SDLD30 - Spectral seismic response of a system 2 masses and 3 springs multimedia

Summarized:

The problem consists in calculating the spectral response of a system 2 masses - 3 springs subjected to a multiple seismic excitation.

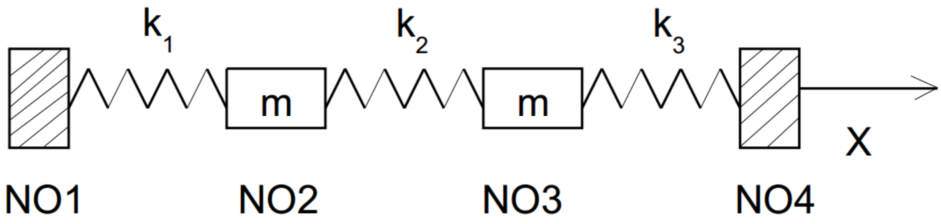
One tests the discrete element in tension, the computation of the eigen modes, the static modes and the spectral response by modal superposition via operator `COMB_SISM_MODAL`. Various office pluralities are tested during the computation of the responses of bearings.

The got results are in very good agreement with the analytical results of reference.

1 Problem of reference

1.1 Geometry

the structure is modelled by a set of 3 springs and of 2 point masses.



1.2 Material properties

Stiffness of connection: $k_1 = k_2 = k = 1000\ N/m$; $k_3 = 10k = 10000\ N/m$
point mass: $m_2 = m_3 = m = 10\ kg$.

1.3 Boundary conditions and loadings

•boundary conditions

only authorized displacements are the translations according to the axis x .
The points $NO1$ and $NO4$ are clamped: `DX=DY=DZ=DRX=DRY=DRZ=0`.
The other points are free in translation according to the direction x : `DY=DZ=DRX=DRY=DRZ=0`.

•loading

the structure is subjected to a multiple spectral seismic excitation and differential displacements.
The response sprectrums of oscillator in pseudonym acceleration are simplified. Only the values corresponding to the 2 eigenfrequencies of the system are mentioned. They do not depend on damping:

•with the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

$$SRO_{NO1}(f_1) = A_{11} = 7\ m/s^2$$
$$SRO_{NO1}(f_2) = A_{21} = 5\ m/s^2$$
$$DDS_{NO1} = D1 = -0.04\ m$$

•with the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

$$SRO_{NO4}(f_1) = A_{12} = 12\ m/s^2$$
$$SRO_{NO4}(f_2) = A_{22} = 6\ m/s^2$$
$$DDS_{NO4} = D_2 = 0.06\ m$$

1.4 Initial conditions

the system is initially at rest.

2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the spectral response by modal superposition of a system masses spring subjected to two distinct excitations. One determines the displacement of the masses and the reactions of bearing to the nodes $NO1$ and $NO4$ along the axis x.

One calculates analytically:

- eigenfrequencies f_i,
- associated eigenvectors φ_{Ni} standardized compared to the modal mass,
- static modes of bearings ψ_j of the system,
- participation factors modal P_{ij} relating to the bearings,
- Rm_{ij} the maximum of response of each mode starting from the excitation spectrums,
- Re_j the contribution of the motion of training of each bearing starting from differential displacements,
- Rc_j the term of static correction,
- the primary and secondary components of the response according to the adopted rules of office plurality.

2.2 Results of reference

•stiffness matrix K

$$K=\begin{bmatrix} k & -k & 0 & 0 \\ -k & 2\text{k} & -k & 0 \\ 0 & -k & 11\text{k} & -10\text{k} \\ 0 & 0 & -10\text{k} & 10\text{k} \end{bmatrix}$$

$$K^p=\begin{bmatrix} 2\text{k} & -k & -k & 0 \\ -k & 11\text{k} & 0 & -10\text{k} \\ -k & 0 & k & 0 \\ 0 & -10\text{k} & 0 & 10\text{k} \end{bmatrix}$$

stamps partitionnée degrees of freedom of structure 2,3, degrees of freedom of support 1,4

$$K^p=\begin{bmatrix} k_{xx} & k_{xs} \\ k_{sx} & k_{ss} \end{bmatrix} \qquad k_{xx}=\begin{bmatrix} 2\text{k} & -k \\ -k & 11\text{k} \end{bmatrix} \qquad k_{xs}=\begin{bmatrix} -k & 0 \\ 0 & -10\text{k} \end{bmatrix}$$

$$k_{sx}=\begin{bmatrix} -k & 0 \\ 0 & -10\text{k} \end{bmatrix} \qquad k_{ss}=\begin{bmatrix} k & 0 \\ 0 & 10\text{k} \end{bmatrix}$$

•mass matrix M

$$M=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

•**modal computation in clamped base**

$$k_{xx}=\begin{bmatrix} 2k & -k \\ -k & 11k \end{bmatrix}$$

$$\left(k_{xx}-\lambda_i m_{xx}\varphi_i\right)=0 \qquad \lambda_i=\omega_i^2$$

is $\lambda_1=\frac{k}{2m}\left(13-\sqrt{85}\right) \quad \lambda_2=\frac{k}{2m}\left(13+\sqrt{85}\right)$

1) eigenfrequencies:

that is to say $f_1=\frac{\omega_1}{2\pi} \; f_2=\frac{\omega_2}{2\pi}$

1) not normalized eigen modes:

that is to say $\varphi_1=\begin{pmatrix} 0 \\ 1 \\ \left(-9+\sqrt{85}\right)/2 \\ 0 \end{pmatrix} \quad \varphi_2=\begin{pmatrix} 0 \\ -1 \\ \left(9+\sqrt{85}\right)/2 \\ 0 \end{pmatrix}$

•generalized modal masses $\mu_i={}^T\varphi_i M \varphi_i$:

that is to say $\mu_1=\frac{m}{4}\left(170-18\sqrt{85}\right) \qquad \mu_2=\frac{m}{4}\left(170+18\sqrt{85}\right)$

[1] eigen modes normalized with the unit generalized modal mass φ_{Ni} :

that is to say $\varphi_{N1}=\frac{\varphi_1}{\sqrt{\mu_1}} \quad \varphi_{N2}=\frac{\varphi_2}{\sqrt{\mu_2}}$

•modal reactions Fm_i :

$$r_i=k_{sx}\varphi_{Nis} \qquad \varphi_{Ni}^p=\begin{pmatrix} \varphi_{Nix} \\ \varphi_{Nis} \end{pmatrix} \qquad Fm_i^p=\begin{pmatrix} 0 \\ r_i \end{pmatrix}$$

that is to say $Fm_1=\frac{k}{\sqrt{\mu_1}}\begin{pmatrix} -1 \\ 0 \\ 0 \\ 5\left(9-\sqrt{85}\right) \end{pmatrix} \qquad Fm_2=\frac{k}{\sqrt{\mu_2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -5\left(9+\sqrt{85}\right) \end{pmatrix}$

•participation factors modal $P_{ij}={}^T\varphi_i M \psi_j$:

•contribution of the dynamic mode 1 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

$$P_{11}={}^T\varphi_1 M \psi_1=\frac{m}{42\sqrt{\mu_1}}\left(13+\sqrt{85}\right)$$

•contribution of the dynamic mode 1 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

$$P_{12}={}^T\varphi_1 M\,\psi_2=\frac{10\,m}{21\sqrt{\mu_1}}\left(-8+\sqrt{85}\right)$$

•contribution of the dynamic mode 2 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

$$P_{21}={}^T\varphi_2 M\,\psi_1=\frac{m}{42\sqrt{\mu_2}}\left(-13+\sqrt{85}\right)$$

•contribution of the dynamic mode 2 to the motion imposed on the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

$$P_{22}={}^T\varphi_2 M\,\psi_2=\frac{10\,m}{21\sqrt{\mu_2}}\left(8+\sqrt{85}\right)$$

•participation factor of the dynamic mode 1 in the direction X:

$$P_{1X}=P_{11}+P_{12}$$

•participation factor of the dynamic mode 2 in the direction X:

$$P_{2X}=P_{21}+P_{22}$$

•static modes of bearings ψ_j

•static solution to a unit displacement of the node is outside the field of definition with a right profile of the EXCLU type node: $NO1$

displacements: $\psi_1=\frac{1}{21}\begin{pmatrix}21\\11\\1\\0\end{pmatrix}$ nodal reactions: $Fs_1=K\,\psi_1=\frac{10}{21}k\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$

•static solution with a unit displacement of the node is outside the field of definition with a right profile of the EXCLU type node: $NO4$

displacements: $\psi_2=\frac{1}{21}\begin{pmatrix}0\\10\\20\\21\end{pmatrix}$ nodal reactions: $Fs_2=K\,\psi_2=\frac{10}{21}k\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}$

•response of the mode i **with the motion of the bearing** j

$Rm_{ij}=r_i P_{ij}\frac{A_{ij}}{\omega_i^2}$ with $r_i=\varphi_{\mathrm{Ni}}$ or Fm_i

•static correction

•static modes u_j solution of $K\,u_j=M\,\psi_j$:

displacements: $u_1=\frac{m}{441\,k}\begin{pmatrix}0\\122\\13\\0\end{pmatrix}$ nodal reactions: $F\,u_1=\frac{m}{441}\begin{pmatrix}-122\\231\\21\\-130\end{pmatrix}$

displacements: $u_2=\frac{m}{441\,k}\begin{pmatrix}0\\130\\50\\0\end{pmatrix}$ nodal reactions: $F\,u_2=\frac{m}{441}\begin{pmatrix}-130\\210\\420\\-500\end{pmatrix}$

•static correction relating to the motion of the bearing j if mode 2 is not retained:

$$Rc_j=\left(ru_j-\frac{P_{1j}r_1}{\omega_1^2}\right)A_{1j} \text{ with: } ru_j=u_j \text{ ou } Fu_j \text{ and } r_1=\varphi_{NI} \text{ ou } Fm_1$$

•contribution of the bearing j to the motion of training

$$R\,e_j=r_j D_j \text{ with } r_j=\psi_j \text{ ou } Fs_j$$

These analytical computations are described in the file Matlab sdld30a.55.

2.3 Uncertainty on the solution

No (exact analytical solution).

3 Modelization A

3.1 Characteristic of the modelization

the system is modelled by:

- 3 discrete elements `K_T_D_L`,
- 2 discrete elements `M_T_D_N`.

3.2 Characteristics of the mesh

The mesh consists of 3 meshes `SEG2`.

4 Results of the modelization A

4.1 Eigenfrequencies

MODE	Reference
1	2,18815E+00
2	5,30484E+00

4.2 total Response on modal base supplements

modes 1 and 2 are taken into account. The components inertial (primary education) and static (secondary) of the response are directly cumulated on the level of the bearings.

- **computation n°1**

  `COMB_MODE=' SRSS'`

  - response of the bearing $j=1$ (node $NO1$): $R_1=\sqrt{Rm_1^2+Re_1^2}$ with $Rm_1=\sqrt{Rm_{11}^2+Rm_{21}^2}$ (office plurality on the modes)
  - response of the bearing $j=2$ (node $NO4$): $R_2=\sqrt{Rm_2^2+Re_2^2}$ with $Rm_2=\sqrt{Rm_{12}^2+Rm_{22}^2}$
  - total response: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the bearings)

absolute displacements: `DEPL`

NOEUD	Reference
$NO1$	4,00000E-02
$NO2$	5,43820E-02
$NO3$	5,75544E-02
$NO4$	6,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
$NO1$	5,36769E+01
$NO4$	7,44120E+01

4.3 total Response on incomplete modal base without static correction

Only mode 1 is taken into account. The components inertial (primary education) and static (secondary) of the response are directly cumulated on the level of the bearings.

- **computation n°1**

```
COMB_MODE=' SRSS'
```

•response of the bearing $j=1$ (node $NO1$): $R_1=\sqrt{Rm_1^2+Re_1^2}$ with $Rm_1=Rm_{11}$
•response of the bearing $j=2$ (node $NO4$): $R_2=\sqrt{Rm_2^2+Re_2^2}$ with $Rm_2=Rm_{12}$
•total response: $R=\sqrt{R_1^2+R_2^2}$

absolute displacements: `DEPL`

NOEUD	Reference
$NO1$	4,00000E-02
$NO2$	5,43794E-02
$NO3$	5,73536E-02
$NO4$	6,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
$NO1$	5,36743E+01
$NO4$	5,68312E+01

4.4 total Response on incomplete modal base with static correction

Only mode 1 intervenes in the computation of the response. The static contribution of neglected mode 2 is taken into account.

- **computation n°1**

```
COMB_MODE=' SRSS'
```

•response of the bearing $j=1$ (node $NO1$): $R_1=\sqrt{Rm_1^2+Rc_1^2+Re_1^2}$ with $Rm_1=Rm_{11}$
•response of the bearing $j=2$ (node $NO4$): $R_2=\sqrt{Rm_2^2+Rc_2^2+Re_2^2}$ with $Rm_2=Rm_{12}$
•total response: $R=\sqrt{R_1^2+R_2^2}$

absolute displacements: `DEPL`

NOEUD	Reference	Tolerance
$NO1$	4,00000E-02	0.001
$NO2$	0.054389658	0.001
$NO3$	0.058152653	0.001
$NO4$	6,00000E-02	0.001

nodal reactions: `REAC_NODA`

NOEUD	Reference	Tolerance
NO1	53.6846755	0.001
NO4	111.6190600	0.001

4.5 Partition of the components primary and secondary of the response

the components inertial (primary) and static (secondary) are treated separately.

•**primary computation**

•n°1 response on complete modal base (modes 1 and 2)
`COMB_MODE=' SRSS'`

1) response of the bearing $j=1$ (node *NO1*): $RI_1=\sqrt{Rm_{11}^2+Rm_{21}^2}$ (office plurality on modes)

•response of the bearing $j=2$ (node *NO4*): $RI_2=\sqrt{Rm_{12}^2+Rm_{22}^2}$

•primary response: $RI=\sqrt{RI_1^2+RI_2^2}$

relative displacements: `DEPL`

NOEUD	Reference
NO1	0,00000E+00
NO2	4,12562E-02
NO3	6,60152E-03
NO4	0,00000E+00

nodal reactions: `REAC_NODA`

NOEUD	Reference
NO1	4,12562E+01
NO4	6,60152E+01

•secondary response
`COMB_DEPL_APPUI=' QUAD'`

1) response secondary: $RII=\sqrt{Re_1^2+Re_2^2}$

displacements of training: `DEPL`

NOEUD	Reference
NO1	4,00000E-02
NO2	3,54306E-02
NO3	5,71746E-02
NO4	6,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
NO1	3,43386E+01
NO4	3,43386E+01

- **primary computation**

•n°2 response on incomplete modal base without static correction
Only mode 1 intervenes in the computation of response
`COMB_MODE=' SRSS'`

•response of the bearing $j=1$ (node $NO1$): $RI_1=Rm_{11}$
•response of the bearing $j=2$ (node $NO4$): $RI_2=Rm_{12}$
•primary response: $RI=\sqrt{RI_1^2+RI_2^2}$

relative displacements: `DEPL`

NOEUD	Reference
NO1	0,00000E+00
NO2	4,12528E-02
NO3	4,52841E-03
NO4	0,00000E+00

nodal reactions: `REAC_NODA`

NOEUD	Reference
NO1	4,12528E+01
NO4	4,52841E+01

•secondary response
`COMB_DEPL_APPUI=' LINE'`

•response secondary: $RII=Re_1+Re_2$

•displacements of training: `DEPL`

NOEUD	Reference
$NO1$	-4,00000E-02
$NO2$	7,61905E-03
$NO3$	5,52381E-02
$NO4$	6,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
$NO1$	-4,76190E+01
$NO4$	4,76190E+01

- **primary computation**

•n°3 response on incomplete modal base with static correction
Only mode 1 intervenes in the computation of response
`COMB_MODE=' SRSS'`

•response of the bearing $j=1$ (node $NO1$): $RI_1=\sqrt{Rm_{11}^2+Rc_1^2}$
•response of the bearing $j=2$ (node $NO4$): $RI_2=\sqrt{Rm_{12}^2+Rc_2^2}$
•primary response: $RI=\sqrt{RI_1^2+RI_2^2}$

relative displacements: `DEPL`

NOEUD	Reference	Tolerance
$NO1$	0,00000E+00	-
$NO2$	4,1266282E-02	0.001
$NO3$	1.0620582E-02	0.001
$NO4$	0,00000E+00	-

nodal reactions: `REAC_NODA`

NOEUD	Reference	Tolerance
$NO1$	4,12662823E+001	0.001
$NO4$	1.0620581996E+02	0.001

•secondary response
`COMB_DEPL_APPUI=' ABS'`

•response secondary: $RII=|Re_1|+|Re_2|$

displacements of training: `DEPL`

NOEUD	Reference
$NO1$	4,00000E-02
$NO2$	4,95238E-02
$NO3$	5,90476E-02
$NO4$	6,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
NO1	4,76190E+01
NO4	4,76190E+01

- **primary computation**

•n°4 response on incomplete modal base with static correction

Only mode 1 intervenes in the computation of the response.
`COMB_MODE=' SRSS'`

•response of the bearing $j=1$ (node *NO1*): $RI_1=\sqrt{Rm_{11}^2+Rc_1^2}$
•response of the bearing $j=2$ (node *NO4*): $RI_2=\sqrt{Rm_{12}^2+Rc_2^2}$
•primary response: $RI=\sqrt{RI_1^2+RI_2^2}$

•secondary response: test office plurality of $DDSs$

5 loading cases are defined. The 5 associated elementary static responses are:
•case a: $DDSa_{NO1}=-0.04$ either $R_a=r_1\times DDSa_{NO1}$
•case b: $DDSb_{NO4}=0.06$ or $R_b=r_2\times DDSb_{NO4}$
•case C: $DDSc_{NO4}=0.03$ that is to say $R_c=r_2\times DDSc_{NO4}$
•case D: $DDSd_{NO1}=-0.07$ that is to say $R_d=r_1\times DDSd_{NO1}$
•case E: $DDSe_{NO4}=0.05$ that is to say $R_e=r_2\times DDSe_{NO4}$

4 combinations are calculated:

- **combination n°1**

linear office plurality of the cases has and secondary `b: TYPE_COMBI=' LINE'`

NUME_ORDRE=200 response: $RII_1=Ra+Rb$

absolute displacements: `DEPL`

NOEUD	Reference
NO1	-4,00000E-02
NO2	7,61905E-03
NO3	5,52381E-02
NO4	6,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
NO1	-4,76190E+01
NO4	4,76190E+01

- **combination n°2**

absolute office plurality of the cases has and C: `Secondary TYPE_COMBI=' ABS'`

NUME_ORDRE=201 response: $RII_2 = |Ra| + |Rc|$

absolute displacements: `DEPL`

NOEUD	Reference
$NO1$	4,00000E-02
$NO2$	3,52381E-02
$NO3$	3,04762E-02
$NO4$	3,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
$NO1$	3,33333E+01
$NO4$	3,33333E+01

- **combination n°3**

quadratic office plurality of the cases D and E: `Secondary TYPE_COMBI=' QUAD'`

NUME_ORDRE=202 response: $RII_3 = \sqrt{Rd^2 + Re^2}$

absolute displacements: `DEPL`

NOEUD	Reference
$NO1$	7,00000E-02
$NO2$	4,37189E-02
$NO3$	4,77356E-02
$NO4$	5,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
$NO1$	4,09635E+01
$NO4$	4,09635E+01

- **combination n°4**

linear office plurality of the cases has and E: `Secondary TYPE_COMBI=' LINE'`

NUME_ORDRE=203 response: $RII_4 = Ra + Re$

absolute displacements: `DEPL`

NOEUD	Reference
$NO1$	-4,00000E-02
$NO2$	2,85714E-03

NO3	4,57143E-02
NO4	5,00000E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
NO1	-4,28571E+01
NO4	4,28571E+01

the total secondary response is established by the quadratic office plurality of the 4 preceding combinations:

$$RII=\sqrt{RII_1^2+RII_2^2+RII_3^2+RII_4^2}$$ `NUME_ORDRE=204`

absolute displacements: `DEPL`

NOEUD	Reference
NO1	9,84886E-02
NO2	5,67386E-02
NO3	9,13703E-02
NO4	9,74679E-02

nodal reactions: `REAC_NODA`

NOEUD	Reference
NO1	8,30266E+01
NO4	8,30266E+01

5 Summary of the results

the results got with *Code_Aster* are in conformity with the analytical results of reference.