SDLD102 - Under transitory structuring: System 3 masses-4 springs

Summarized:

The scope of application of this test relates to the dynamics of structures. It makes it possible to time step validate the diagrams of integration with adaptive "ADAPT_ORDRE2" and "RUNGE_KUTTA_54" of the operator of transient computation on modal base as well as the computation of linear transient response on a modal base calculated by substructuring (for the 5 diagrams of integration : "EULER", "DEVOGE", "NEWMARK", "RUNGE_KUTTA_32" and "ADAPT_ORDRE2"). In particular, the case of the application of a reduced damping to the dynamic modes of projection bases of substructures is treated.

It is a question of determining the transient response of a system made up of 3 masses and 4 springs, embedded at its ends and subjected to a constant force from initial time. Springs are modelled by elements of the type "DIS_TR" and the masses by elements of the type "DIS_T".

Three modelizations are proposed. In the 2 first, the structure is undamped. The méthodes de calcul transitory by substructuring with interfaces of the type Craig-Bampton ("CRAIGB") and Mac Neal ("MNEAL") are tested. The results of reference which are associated for them result from an analytical computation. In the third, one imposes a reduced damping of 1% on the dynamic modes of projection bases of under - structures. The transitory equation checked by complete structure was obtained analytically. Its resolution, which serves as reference, was carried out by the Maple software.

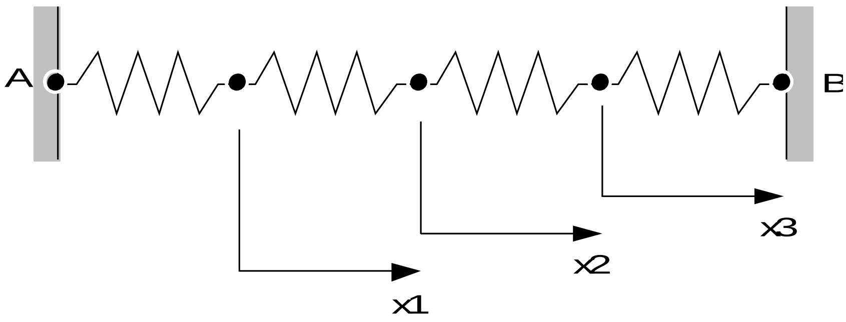
Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

1 Problem of reference

1.1 Geometry

the studied system is composed of 3 masses (m) and 4 springs (k). The group is embedded at its ends.

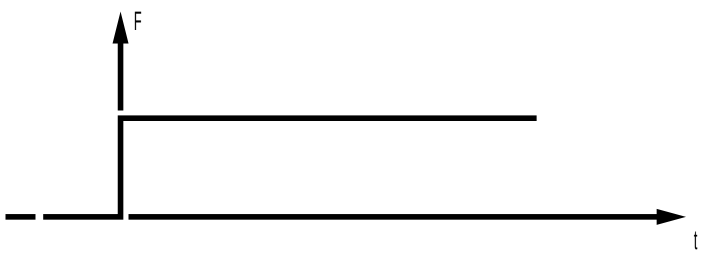


1.2 Material properties

Stiffness of springs: $k=1\ N/m$.

Point masses: $m=1\ kg$.

1.3 Boundary conditions and loadings



Points A and B clamped.

Application to the point $x1$ of a constant force $F=1\ N$, from time $t=0\ s$.

1.4 Initial conditions

Structure initially at rest.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 undamped Structure

In this case, the reference solution can be obtained analytically:

$$m\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{pmatrix}+k\begin{pmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}$$

The own pulsations of the spring-mass system are worth:

$$\omega_1^2=(2-\sqrt{2})\frac{k}{m}\quad\omega_2^2=2\frac{k}{m}\quad\omega_3^2=(2+\sqrt{2})\frac{k}{m}$$

respective modal deformed shapes:

$$\Phi_1=\begin{pmatrix}\sqrt{2}\\2\\\sqrt{2}\end{pmatrix}\quad\Phi_2=\begin{pmatrix}1\\0\\-1\end{pmatrix}\quad\Phi_3=\begin{pmatrix}-\sqrt{2}\\2\\-\sqrt{2}\end{pmatrix}$$

Projected on the basis as of eigen modes, the transitory equation becomes η_i with like generalized coordinates:

$$m\begin{pmatrix}8&0&0\\0&2&0\\0&0&8\end{pmatrix}\begin{pmatrix}\ddot{\eta}_1\\\ddot{\eta}_2\\\ddot{\eta}_3\end{pmatrix}+4\text{k}\begin{pmatrix}4-\sqrt{2}&0&0\\0&1&0\\0&0&4+2\sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\\eta_2\\\eta_3\end{pmatrix}=\begin{pmatrix}+\sqrt{2}\\2\\-\sqrt{2}\end{pmatrix}$$

The system can be solved analytically. One obtains:

$$\{\eta(t)\}=\frac{1}{2\text{m}}\begin{pmatrix}\dfrac{\sqrt{2}}{4\omega_1^2}(1-\cos\omega_1 t)\\\dfrac{1}{\omega_2^2}(1-\cos\omega_2 t)\\\dfrac{\sqrt{2}}{4\omega_3^2}(\cos\omega_3 t-1)\end{pmatrix}$$

The solution on physical base is obtained by means of the transformation of Ritz:

$$x(t)=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\Phi\eta=\begin{pmatrix}\sqrt{2}&1&-\sqrt{2}\\2&0&2\\\sqrt{2}&-1&\sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\\eta_2\\\eta_3\end{pmatrix}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2.1.2 Structure deadened

The damping is applied to the eigen modes of projection bases of clamped substructures (reduced damping). In this case, one leads to the transitory equation in generalized coordinates following (feeding-bottle [1]):

$$m\begin{pmatrix}8 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 8\end{pmatrix}\begin{pmatrix}\ddot{\eta}_1\\ \ddot{\eta}_2\\ \ddot{\eta}_3\end{pmatrix}+4\varepsilon\sqrt{2\mathrm{km}}\begin{pmatrix}3-2\sqrt{2} & 0 & -1\\ 0 & 1 & 0\\ -1 & 0 & 3+2\sqrt{2}\end{pmatrix}\begin{pmatrix}\dot{\eta}_1\\ \dot{\eta}_2\\ \dot{\eta}_3\end{pmatrix}+4\mathrm{k}\begin{pmatrix}4-\sqrt{2} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 4+2\sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\\ \eta_3\end{pmatrix}=\begin{pmatrix}+\sqrt{2}\\ 1\\ -\sqrt{2}\end{pmatrix}$$

This system not being uncoupled, it was solved using the Maple software. One obtained $(\varepsilon=0.01)$:

$$\{\eta(t)\}=\frac{1}{2\mathrm{m}}\begin{pmatrix}\frac{\sqrt{2}}{4\omega_1^2}(1-e^{-\frac{t}{\tau_1}}\cos\omega_1 t)\\ \frac{1}{\omega_2^2}(1-e^{-\frac{t}{\tau_2}}\cos\omega_2 t)\\ \frac{\sqrt{2}}{4\omega_3^2}(e^{-\frac{t}{\tau_3}}\cos\omega_3 t-1)\end{pmatrix}$$

with $\tau_1=1.65\,10^3\,s$, $\tau_2=\frac{1}{\varepsilon\,\omega_2}=\frac{100}{\sqrt{2}}$ and $\tau_3=4.85\,10^1\,s$

One thus obtains a formulation close to the undamped case, but in which intervene of the exponential terms which characterize damping.

The solution on physical base is obtained by means of the transformation of Ritz:

$$x(t)=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\phi\,\eta=\begin{pmatrix}\sqrt{2} & 1 & -\sqrt{2}\\ 2 & 0 & 2\\ \sqrt{2} & -1 & \sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\\ \eta_3\end{pmatrix}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2.2 Results of reference

undamped Structure:

Displacement, velocity and acceleration of the node x_2 at time $t=80\ s$:

$$\begin{aligned} x_2(80) &= 4.1700\,10^{-1}\,m \\ \dot{x}_2(80) &= -4.3011\,10^{-1}\,m.s^{-1} \\ \ddot{x}_2(80) &= 3.3749\,10^{-1}\,m.s^{-2} \end{aligned}$$

Damped structure:

Displacement of the node x_2 at time $t=80\ s$:

$$x_2(80)=4.9867\,10^{-1}\,m$$

2.3 Uncertainty on the solution

undamped Case: analytical solution.

Damped case: semi-analytical solution.

2.4 Bibliographical reference

1.C. VARE - Ratio HP 61/95/025/A - “Implementation of nonlinear transient computation by substructuring in *the Code_Aster*”.

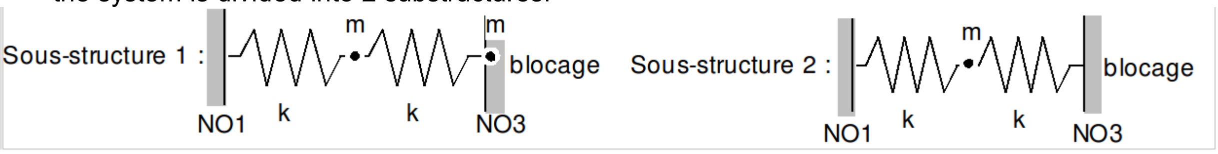
Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

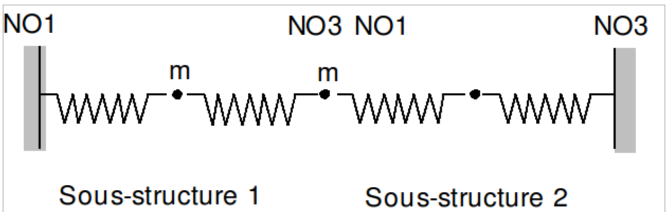
3 Modelization A

3.1 Characteristic of the modelization

the system is divided into 2 substructures:



In situation, the two substructures are connected to the level of the 2nd mass. The dynamic interface of 1st substructure consists of a mass m on the level of the node $NO3$ of the mesh and coincides with the dynamic interface of 2nd substructure which does not comprise any mass and simply is blocked on the level of the node $NO1$.



The eigen modes of the complete system are calculated by means of the method of calculating modal by substructuring with interfaces of the type “Craig-Bampton” (blocked interfaces). The bases of each substructure are made up of a dynamic mode and a constrained mode.

The transient response of the system is calculated on the modal base calculated by substructuring.

Time step used are equal to: $10^{-2} s$ in "EULER", $10^{-2} s$ in "NEWMARK", $10^{-2} s$ "DEVOGE", $10^{-1} s$ "ADAPT_ORDRE2" (for this last, it is time step initial the algorithm and the time step maximum one, is fixed to him at formula $0{,}15 s$ then formula $0{,}2 s$ to optimize the computing time).

3.2 Characteristics of the mesh of the substructure

Many nodes: 3

Number of meshes and types: 2 SEG2

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

3.3 Quantities tested and Computation

results by modal recombination without substructuring: Method `ADAPT_ORDRE2` and `RUNGE_KUTTA_54`

Identification	Reference
Method: `ADAPT_ORDRE2`	
Node x_2, displacement (m)	4.1700 10-1
Node x_2, velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2, acceleration ($m.s^{-2}$)	3.3375 10-1
Method: `RUNGE_KUTTA_54`	
Node x_2, displacement (m)	4.1700 10-1
Node x_2, velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2, acceleration ($m.s^{-2}$)	3.3375 10-1

Computation per substructuring

Method: `EULER`

Node x_2, displacement (m)	4.1700 10-1
Node x_2, velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2, acceleration ($m.s^{-2}$)	3.3375 10-1

Method: `DEVOGE`

Node x_2, displacement (m)	4.1700 10-1
Node x_2, velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2, acceleration ($m.s^{-2}$)	3.3375 10-1

Method: `NEWMARK`

Node x_2, displacement (m)	4.1700 10-1
Node x_2, velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2, acceleration ($m.s^{-2}$)	3.3375 10-1

Method: `RUNGE_KUTTA_32`

Node x_2, displacement (m)	4.1700 10-1
Node x_2, velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2, acceleration ($m.s^{-2}$)	3.3375 10-1

Method: `ADAPT_ORDRE2`

Node x_2, displacement (m)	4.1700 10-1
Node x_2, velocity ($m.s^{-1}$)	– 4.3011 10-1

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Node x_2, acceleration ($m.s^{-2}$)	3.3375 10-1

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

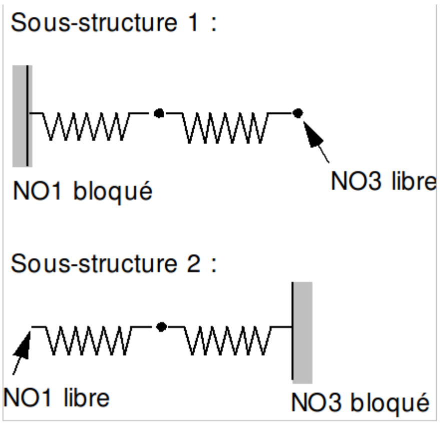
4 Modelization B

4.1 Characteristic of the modelization

This modelization is identical to the precedent if it is not that the eigen modes of the complete system are calculated by means of the méthode de calcul modal by substructuring with interfaces of the type “Mac Neal” (free interfaces). The bases of each substructure are made up of a dynamic mode and an attach mode.

The transient response of the system is calculated on the modal base calculated by under - structuring.

More precisely, the studied substructures have their free interfaces:



Time step used are worth: $10^{-2} s$ in EULER, $10^{-2} s$ in NEWMARK, $10^{-2} s$ DEVOGE, $10^{-2} s$ ADAPT_ORDRE2 (for this last, it is time step initial the algorithm and the time step maximum one, is fixed to him at formulepour $0,1\ s$ to optimize the computing time).

4.2 Characteristics of the mesh of the substructure

Many nodes: 3

Number of meshes and types: 2 SEG2

4.3 Quantities tested and results

Identification	Reference
Method: EULER	
Node x_2 , displacement (m)	4.1700 10-1
Node x_2 , velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2 , acceleration ($m.s^{-2}$)	3.3749 10-1
Method: NEWMARK	
Node x_2 , displacement (m)	4.1700 10-1

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Node x_2 , velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2 , acceleration ($m.s^{-2}$)	3.3749 10-1

Method: DEVOGE

Node x_2 , displacement (m)	4.1700 10-1
Node x_2 , velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2 , acceleration ($m.s^{-2}$)	3.3749 10-1

Method: ADAPT_ORDRE2

Node x_2 , displacement (m)	4.1700 10-1
Node x_2 , velocity ($m.s^{-1}$)	– 4.3011 10-1
Node x_2 , acceleration ($m.s^{-2}$)	3.3749 10-1

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

5 Modelization C

5.1 Characteristic of the modelization

the eigen modes of the complete system are calculated by means of the méthode de calcul modal by substructuring with interfaces of the type "Craig-Bampton" (blocked interfaces). The bases of each substructure are made up of a dynamic mode and a constrained mode.

With the dynamic mode of each substructure a reduced damping is associated with 1% .

The transient response of the damped system is calculated on the modal base calculated by under - structuring.

Time step taken are equal to: $10^{-2} s$ in EULER, $10^{-2} s$ in NEWMARK, $10^{-2} s$ ADAPT_ORDRE2 (for this last, it is time step initial the algorithm and the time step maximum one, is fixed to him at formula $0,1 s$ to optimize the computing time).

5.2 Characteristics of the mesh of the substructure

Many nodes: 3

Number of meshes and types: 2 SEG2

5.3 Quantities tested and results

Identification	Reference
Method: EULER	
Node x_2 , displacement (m)	4.9867 10-1
Method : NEWMARK	
Node x_2 , displacement (m)	4.9867 10-1
Method: ADAPT_ORDRE2	
Node x_2 , displacement (m)	4.9867 10-1

Note:: the tests of the modelization C are duplicated because one wants to test modal computation on the matrixes projected with automatic transition on the MUMPS solver whereas the user asked for the solver MULT_FRONT, who is not available in this case. The results are identical.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

6 Modelization D

6.1 Characteristic of the modelization

the characteristics of the modelizations D are exactly identical to that of selected modelization A. On the other hand, one method " ELIMINE" to manage the boundary conditions and the assembly of under structures.

6.2 Quantities tested and results

Identification	Reference
Method: EULER	
Node x_3 , displacement (m)	4.17 10-1
Method: EULER	
Node x_3 , velocity ($m.s^{-1}$)	-4.3011 10-1
Method: EULER	
Node x_3 , Acceleration ($m.s^{-2}$)	3.3749 10-1

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

7 Modelization E

7.1 Characteristic of the modelization

the characteristics of the modelizations E are exactly identical to that of modelization A. On the other hand, one uses commands CREA_ELEM_SSD and ASSE_ELEM_SSD for the creation and the assembly of the dynamic macro-elements. One also presents how one creates the initial conditions for a transient computation on modal base by dynamic substructuring. The computation of the direct response (without substructuring) is not included in this modelization.

7.2 Quantities tested and results

Identification	Reference
Method: EULER	
Node x_3 , displacement (m)	4.17 10-1
Method: EULER	
Node x_3 , velocity ($m.s^{-1}$)	-4.3011 10-1
Method: EULER	
Node x_3 , Acceleration ($m.s^{-2}$)	3.3749 10-1

8 Summary of the results

the accuracy on displacement, the velocity and the acceleration of the node x_2 at time $t=80\ s$ is excellent (relative error $< 1\%$).

This test thus validates the operators of linear computation of transient response on modal base calculated by dynamic substructuring (with and without damping), as well as the diagram of integration to time step adaptive of order 2 of the operator of transient computation on modal base.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)