SDLD31 - Elementary validation of the time schemes in dynamics

Summarized:
This benchmark makes it possible to validate the programming of the diagrams of integration in time in DYNA_NON_LINE and DYNA_TRAN_MODAL.

More precisely, for DYNA_NON_LINE one tests the following implicit schemes:

1) average acceleration (key word NEWMARK) with resolution in displacement and acceleration;
2) modified average acceleration (key word HHT with MODI_EQUI='NON');
3) HHT complete (key word HHT with MODI_EQUI='OUI');
4) θ - diagram (key word THETA_SCHEMA with THETA = 0.61) and resolution in displacement and of velocity;
5) Krenk (key word KRENK with KAPPA = 1.22) with resolution in displacement and of velocity.

With complete diagram HHT, one tests also the poursuites because this diagram requires a particular initialization. In the same way, one validates also the poursuites for the average acceleration with resolution in acceleration because it is not tested in other benchmarks.

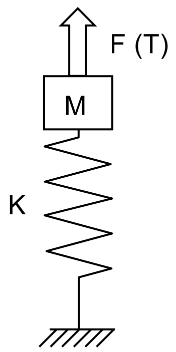
As for DYNA_TRAN_MODAL, the diagrams with time step constant are tested, i.e.:

1) diagram of order 1 clarifies known as EULER ;
2) implicit diagram NEWMARK of order 2;
3) explicit diagram of order 4 known as DEVOGELAERE

the goal being studying the behavior of the time schemes, the selected problem is voluntarily very simple: it is about a linear system 1 mass-spring degree of freedom which is subjected to a sinusoidal force.
The reference solution is obtained by reprogramming of the diagrams of integration in Matlab and by computation of the analytical solution.

1 Problems of reference

1.1 Geometry



the system to 1 degree of freedom is of the type masses M at the end of a come out from stiffness K directed according to the vertical direction z .

1.2 Properties of the material

Stiffness arises K : $36.\pi^2 N/m$
Point mass M : $1\,kg$
The values are selected so as to have an own pulsation of the spring-mass system ω_0 such as: $\omega_0=6.\pi\,rad/s$ because $\omega_0^2=K/M$.

1.3 Geometrical characteristics

displacement is done according to the vertical direction z .

1.4 Boundary conditions and loadings

the base of spring is clamped, the only degree of freedom is thus the following displacement z of the point mass M which is fixed at the other end of spring.

The imposed loading is a vertical sinusoidal $F(t)$ force imposed on the point mass M: $F(t)=\sin(1{,}1.\omega_0.t)$.

1.5 Initial conditions

the system is initially at rest.

2 Reference solution

2.1 Method of calculating

By this benchmark one wants to study the behavior of the various diagrams of integration in implicit times of operator DYNA_NON_LINE. It is not thus a question of seeking most accurately to reproduce possible an analytical solution.

One thus chooses one time step $dt=10^{-2}s$, sufficiently small compared to the own pulsation of the system and one will solve the linear transitory problem with operator DYNA_NON_LINE.

For the nondissipative diagram of the average acceleration (key word `NEWMARK`) it would be possible to calculate the analytical solution to compare itself with it. One tests the resolution in displacement or acceleration which must obviously give the same results.

For the other diagrams which one wishes to test and which are dissipative, obtaining an analytical solution is not very easy.

We thus chose to compare all computations with a numerical solution obtained with the Matlab code. For that, the various diagrams were programmed in Matlab.

One will thus carry out several transient computations in only one stage: with the diagram of average acceleration, the diagram of modified average acceleration, complete diagram HHT, it θ - diagram and the diagram of Krenk. For these the last two dissipative diagrams (with the selected values of parameters), one validates the resolution in displacement like of velocity.

Then one tests the resumption of computation with complete diagram HHT, to validate the mechanism of poursuite with this diagram (one makes two poursuites, the first with $0{,}2\,s$ and the second with $0{,}35\,s$).

2.2 Quantities and results of reference

the comparisons will relate to the displacement and the acceleration of the point mass M at following times: $0{,}5\,s$, $0{,}7\,s$ and $1\,s$.

3 Modelization A - `DYNA_NON_LINE`

3.1 Quantities tested and results

the quantities tested are displacements and accelerations of the point mass M .

Standard	time scheme of the Urgent	field	Values of reference	tolerance
`NEWMARK` Solved in displacement or acceleration (with and without poursuite)	`DEPL`	0,5 S	1.0804500210685E-02	1.E-5%
		0,7 S	-4.0671779495390E-03	1.E-5%
		1,0 S	-1.3026189840935E-02	1.E-5%
	`ACCE`	0,5 S	-4.6479181362891E+00	1.E-5%
		0,7 S	2.3748682319566E+00	1.E-5%
		1,0 S	5.5793367773016E+00	1.E-5%
`HHT` MODI_EQUI='NON'	`DEPL`	0,5 S	9.0224842641940E-03	1.E-5%
		0,7 S	-2.0242152707660E-03	1.E-5%
		1,0 S	-9.074477606657 E-03	1.E-5%
	`ACCE`	0,5 S	-4.0147576088701E+00	1.E-5%
		0,7 S	1.6489918279122E+00	1.E-5%
		1,0 S	4.022313059734449E+00	1.E-5%
`HHT` MODI_EQUI='OUI' With or without poursuites	`DEPL`	0,5 S	1.0775515187707E-02	1.E-5%
		0,7 S	-4.1787420850760E-03	1.E-5%
		1,0 S	-1.3121050364360E-02	1.E-5%
	`ACCE`	0,5 S	-4.6864764249454E+00	1.E-5%
		0,7 S	2.7540329873126E+00	1.E-5%
		1,0 S	5.9586276847714E+00	1.E-5%
`THETA_SCHEMA` Solved in displacement or velocity	`DEPL`	0,5 S	9.4664592252170E-03	1.E-5%
		0,7 S	-2.4964363793720E-03	1.E-5%
		1,0 S	-9.0744776066570E-03	1.E-5%
	`ACCE`	0,5 S	-4.1642290444260E+00	1.E-5%
		0,7 S	1.6728854044803E+00	1.E-5%

		1,0 S	4.0223130597344E+00	1.E-5%
KRENK Solved in displacement or velocity	DEPL	0,5 S	9.5870021341210E-03	1.E-5%
		0,7 S	-2.8112460401650E-03	1.E-5%
		1,0 S	-9.4157749054510E-03	1.E-5%
	ACCE	0,5 S	-4.1725044691246E+00	1.E-5%
		0,7 S	1.8167747070564E+00	1.E-5%
		1,0 S	4.1752706647681E+00	1.E-5%

4 Modelization B - DYNA_VIBRA

In the tests on the diagrams of DYNA_VIBRA one introduces a light viscous damping of for thousand. The good processing of damping is thus validated. It is also the occasion to validate quadratic modal computation.

4.1 Analytical solution

William Weaver Jr. Stephen P. Timoshenko and Donovan H. Young provide to chapter 1.9 of "Vibration Problems in Engineering" the solution with the problem of a system masses/spring with viscous damping subjected to a harmonic excitation.

The equation to be solved is an equation of the second order in time on only one degree of freedom in space:

$$\ddot{x}+2\eta\dot{x}+\omega_0^2 x=\frac{F}{m}\sin(\omega_e t)$$

where x is the displacement of the mass, $\dot{x}$ its velocity and $\ddot{x}$ its acceleration.

$\omega_0^2=\frac{k}{m}$ is the own pulsation of the system, m being its mass and k its stiffness.

η is reduced damping.

Finally F is the amplitude of the force of excitation whereas ω_e is its pulsation.

4.2 Quantities tested and Standard

Computation	results of the Urgent	field (or modal method)	Values of reference	tolerance
MODE_ITER_SIMULT		"SORENSEN"		
	FREQ		3 Hz	1.E-4%
	AMOR_REDUIT		1E-03	1.E-4%
		"TRI_DIAG"		
	FREQ		3 Hz	1.E-4%
	AMOR_REDUIT		1E-03	1.E-4%
EULER (with and without REST_GENE_PHYS)	DEPL	0,5 S	0.010785	1.E-2%
		0,7 S	-3.745074E-03	1.E-1%
		1,0 S	-0.0125639	1.E-1%
NEWMARK	DEPL	0,5 S	0.010785	1.E-2%
		0,7 S	-3.745074E-03	1.E-1%
		1,0 S	-0.0125639	1.E-1%
DEVOG	DEPL	0,5 S	0.010785	1.E-2%
		0,7 S	-3.745074E-03	1.E-1%
		1,0 S	-0.0125639	1.E-1%

5 Summary of the results

This case test makes it possible to validate, into linear, the implicit time schemes (average acceleration, modified average acceleration, HHT complete, θ - diagram and Krenk) of operator DYNA_NON_LINE, in the case of a variable loading imposed, for resolutions in displacement, velocity or acceleration according to the cases.

In this frame one validates also the poursuite with complete diagram HHT.

On the same model one validates also the diagrams d'EULER, of NEWMARK and DEVOGELAERE of operator DYNA_VIBRA.