\sqrt{2})m/2\mathrm{k}}} = 9.30\,Hz \end{cases}, \quad \Phi_N = \frac{1}{2\sqrt{m}}\begin{bmatrix} 1 & -\sqrt{2} & -1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & \sqrt{2} & 1 \end{bmatrix} \text{ and } \Psi = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}.$$

- Computation of the generalized response of the system multimedia

the fundamental equation of the dynamics, in the relative reference on the active degrees of freedom is written:

$M\ddot{X}_r + K X_r = (M\Psi + M_{XS})\ddot{X}_S$ with $\ddot{X}_S = \begin{bmatrix} at^2 \\ 0 \end{bmatrix}$, the vector of the accelerations imposed on the level of the various points of anchorage.

The equation of motion projected on the basis of dynamic mode standardized compared to the modal mass Φ_N is written, by considering only the active degrees of freedom:

$$\ddot{q}(t) + K_G q(t) = -\Phi_N^T M \Psi \ddot{X}_S = \frac{a\sqrt{m}t^2}{4}\begin{bmatrix} 2+\sqrt{2} \\ \sqrt{2} \\ 2-\sqrt{2} \end{bmatrix}$$

The response of this linear system, at one time t, then consists in calculating the integral of Duhamel:

$$q(t) = -\frac{a\sqrt{m^3}}{4k}\begin{bmatrix}(3+2\sqrt{2})(t^2+(2+\sqrt{2})(\cos\omega_1 t-1)m/k)\\(t^2+(\cos\omega_2 t-1)m/k)/\sqrt{2}\\(3-\sqrt{2})(t^2+(2-\sqrt{2})(\cos\omega_3 t-1)m/k)\end{bmatrix}$$

- Computation of displacement relating to the active degrees of freedom: $X_r = \sum_i \Phi_{\mathrm{Ni}} q_i$ that is to say:

$$X_r = -\frac{am}{8k}\begin{bmatrix}7t^2+\left[(10+7\sqrt{2})\frac{m}{k}(\cos\omega_1 t-1)+(\cos\omega_2 t-1)+(10-7\sqrt{2})(\cos\omega_3 t-1)\right]m/k\\8t^2+\left[(10\sqrt{2}+14)\frac{m}{k}(\cos\omega_1 t-1)+(-10\sqrt{2}+14)(\cos\omega_3 t-1)\right]m/k\\5t^2+\left[(10+7\sqrt{2})\frac{m}{k}(\cos\omega_1 t-1)-(\cos\omega_2 t-1)+(10-7\sqrt{2})(\cos\omega_3 t-1)\right]m/k\end{bmatrix}.$$

- Computation of displacements of training to the active degrees of freedom: $X_e = \Psi X_S = a\frac{t^4}{48}\begin{bmatrix}3\\2\\1\end{bmatrix}$.

- Computation of absolute displacements to the active degrees of freedom: $X_a = X_r + X_e$.

2.3 Uncertainty on the solution

No if one calculates the integral of Duhamel analytically [bib1].

2.4 Bibliographical references

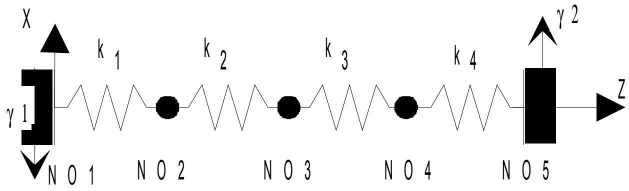
1) J.S. PRZEMIENIECKI: Theory of matrix structural analysis. New York, Mac Graw-Hill, 1968, pages 351-357.

2) Fe WAECKEL: Documentations use and validation of the developments carried out to compute: the seismic multimedia structure response. HP-52/96/002

3 Modelization A

3.1 Characteristic of the modelization

the elements are modelled by discrete elements with 3 degrees of freedom `DIS_T`.



The node $NO1$ is subjected to an imposed acceleration $\gamma_1(t)$, the node $NO5$ with $\gamma_2(t)$. One calculates relative displacement of the nodes $NO2$, $NO3$ and $NO4$ compared to their static deformed shape, their displacement of training and their absolute displacement.

Temporal integration is carried out with the algorithms of Eulerian (time step: 10^{-3} second), of Devogelaere (time step: 10^{-3} second) and with an algorithm with time step adaptive of order 2.

3.2 Characteristics of the mesh

The mesh consists of 5 nodes and 4 discrete elements (`DIST_T`).

4 Results of the modelization A

4.1 Values tested of the modelization A

4.1.1 relative Displacements of the nodes $NO2$, $NO3$ and $NO4$

- relative Displacements of the node $NO2$ with the algorithms of numerical integration of Eulerian, Devogelaere, adaptive of order 2 and Runge-Kutta (32 and 54):

Time (S)	Reference
0,1	– 8,47734E-01
0,3	– 1,55202E+01
0,5	– 4,36449E+01
0,7	– 8,50830E+01
1,0	– 1,74790E+02

- relative Displacements of the node $NO3$ with the algorithm of numerical integration of Eulerian:

Time (S)	Reference
0,01	9,87666E-10
0,02	2,49501E-07
0,03	6,25468E-06
0,04	6,05829E-05
0,05	3,47191E-04
0,06	1,42349E-03
0,07	4,62144E-03
0,08	1,26245E-02
0,09	3,01825E-02
0,1	– 7,68449E-01

0,3	– 1,76923E+01
0,5	– 4,99310E+01
0,7	– 9,70711E+01
1,0	– 1,99722E+02

- relative Displacements of the node $NO3$ with the algorithms of numerical integration of Devogelaere, adaptive of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0,1	– 7,68449E-01
0,3	– 1,76923E+01
0,5	– 4,99310E+01
0,7	– 9,70711E+01
1,0	– 1,99722E+02

- relative Displacements of the node $NO4$ with the algorithm of numerical integration of Eulerian, Devogelaere, adaptive of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0,1	– 4,09632E-01
0,3	– 1,10372E+01
0,5	– 3,12415E+01
0,7	– 6,05833E+01
1,0	– 1,24803E+02

4.1.2 absolute Displacements of the nodes $NO2$, $NO3$ and $NO4$

- absolute Displacements of the node $NO2$ with the algorithm of numerical integration of Eulerian, Devogelaere, adaptive of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0,1	4,02266E-01
0,3	8,57298E+01
0,5	7,37605E+02
0,7	2,91617E+03
1,0	1,23252E+04

- absolute Displacements of the node $NO3$ with the algorithm of numerical integration of Eulerian, Devogelaere, adaptive of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0,1	6,48847E-02
0,3	4,98077E+01
0,5	4,70902E+02
0,7	1,90376E+03
1,0	8,13361E+03

- absolute Displacements of the node $NO4$ with the algorithm of numerical integration of Eulerian, Devogelaere, adaptive of order 2 and Runge-Kutta (54 and 32):

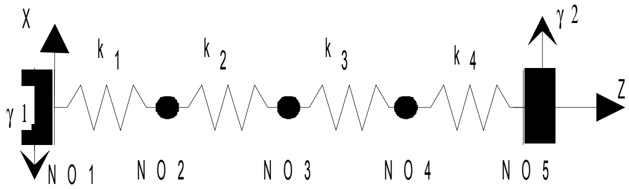
Time (S)	Reference
0,1	7,03506E-03
0,3	2,27128E+01
0,5	2,29175E+02
0,7	9,39833E+02
1,0	4,04186E+03

5 Modelization B

It is the same modelization as the preceding one except for the loading which is an accelerogram of rotation.

5.1 Characteristics of the modelization

the elements are modelled by discrete elements with 3 degrees of freedom `DIS_T`.



The node $NO1$ is subjected to an imposed acceleration $\gamma_1(t)$, the node $NO5$ with $\gamma_2(t)$. One calculates relative displacement of the nodes $NO2$, $NO3$ and $NO4$ compared to their static deformed shape, their displacement of training and their absolute displacement.

Temporal integration is carried out with the algorithm of Eulerian (time step: 10^{-3} second).

5.2 Characteristics of the mesh

The mesh consists of 5 nodes and 4 discrete elements (`DIST_TR`).

6 Results of the modelization B

6.1 Values tested of the modelization B

6.1.1 relative Displacements of the nodes $NO2$, $NO3$ and $NO4$

- relative Displacements of the node is outside the field of definition with a right profile of the EXCLU type node: $NO2$

Time (S)	Reference
0,1	– 8,47734E-01
0,3	– 1,55202E+01
0,5	– 4,36449E+01
0,7	– 8,50830E+01
1,0	– 1,74790E+02

- relative Displacements of the node is outside the field of definition with a right profile of the EXCLU type node: $NO3$

Time (S)	Reference
0,1	– 7,68449E-01
0,3	– 1,76923E+01
0,5	– 4,99310E+01
0,7	– 9,70711E+01
1,0	– 1,99722E+02

- relative Displacements of the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

Time (S)	Reference
0,1	– 4,09632E-01
0,3	– 1,10372E+01
0,5	– 3,12415E+01
0,7	– 6,05833E+01
1,0	– 1,24803E+02

6.1.2 absolute Displacements of the nodes $NO2$, $NO3$ and $NO4$

•absolute Displacements of the node is outside the field of definition with a right profile of the EXCLU type node: $NO2$

Time (S)	Reference
0,1	4,02266E-01
0,3	8,57298E+01
0,5	7,37605E+02
0,7	2,91617E+03
1,0	1,23252E+04

• absolute Displacements of the node is outside the field of definition with a right profile of the EXCLU type node: *NO3*

Time (S)	Reference
0,01	9,87666E-10
0,02	2,49501E-07
0,03	6,25468E-06
0,04	6,05829E-05
0,05	3,47191E-04
0,06	1,42349E-03
0,07	4,62144E-03
0,08	1,26245E-02
0,09	3,01825E-02
0,10	6,48847E-02
0,30	4,98077E+01
0,50	4,70902E+02
0,70	1,90376E+03
1,0	8,13361E+03

• absolute Displacements of the node is outside the field of definition with a right profile of the EXCLU type node: *NO4*

Time (S)	Reference
0,1	7,03506E-03
0,3	2,27128E+01
0,5	2,29175E+02
0,7	9,39833E+02
1,0	4,04186E+03

7 Summary of the results

the results got with *Code_Aster* are in conformity with the results of reference (the error is in general lower than 0,03%).