

SDLD33 - Spectral seismic response of a system 2 masses - 4 springs multimedia by 2 uncorrelated groups of bearings

Summarized:

The problem consists in calculating the spectral response of a system 2 masses – 4 springs multimedia, subjected to a multiple seismic excitation, by regarding the 3 bearings as 2 groups of uncorrelated bearings.

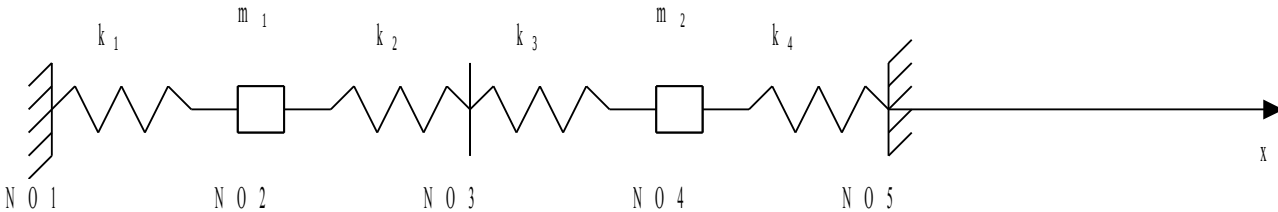
One tests the discrete element in tension, the computation of the eigen modes, the static modes and the spectral response by modal superposition via operator `COMB_SISM_MODAL`. This benchmark makes it possible to validate the order of the combinations to be considered in this case, namely the office plurality intra-group then the office plurality joint committee.

The got results are in very good agreement with the analytical results of reference.

1 Problem of reference

1.1 Geometry

the system is composed of a set of 4 springs, 2 point masses, is supported by 3 bearings.



1.2 Material properties

Stiffness of connection: $k = k_1 = k_2 = 10^3 \text{ N/m}$ $k_3 = k_4 = 2.10^3 \text{ N/m}$;
point mass: $m = m_1 = m_2 = 10 \text{ kg}$.

1.3 Boundary conditions and loadings

Boundary conditions :

Only authorized displacements are the translations according to the axis x .

The points $NO1$, $NO3$ and $NO5$ are clamped: $dx = dy = dz = drx = dry = drz = 0$.

The other points are free in translation according to the direction x : $dy = dz = drx = dry = drz = 0$.

Loading :

The structure is subjected to a multiple spectral seismic excitation and differential displacements.

The response spectrums of oscillator in pseudo-acceleration are simplified. Only the values corresponding to the 2 eigenfrequencies of the system are mentioned. They do not depend on damping:

with the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

$$SRO_{NO1}(f_1) = A_{11} = 7 \text{ m/s}^2$$

$$SRO_{NO1}(f_2) = A_{21} = 5 \text{ m/s}^2$$

$$DDS_{NO1} = D_1 = -0.04 \text{ m}$$

with the node is outside the field of definition with a right profile of the EXCLU type node: $NO3$

$$SRO_{NO3}(f_1) = A_{11} = 7.7 \text{ m/s}^2$$

$$SRO_{NO3}(f_2) = A_{12} = 5.5 \text{ m/s}^2$$

$$DDS_{NO3} = D_2 = -0.044 \text{ m}$$

with the node is outside the field of definition with a right profile of the EXCLU type node: $NO5$

$$SRO_{NO5}(f_1) = A_{21} = 12 \text{ m/s}^2$$

$$SRO_{NO5}(f_2) = A_{22} = 6 \text{ m/s}^2$$

$$DDS_{NO5} = D_3 = 0.06 \text{ m}$$

The excitations with the nodes *NO1* and *NO3* are correlated. One sets up 2 groups of uncorrelated supports: group 1 is composed of the nodes *NO1* and *NO3* ; group 2 is made up by the only node *NO5* .

1.4 Initial conditions

the system is at rest.

2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the spectral response by modal superposition of a system 2 masses – 4 springs subjected to three distinct excitations. One determines the displacement of the masses to the nodes *NO2* and *NO4* along the axis *x*.

One calculates analytically:

- the eigenfrequencies f_i ,
- the associated eigenvectors ϕ_{N_i} standardized compared to the modal mass,
- the static modes of bearings Ψ_i of the system,
- the participation factors modal P_{ij} relating to the bearings,
- Rm_{ij} the maximum of response of each mode starting from the excitation spectrums,
- Re_{ij} the contribution of the motion of training of each bearing starting from differential displacements,

2.2 Results of reference

2.2.1 Matrixes and vectors characteristic

•stiffness matrix K

$$K = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \quad \text{stamps relating to degrees of freedom 1,2,3,4,5}$$

$$K^p = \begin{bmatrix} k_1+k_2 & 0 & -k_1 & -k_2 & 0 \\ 0 & k_3+k_4 & 0 & -k_3 & -k_4 \\ -k_1 & 0 & k_1 & 0 & 0 \\ -k_2 & -k_3 & 0 & k_2+k_3 & 0 \\ 0 & -k_4 & 0 & 0 & k_4 \end{bmatrix}$$

stamps partitionnée degrees of freedom of structure 2,4, degrees of freedom of support 1,3,5

$$K^p = \begin{bmatrix} k & k_{xs} \\ k_{sx} & k_{ss} \end{bmatrix} \quad k = \begin{bmatrix} k_1+k_2 & 0 \\ 0 & k_3+k_4 \end{bmatrix} \quad k_{xs} = \begin{bmatrix} -k_1 & 0 & -k_2 & 0 \\ 0 & -k_3 & 0 & -k_4 \end{bmatrix}$$

•mass matrix M

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

stamps relating to degrees of freedom 1,2,3,4,5

$$M^p = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

stamps partitionnée degrees of freedom of structure 2,4, degrees of freedom of support 1,3,5

- modal computation in clamped base

$$(K - \lambda_i M) \phi_i = 0 \quad \lambda_i = \omega_i^2$$

$$\det(K - \lambda_i M) = 0 \Leftrightarrow \lambda_i^2 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_3 + k_4}{m_2} \right) \lambda_i + \frac{(k_1 + k_2)(k_3 + k_4)}{m_1 m_2} = 0$$

•

$$\lambda_1 = \frac{2k}{m} \quad \lambda_2 = \frac{4k}{m}$$

- eigenfrequencies :

$$\Rightarrow f_1 = \frac{\omega_1}{2\pi} \quad f_2 = \frac{\omega_2}{2\pi}$$

- not normalized eigen modes:

$$\phi_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

- generalized modal masses $\mu_i = \phi_i^T M \phi_i$:

$$\bullet \mu_1 = \mu_2 = m$$

- eigen modes normalized with the unit generalized modal mass ϕ_{Nk} :

$$\bullet \Rightarrow \phi_{N1} = \frac{\phi_1}{\sqrt{\mu_1}} \quad \phi_{N2} = \frac{\phi_2}{\sqrt{\mu_2}}$$

- **static modes of bearings** ψ_{Sj}

Stamps static modes reduced with the ddls of static $\varphi_S = -\mathbf{k}^{-1} \mathbf{k}_{xs}$

$$\varphi_S = \frac{-1}{4k} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -k & -k & 0 \\ 0 & -2k & -2k \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

- structure solution to a unit displacement of the node is outside the field of definition with a right profile of the EXCLU type node: *NO1*

displacements: $\psi_{S1} = \begin{bmatrix} 1 \\ 1/2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

- static solution with a unit displacement of the node is outside the field of definition with a right profile of the EXCLU type node: *NO3*

displacements: $\psi_{S2} = \begin{bmatrix} 0 \\ 1/2 \\ 1 \\ 1/2 \\ 0 \end{bmatrix}$

- static solution with a unit displacement of the node is outside the field of definition with a right profile of the EXCLU type node: *NO5*

displacements: $\psi_{S3} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/2 \\ 1 \end{bmatrix}$

- **mode of rigid body** ψ_{RI}

Stamps rigid modes reduced with the structure ddls: $\varphi_R = \varphi_S \mathbf{S}_R$

Mode of rigid body $\psi_{RI} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

One checks well that: $\psi_{RI} = \psi_{S1} + \psi_{S2} + \psi_{S3}$

2.2.2 Loading 1 multi-bearings

• participation factors modal $P_{kj} = \phi_{Nk}^T M \psi_j$:

• contribution of the dynamic mode 1 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: *NO1*

$$\Rightarrow P_{11} = \phi_{N1}^T M \psi_1 = \frac{\sqrt{m}}{2}$$

• contribution of the dynamic mode 1 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: *NO3*

$$\Rightarrow P_{12} = \phi_{N1}^T M \psi_2 = \frac{\sqrt{m}}{2}$$

• contribution of the dynamic mode 1 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: *NO5*

$$\Rightarrow P_{13} = \phi_{N1}^T M \psi_3 = 0$$

• contribution of the dynamic mode 2 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: *NO1*

$$\Rightarrow P_{21} = \phi_{N2}^T M \psi_1 = \frac{\sqrt{m}}{2}$$

• contribution of the dynamic mode 2 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: *NO3*

$$\Rightarrow P_{22} = \phi_{N2}^T M \psi_2 = \frac{\sqrt{m}}{2}$$

• contribution of the dynamic mode 2 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: *NO5*

$$\Rightarrow P_{23} = \phi_{N2}^T M \psi_3 = \frac{\sqrt{m}}{2}$$

- **response of the mode i with the motion of the bearing j**

$$Rm_{kj} = \phi_{Nk} P_{kj} \frac{A_{kj}}{\omega_i^2}$$

Responses combined of the modal oscillators

Response of mode 1 to the motion of bearing 1:

$$Rm_{11} = \phi_{N1} P_{11} \frac{A_{11}}{\omega_1^2} = \frac{A_{11}}{2\omega_1^2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Response of mode 1 with the motion of bearing 2:

$$Rm_{12} = \phi_{N1} P_{12} \frac{A_{12}}{\omega_1^2} = \frac{A_{12}}{2\omega_1^2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Response of mode 1 with the motion of bearing 3:

$$Rm_{13} = \phi_{N1} P_{13} \frac{A_{13}}{\omega_1^2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Response of mode 2 with the motion of bearing 1:

$$Rm_{21} = \phi_{N2} P_{21} \frac{A_{21}}{\omega_2^2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Response of mode 2 with the motion of bearing 2:

$$Rm_{22} = \phi_{N2} P_{22} \frac{A_{22}}{\omega_2^2} = \frac{A_{22}}{2\omega_2^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Response of mode 2 with the motion of bearing 3:

$$Rm_{23} = \phi_{N2} P_{23} \frac{A_{23}}{\omega_2^2} = \frac{A_{23}}{2\omega_2^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **office plurality intra-group (algebraic sum)**

mode 1:

$$Rm_{1\text{groupe1}} = Rm_{11} + Rm_{12} = \frac{A_{11} + A_{12}}{2\omega_1^2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

mode 2:

$$Rm_{2\text{groupe1}} = Rm_{21} + Rm_{22} = \frac{A_{22}}{2\omega_2^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **contribution of the bearing j to the motion of training**

$$Re_j = \psi_j D_j$$

contributions of groups 1 and 2 to the motion of training

$$Re_{\text{groupe1}} = Re_1 + Re_2 = \frac{1}{2} \begin{pmatrix} D_1 + D_2 \\ D_2 \end{pmatrix} \quad Re_{\text{groupe2}} = Re_3 = \frac{1}{2} \begin{pmatrix} 0 \\ D_3 \end{pmatrix}$$

- **office plurality on the modes (quadratic)**

$$Rm_{\text{groupe1}} = \sqrt{(Rm_{11} + Rm_{12})^2 + (Rm_{21} + Rm_{22})^2} = \begin{pmatrix} \frac{A_{11} + A_{12}}{2\omega_1^2} \\ \frac{A_{22}}{2\omega_2^2} \end{pmatrix}$$

$$Rm_{\text{groupe2}} = \sqrt{Rm_{13}^2 + Rm_{23}^2} = \frac{A_{23}}{2\omega_2^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **response of the groups of bearings 1 and 2**

$$R_1 = \sqrt{Rm_{\text{groupe1}}^2 + Re_{\text{groupe1}}^2}$$

$$R_1^2 = \frac{1}{4} \begin{bmatrix} \frac{(A_{11} + A_{12})^2}{\omega_1^2} + (D_1 + D_2)^2 \\ \frac{A_{22}^2}{\omega_2^2} + D_2^2 \end{bmatrix}$$

$$R_2 = \sqrt{Rm_{\text{groupe2}}^2 + Re_{\text{groupe2}}^2}$$

$$R_2^2 = \frac{1}{4} \begin{bmatrix} 0 \\ \frac{A_{23}^2}{\omega_2^2} + D_3^2 \end{bmatrix}$$

- **office plurality joint committee (quadratic)**

$$R = \sqrt{R_1^2 + R_2^2}$$

$$R^2 = \frac{1}{4} \left[\frac{(A_{11} + A_{12})^2}{\omega_1^2} + (D_1 + D_2)^2 + \frac{A_{22}^2 + A_{23}^2}{\omega_2^2} + D_2^2 + D_3^2 \right]$$

2.3 Uncertainty on the solution

No (analytical solution)

3 Modelization A

3.1 Characteristic of the modelization

the system is modelled by:

- 4 discrete elements K_T_D_L,
- 2 discrete elements M_T_D_N.

3.2 Characteristics of the mesh

The mesh consists of 4 meshes SEG2.

3.3 Parameters of modelization

Response on the first 2 modes without static correction (combination of modal responses SRSS)

4 Results of the modelization A

4.1 Eigenfrequencies

MODE	Reference	Code_Aster	relative Error (%)
1	2.2507900E+00	2.25079079E+00	3.5E-05
2	3.1830000E+00	3.18309886E+00	0.003

4.2 Response total on modal base supplements

modes 1 and 2 are taken into account. The components inertial (primary education) and static (secondary) of the response are directly cumulated on the level of the bearings.

COMB_MODE=' SRSS '

•response of the bearing $j=1$ (node NO1): $R_1 = \sqrt{Rm_1^2 + Re_1^2}$ with

$$Rm_1 = \sqrt{Rm_{11}^2 + Rm_{21}^2}$$

•response of the bearing $j=2$ (node NO4): $R_2 = \sqrt{Rm_2^2 + Re_2^2}$ with

$$Rm_2 = \sqrt{Rm_{12}^2 + Rm_{22}^2}$$

•total response: $R = \sqrt{R_1^2 + R_2^2}$

•absolute displacements: DEPL

NOEUD	Reference	Code_Aster	relative Error (%)
NO1	4.00000E-02	4.00000E-02	0.000
NO2	5.58000E-02	5.58083E-02	0.015
NO3	4.40000E-02	4.40000E-02	0.000
NO4	3.85600E-02	3.85683E-02	0.022
NO5	6.00000E-02	6.00000E-02	0.000

•nodal reactions: REAC_NODA

NOEUD	Reference	Code_Aster	relative Error (%)
NO1	3.68000E+01	3.68044E+01	0.012
NO2	0.00000E+00	2.23756E-14	2.2E-14
NO3	8.64900E+01	8.64906E+01	7.0E-04
NO4	0.00000E+00	3.071843E-14	Perfect
3.1E-14	NO5	7.71400E+01	7.71367E+01

5 -0.004 Summary of

the results agreement of the Aster *results* with the analytical values of reference.